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Research School for Operations Management and Logistics

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Beta Working Paper series 356

BETA publicatie	WP 356 (working paper)
ISBN	
ISSN	
NUR	804
Eindhoven	September 2011

Last time buy decisions for products sold under warranty

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Abstract

Manufacturers supplying products under warranty need a strategy to deal with failures during the warranty period: repair the product or replace it by a new one, depending on e.g. age and/or usage of the failed product. An (implicit) assumption in virtually all models is that new products to replace the failed ones are immediately available at given replacement costs. Because of the short life cycles of many products, manufacturing may be discontinued before the end of the warranty period. At that point in time, the supplier has to decide how many products to put on the shelf to replace failed products under warranty that will be returned from the field (the *last time buy* decision). This is a trade-off between product availability for replacement and costs of product obsolescence. In this paper, we consider the joint optimization of repair-replacement decisions and the last time buy quantity for products sold under warranty. We develop approximations to estimate the total relevant costs and service levels for this problem, and show that we can easily find near-optimal last time buy quantities using a numerical search. Comparison to discrete event simulation results shows an excellent performance of our methods.

Key words: warranty, repair, spare parts, inventory, last time buy

1. Introduction

Many products in the consumer market are sold with a guarantee that the product will be operational during a certain warranty period (or an *extended* warranty period if the consumer pays for such an extension). A servicing strategy for these products usually consists of some decision rule when to replace and when to repair a failed product based on criteria like the age and usage of the product.

Many models focus on the analysis of a single product and assume that a new product to replace a failed one is always available if needed and that replacement occurs in negligible time. However, new product versions appear quickly on the market, and therefore manufacturing of the previous version may be discontinued far before the end of the warranty period of the products in the market. To facilitate product replacement during the remaining warranty period, the supplier needs to procure a certain amount of spare products (or spare parts) at once to cover the demand during the remaining warranty period[3]. This is called the *last time buy* decision.

The basic trade-off for the last time buy decision is between the costs of spare part unavailability versus the costs of obsolescence of spare parts that have not been used at the end of the warranty period. If the warranty strategy prescribes that a failed product should be replaced whereas no spare is available, the supplier is forced to repair the product at high costs or to replace the product by a different (usually more expensive) version. The last time but decision interacts with the warranty strategy: The decision rule when to repair or replace a product determines the demand for spare parts, whereas the availability of spare parts determines the possibility to replace a product.

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[3] We will use the phrase "spare parts" throughout this paper, noting that this may be a complete product.

A complication is that spare parts are kept on stock for the complete set of products in the field, thereby taking advantage of the risk pooling effect. As a consequence, the problem is intrinsically a *multi-product* problem, whereas the analysis of a single product is typically sufficient to optimize the warranty strategy given unlimited supply of replacement parts. This complicates the analysis of the problem. As we will see in the next section, the joint problem of choosing the last time buy quantity and the repair/replacement decision of products sold under warranty has only been studied in the literature before as a deterministic optimization problem (Sayouni et al. [2010]).

In this paper, we study stochastic models for the joint problem of finding the last time buy quantity and the warranty strategy. We develop approximations and optimization heuristics to solve this problem. We perform numerical experiments to get insight in the type of policies. Also, we check the accuracy of our approximations by comparison to results from discrete event simulation.

In the next section, we discuss the related literature and define our contribution more detail. In Section 3, we give an overview of model assumptions and basic notation to be used throughout our paper. Next, we formulate a first simple model for a single product with dedicated spares and show that we can solve this using stochastic dynamic programming (Section 4). In Section 5, we develop a basic model for multiple products sharing a common pool of spare parts. Because of the dimension of the state space, we cannot solve this problem easily anymore using stochastic dynamic programming. Therefore, we use a decomposition in single-product models. Next, we derive demand characteristics to estimate the total relevant costs and service levels for a multi-product model under various stock levels. Also, we show the impact of an iterative heuristic in which we alternately optimize the repair / replacement decision and the last time buy quantity of spare parts. In Section 6, we extend our model to the situation where each product has a certain age and only a part of the initial warranty period is remaining when the last time buy decision has to be taken. In Section 7, we conduct a numerical experiment to study the interaction between the last time buy decision and the repair / replacement strategy. Finally, we give our conclusions and directions for further research in Section 8.

2. Literature

Related literature can be found in warranty strategies (repair/replacement decisions) and last time buy decisions for spare parts. We will discuss these two areas below. Next, we discuss the only paper that we found on the integration of last time buy and warranty strategy. Finally, we state our contribution to the literature.

2.1. Repair / replacement decisions under warranty

The literature on warranty strategies in general is rich, see e.g. Murhy et al. [2004], Murthy and Blischke [2005] and the references therein. Below, we give a sample of relevant papers to position our research.

A part of the warranty literature deals with repair / replacement decisions: When should we repair a product upon failure within the warranty period and when should we replace the product by a new one? Such a decision depends on many factors, factors like the costs of repair and replacement, the product failure rate function, and the impact of repair. Regarding the latter, we will typically have imperfect repair that improves the failure rate to a state worse than as-good-as-new (Rao [2011]) with as special cases: (1) perfect repair that restores a product to the as-good-as-new state (ii) minimal repair that restores the product to the same state as just before the failure (Jack and Van der Duyn Schouten [2000]). The level of repair may also be part of the warranty strategy, so that the level of improvement of the failure rate depends on the age of the product (Yun et al. [2008]). Another model assumes that the product's failure rate that only depends on the *number* of repairs thus far and not on the age (Lugtigheid et al. [2008]).

Warranties can be either *one-dimensional* or *multi-dimensional* (usually two-dimensional). Under a one-dimensional warranty, the warranty ends when a threshold of a single variable (age or usage) has been passed (see e.g. Jack and Van der Duyn Schouten [2000] and Iskandar et al [2011]). Under a two-dimensional warranty, the warranty depends on two variables (e.g. both usage and age), and the warranty expires if the first threshold has been passed (see e.g. Iskandar et al. [2005] and Jack et al. [2009]). For example, the warranty of a car may expire after 50.000 km. or 5 years, whatever comes first. Rules for repair/replacement decisions depend on the type of warranty: Under a two-dimensional warranty, the decision will generally also depend on both variables (e.g. age and usage).

The decision rules for repair/replacement can be classified as *dynamic* and *static* rules. Dynamic rules depend on the remaining warranty period, whereas static rules do not. For example, consider a product with one dimensional warranty based on age and minimal repair. Jack and Van der Duyn Schouten [2000] developed a dynamic decision rule for this model in which a minimal repair is performed if the remaining warranty period is t and the product age is less than or equal to a control limit $L(t)$. After discretization of time, they find the optimal control limits using stochastic dynamic programming on a two-dimensional state space. They conjecture that their rule is optimal, which has been proven later on by Jiang et al. [2006]. A much simpler static rule may prescribe minimal repair if the product age is less than some constant threshold value, or if the number of minimal repairs performed is less than some threshold value, see e.g. Iskandar et al. [2011]. These threshold values do not depend on the remaining warranty period. Samatlı-Paç and Taner [2009] show that dynamic policies are particularly better for highly reliable products.

To the best of our knowledge, the basic assumption in all papers is that a replacement is always possible and not restricted by a limited inventory of spare parts.

2.2. Last time buy decisions for spare parts

A last time buy decision has to be taken when manufacturing of products or components needed to cover a warranty or service period is discontinued before the end of the period. Most literature on last time buy decisions focuses on a business-to-business environment, particularly components for (advanced) capital goods like computer systems, medical systems or manufacturing equipment. Therefore, the service period tends to be longer than most warranty periods, say 3-10 years. Note that the replacement level is usually a component within a product and not the product itself, as is the case in the warranty models as described above.

The basis paper in this line of research is by Moore [1971] who uses time series analysis and curve fitting methods to estimate the so-called all-time requirement for spare parts in the car industry, as basis for the last time buy decision. Fortuin [1980] developed a model for the last time buy quantity of non-repairable parts based for a case study at Philips Electronics. Later refinements have a .o. been described in Teunter and Klein Haneveld [1998]. Teunter and Klein Haneveld [2002] allow for reordering of parts later in the service period, but at a higher price.

Initially, most papers considered the last time buy as the only source of spare parts. More recent papers include the repair option as an additional source, as well as asset recovery (complete systems that are returned from the field and that are stripped for reusable spare parts). Asset recovery as source of spare parts has recently been addressed by Inderfurth and Mukherjee [2008], Kleber and Inderfurth [2009], Krikke and Van der Laan [2011], and Pourakbar et al. [2011]. Repair of failed parts that are returned from the field is considered as an alternative source of supply in Van Kooten and Tan [2009]. Using such alternative supply options, the last time buy quantity can be reduced and obsolescence risk at the end of the service period can be reduced.

More closely related to our research, Pourakbar et al. [2009] focus on last time buy decisions for spare parts of consumer goods rather than spare parts of expensive capital goods. The last time buy decision refers to the spare parts. An alternative for repairing failed products is replacing the entire product by a new one, possible a new (improved) product version. Because of significant price erosion in the market for consumer electronics, this may be a more profitable option from some point in time to the end of the service life cycle. They consider the last time buy quantity and the switching time from repair to replacement as key decision variables.

A common assumption in the models for last time buy decisions is that demand for replacement parts arises from a (nonstationary) Poisson process. An explicit failure rate model is usually not taken into account, and therefore there is also no decision related to repair or replacement of parts depending on, for example, age of the product.

2.3. Last time buy decisions for products sold under warranty

To the best of our knowledge, Sahyouni et al. [2010] is the only paper that integrates last time buy and warranty repair operations. The authors develop a deterministic optimization model with as decision variables (i) the last time buy quantity, and (ii) the point in time after which failed products are not repaired anymore but replaced by new ones. Although the failure process in stochastic by nature, such a deterministic model may be suitable in very large scale operations where the risk pooling effect justifies an approximation by a deterministic model. The authors suggest as further research to develop a stochastic version of their basic model.

2.4. Our contribution

In this paper, we consider the interaction of the last time buy decision for products or parts replacing failed ones in relation to the repair/replacement policy. To the best of our knowledge, this problem has not been studied as a stochastic model in the literature before. The repair/replace policy determines the demand process for new products replacing the failed ones, which is input for the last time buy decision when manufacturing of products is discontinued. Reversely, the last time buy decision determines the availability of products to replace the failed ones, and so it influences the repair/replacement policy: The decision rule may tell that a failed product should be replaced, but an alternative solution (in our model repair) is still needed if a new product I not available anymore.

We focus on one-dimensional polices based on the product age upon failure. We assume minimal repair: The failure rate just after repair is equal to the failure rate just before failure, so the effective product age remains the same. We first develop a model for a single product with dedicated spare parts and develop a simple dynamic heuristic rule to solve the problem. Because the standard repair/replacement decisions rule for a one-dimensional policy is a special case of our model with limited supply, we can analyze the quality of our heuristics by comparison to the optimal solution for problem instances for this special case. Next, consider a multi-product problem with a shared inventory of spares. The analysis of this model is considerably more complicated, because it is hard to identify clear and simple renewal point in the stochastic process. Therefore, we propose a simple heuristic based on decomposition by product to solve the problem. We test our heuristic in a numerical experiment and provide insights in the impact of the key parameters.

3. General model outline and notation

We consider a repairable product that is sold with a fixed warranty period W. Upon failure, the product can either be repaired or be replaced by a new one, provided that a new product is available. Any repair is a minimal repair, so the failure rate remains unchanged (or: the effective product age is unaltered). We assume that the product has in increasing failure rate function to avoid uninteresting problem instances: In general, replacement will be more expensive than repair, so the product will always be repaired if the failure rate function is non-increasing.

For the repair / replacement decision, we use a simple time-based rule that is known to be efficient: We decide to perform a minimal repair if the product age upon failure is at most equal to some critical age τ, otherwise we replace the product by a new one. We use a dynamic rule, where the value of τ depends on the remaining warranty period. We do this for two reasons: (i) dynamic rules tend to be better than static rules, see Samatlı-Paç and Taner [2009], and (ii) it is computationally more efficient, since we can apply stochastic dynamic programming after discretization of time, whereas we repeatedly need to solve a renewal equation numerically, as we have to perform a numerical search over τ, see Iskandar et al. [2011].

We assume that the times for repair and replacement are negligible. At a certain point in time, manufacturing of products is discontinued, and the supplier can order a last time buy quantity to replace failed products during the remaining warranty period. Buying too many spare products leads to obsolescence costs if the products have not been used during the remaining warranty period. Buying insufficient spares means that replacement is not possible if it would have been economically better than repair. In the latter case, we assume that we perform a minimal repair as the only option left. This may be costly, particularly if we run out of stock a significant time before the end of the warranty period and if the failure rate is strongly increasing.

In the next sections, we describe a sequence of models to deal with the joint problem of finding the last time buy quantity and the repair/replace policy during the warranty period:

1. A single product with a dedicated stock of spares; the last time but decision has to be taken when the product is as good as new (Section 4)
2. A set of multiple identical products with a shared stock of spares; the last time but decision has to be taken when all products are as good as new and so the entire warranty period W has to be covered for all products (Section 5).
3. A set of multiple identical products with a shared stock of spares; at the time of the last time buy decision, the products have a certain age and only a part of the warranty period W is remaining; both age and remaining warranty period may vary over products (Section 6).

For these models, we will use the following general notation:

Input data:

W = length of the warranty period

$f(t)$ = Probability density function of the time to failure of the product that is as good as new; we denote the corresponding distribution function by $F(t)$, the survival function by $\bar{F}(t)$, the hazard function by $h(t) = f(t)/\bar{F}(t)$, and the cumulative hazard function by $H(t)$.

C_m = costs of a single minimal repair

C_p = procurement costs of a spare product at time 0

C_s = scrap costs of an operational spare product remaining at the end of the warranty period; in case $C_s<0$, we have a salvage value

C_d = cost of replacement of a failed product by a new one (replacement costs, disposal costs of the failed part that is not repaired)

Decision variables:

τ = critical age limit for minimal repair (which may depend on the remaining warranty period and the number of spare products available)

s = last buy quantity, i.e., the total number of spare products procured at t=0 to replace failed products throughout their remaining warranty period

Auxiliary notation:

We denote the hazard function of the time to failure by $h(t) = f(t)/\bar{F}(t)$, and the cumulative hazard function by $H(t)$, the primitive of $h(t)$. Further, we need the probability distribution function of the time until the first need of a spare product, defined as $Z=\tau+Y$, where Y is the time to failure of a product with age τ. The density function $g(t;\ \tau)$ of Z is given by $g(t;\tau)=\frac{f(t)}{\bar{F}(\tau)}, t \geq \tau$.

4. Single product model

First, we consider a simple model for a single product subject to failure supported by s spare products procured at t=0 to support product replacements throughout warranty period W. At t=0, the product is as good as new. To facilitate the application of stochastic dynamic programming, we split the time in discrete intervals. Without loss of generality, we assume that each time interval has length 1 (normalization of time). We define the discrete form of the density function, distribution function and survival function of the time to first failure as:

f_t = probability that a failure occurs in time interval t.
F_t = probability that a failure occurs in time interval t or earlier
$\overline{F}_t$= probability that a failure occurs after time interval t (so in interval t+1 or later)
We analogously define the discretized characteristics of the conditional time to failure of a product with age τ by density $g_{t,\tau}$, distribution $G_{t,\tau}$ and survival function $\overline{G}_{t,\tau}$, $t \geq \tau$.

4.1. Basic repair/replacement decisions

A dynamic repair/replacement policy implies that for each remaining time horizon w we decide to replace the product upon the next failure after time $\tau(w;s)$ if we still have s products on stock. Actually, this is a heuristic, because an optimal dynamic decision rule consists of a control limit $L(t)$, indicating that minimal repair is performed if the remaining warranty period is t and the product age is less than or equal to $L(t)$, see Jack and Van der Duyn Schouten [2000]. This yields a two-dimensional value function in both the age of the failing component and the remaining time horizon. Hence, it is rather complex and time consuming to find an optimal solution. We choose a simpler heuristic, because (i) we aim to further extend our model further to the multi-product case, and (ii) a static variant of this rule has shown to be effective in Iskandar et al. [2011].

Now we define the value function $V(w;s)$ as the minimum expected costs if the product has just been replaced, there are w periods to go until the end of the warranty period, and we have still s spare products on stock. We can state a recursive equation by minimizing the value function $V(w;s)$ over the critical age $\tau(w;s)$. For a given time horizon w, stock level $s \geq 1$, and critical age $\tau(w;s)$, we face the following costs:

- The costs of minimal repair during $\tau(w;s)$, at costs C_m per event, where the number of minimal repairs in t periods is given by the cumulative hazard function $H(t)$
- The costs of replacing a failed product by a spare one upon first failure after time $\tau(w;s)$), if it occurs before the end of the planning horizon w. This happens with probability $G_{w,\tau(w;s)}$ and yields costs of initial procurement of the replacement product and the replacement costs itself, $C_p + C_d$.
- If the failure occurs at some time t ($\tau<t<w$), for which the density $g_{t,\tau(w;s)}$ applies, we face costs $V(w-t;s-1)$ over the remaining time horizon w-t.
- If no failure occur in the remaining planning horizon, we face the costs of initial procurement and scrapping the s products, $C_p + C_s$.

For sake of simplicity, we assume that minimal repair or replacement occurs at the end of the time interval. As a consequence, discretization leads to a slight underestimation of the total costs that decreases with the number of time intervals in the planning horizon W. Now we find the following recursive expression for the value functions:

$$V(w;s) = \min_{0\le\tau(w;s)\le w}\left\{C_m H\big(\tau(w;s)\big) + \big(C_d + C_p\big)G_{w,\tau(w;s)} + \sum_{t=\tau(w;s)+1}^{w} g_{t,\tau(w;s)} V(w-t;s-1) + \overline{G}_{w,\tau(w;s)}\big(C_s + C_p\big)s\right\}, s \ge 1$$

(1)

with the initial conditions:

- $V(w;0) = C_m H(w)$ for all w: we only perform minimal repair if no spare products are available for replacement
- $V(0;s) = (C_s + C_p)s$ for all s: If we have s products left at the end of the life cycle, we face the costs of initial procurement and scrapping these products.

This is easy to solve, since the number of spare parts will generally be small. We have all value functions for i=0 spares. If we have all value functions $V(w;s)$ for all time intervals $0\le w\le W$, then we can derive all value functions for $V(w;s+1)$ for all time intervals $0\le w\le W$. Although we have not formally proven that $V(W;s)$ is convex in s for a fixed length of the warranty period W, this seems to be logical and also appeared to true in all our numerical experiments. This means that we can recursively calculate all value functions for s=1, 2, … until the value function $V(W;s)$ does not decrease anymore.

We can gain efficiency because for each combination (w, s) the value function to be minimized is convex in $\tau(w;s)$. So, we can stop evaluating the cost function when the total expected costs as function of $\tau(w;s)$ starts to increase.

Remark:
We obtain a lower bound for the costs, since we assume that minimal repair or replacement occurs at the end of the time interval. If we move the costs to the start of the interval, we find an upper bound for the costs using the recursion. Then we replace $V(w-t;s-1)$ in (1) by $V(w-t+1;s-1)$. However, the value function converges fastest if we take the average of the two value functions. That is, we replace $V(w-t;s-1)$ in (1) by $\big(V(w-t;s-1)+V(w-t+1;s-1)\big)/2$. In our numerical experiments, this appeared to be far the most efficient procedure, since we do not need many time intervals to get a very accurate cost estimate. As an example, we got accurate results (<0.01% cost deviation) with discretization of time in about 25 intervals, whereas we needed more than 1000 intervals to get a similar accuracy with recursion (1).

4.2. Improvement of repair/replacement decisions

Iskandar et al. [2011] show that a static version of the critical age rule has one drawback: It may prescribe an expensive replacement just before the end of the warranty period, whereas a cheaper repair is better given the short time remaining. They improve their static rule by adding a control variable a, indicating that a failed product is not replaced anymore if the remaining planning horizon upon failure is at most equal to a.

In our dynamic rule with limited stock level s, we can apply the same logic. In addition to the critical age $\tau(w;s)$. When the remaining time horizon equals w and s products are still on stock for replacement, we decide to perform the next replacement if the failure occurs at an age exceeding $\tau(w;s)$ and the time until the end of the service period is more than $a(w;s)$. Similar to line of reasoning used to find equation (1), we find the following stochastic dynamic programming recursion ($s\ge 1$):

$$V(w;s) = \min_{\substack{0\le \tau(w;s)\le w \\ 0\le a(w;s)\le w-\tau(w;s)}} \begin{cases} C_m\{H(\tau(w;s)) + [H(w) - H(w-a(w;s))] * \overline{G}_{w-a\tau(w;s),\tau(w;s)}\} \\ + \overline{G}_{w-a(w;s),\tau(w;s)}(C_s + C_p)s + (C_d + C_p)G_{w-a(w;s),\tau(w;s)} + \sum_{t=\tau(w;s)+1}^{w-a(w;s)} g_{t,\tau(w;s)}V(w-t;s-1) \end{cases} \quad (2)$$

The initial conditions remain the same as shown under equation (1).

4.3. Numerical results

4.3.1. Special case: no obsolescence costs

In fact, the well-known problem of finding an optimal repair/replacement decision under one-dimensional warranty and minimal repair (Jack and Van der Duyn Schouten [2000]) is a special case of our model, namely if C_s = -C_p (we can sell the spare parts left bovver at the end of the warranty period back for the original price). Then, we can compare our method to known results in the literature. First, we compare our rule to the optimal dynamic rule for the cases presented in Jack and Van der Duyn Schouten [2000]. We consider the following problem instance: α = 1 (year), β = 2, W = 2 (years), C_m=1, C_d=0, and C_p is varied as C_p=1.01, 1.5, 2.0, 2.5 and 5.0. We discretize the warranty period in 100 time intervals. Table 1 compares the optimal total relevant costs found under the optimal dynamic policy, TR*C(opt)*, to the minimum costs found under heuristic $\tau_s(w)$, denoted by *TRC(H₁)*, and under heuristic $\tau(w;s)$, $a(w;s)$, denoted by *TRC(H₂)*.

	C_p=1.01	C_p=1.5	C_p=2.0	C_p=2.5	C_p=5.0
TRC(opt)	1.913	2.694	3.228	3.668	4.000
TRC(H_1)	1.913	2.714	3.250	3.736	4.000
TRC(H_2)	1.913	2.695	3.228	3.668	4.000

Table 1. Value function comparison of our heuristics to the optimal rules for the special case C_s = -C_p

We see that the expected costs of heuristic 1 are only slightly higher than the optimal costs. The costs of heuristic 2 are almost the same as for the optimal solution.

Because the number of experiments is small, we also compare the performance of our dynamic rule to the best static heuristic for each problem instance as presented in Iskandar et al. [2011]. We denote these minimum costs by *TRC(Isk)*, see Table 2. We consider the following problem instance: α = 1 (year), β = 2, W = 3 (years), C_m=1, C_d=0, and C_p is varied between 1.2 and 10.

	C_p=1.2	C_p=1.4	C_p=1.6	C_p=1.8	C_p=2.0	C_p=3.0	C_p=4.0	C_p=5.0	C_p=6.0
TRC(Isk)	3.6120	4.1759	4.6885	5.1522	5.5653	6.7713	7.7609	8.7204	9.0000
TRC(H_1)	3.6088	4.1405	4.6113	5.0267	5.3947	6.6654	7.6648	8.6643	9.0000
TRC(H_2)	3.6017	4.1241	4.5885	5.0010	5.3683	6.6628	7.6536	8.6129	9.0000

Table 2. Value function comparison of our heuristics to Iskandar et al. [2011] for the special case C_s = -C_p

In Table 3, we make a similar comparison where we vary the length of the warranty period W between 0.5 and 7.0. We observe that the dynamic rules are better than the best static rule for most problem instances, which corresponds to the findings of Samatlı-Paç and Taner [2009]. This only does not hold if the warranty period is short (W=1 year) and the cost ratio of product replacement to minimal repair is small. The cost difference between the two dynamic heuristics is always (very) small.

W		C_p=1.2	C_p=1.4	C_p=1.6	C_p=1.8	C_p=2.0
1.0	*TRC(Isk)*	0.8790	0.9714	1.00	1.00	1.00
	$TRC(H_1)$	0.8936	1.0000	1.00	1.00	1.00
	$TRC(H_2)$	0.8843	0.9754	1.00	1.00	1.00
2.0	*TRC(Isk)*	2.2562	2.5986	2.9005	3.1321	3.3152
	$TRC(H_1)$	2.2574	2.5740	2.8420	3.0588	3.2503
	$TRC(H_2)$	2.2503	2.5581	2.8204	3.0409	3.2285
3.0	*TRC(Isk)*	3.6120	4.1759	4.6885	5.1522	5.5653
	$TRC(H_1)$	3.6088	4.1405	4.6113	5.0267	5.3947
	$TRC(H_2)$	3.6017	4.1241	4.5885	5.0010	5.3683
5.0	*TRC(Isk)*	6.3207	7.3369	8.2678	9.1225	9.9089
	$TRC(H_1)$	6.3130	7.2737	8.1483	8.9424	9.6657
	$TRC(H_2)$	6.3059	7.2572	8.1254	8.9163	9.6392
7.0	*TRC(Isk)*	9.0288	10.5092	11.8662	13.1187	14.2775
	$TRC(H_1)$	9.0171	10.4068	11.6854	12.8579	13.9363
	$TRC(H_2)$	9.0099	10.3906	11.6624	12.8315	13.9093

Table 3. Value function comparison of our heuristics to Iskandar et al. [2011] for the special case $C_s = -C_p$

Because of its simplicity and good performance, we select Heuristic 1 for further model development in Section 5 and 6. In principle, the same line of reasoning can be followed and similar expressions can be derived for Heuristic 2.

4.3.2. Positive obsolescence costs

As an illustration of the impact of positive obsolescence costs, we consider a similar example as in the previous subsection: $\alpha = 1$ (year), $\beta = 2$, $W = 3$ (years), C_m=1, C_d=0, C_s=0, and C_p=2. We find for the value functions $V(3;0) = 9.000$, $V(3;1) = 5.667$, $V(3;2) = 5.569$, $V(3;3) = 6.5210$ using heuristic 1 (heuristic 2 yields the same values for this instance). We find the minimum expected costs for *V(3;2)*, so it is optimal to buy 2 spares. Figure 1 displays both the value functions *V(W;s)* and the values of the critical age $\tau(W;s)$ as function of the number of spares *s* and the warranty period *W*.

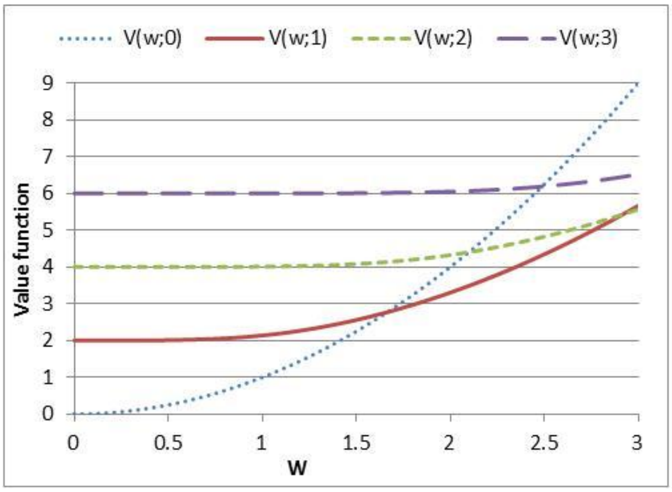

Figure 1a. Value functions *V(W;s)* as function of planning horizon *W* and stock level *s* for the example.

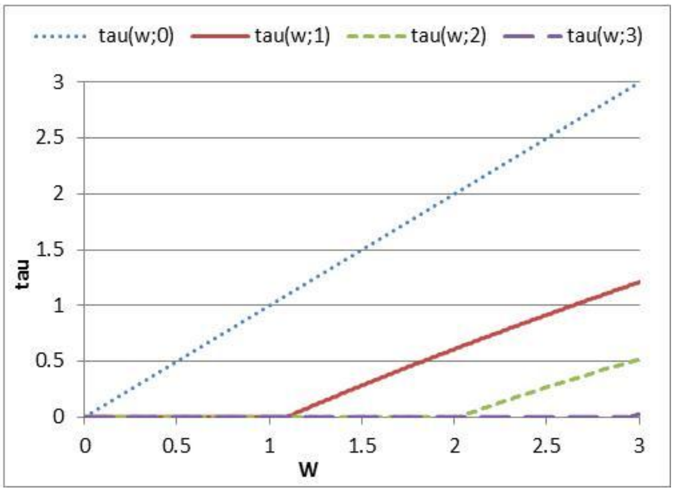

Figure 2b. Critical age limits $\tau(W;s)$ as function of planning horizon *W* and stock level *s* for the example.

Regarding the replacement policy, we observe that the critical age decreases with the time horizon and the stock level. If we have plenty of spares (s=3), we do not perform any minimal repair anymore. However, this is not the minimum cost solution that prescribes some minimal repairs early in the warranty period, namely during the first half year: $\tau(3;2)=0.516$. Our heuristic also yields the minimum cost solutions for all smaller warranty periods. We see from the Figure 1a that the optimal number of spares reduces from s=2 to s=1 spare if $W\leq 2.92$. For $W\leq 1.66$, we should not procure any spares and perform minimal repairs only.

5. Basic multi-product model

In practice, we will use a single pool of spare products to replace failed ones to take advantage of the risk pooling effect. A straightforward extension of our stochastic dynamic approach to a multi-product model is not possible unfortunately: When one product is replaced, we do not have a renewal point since the other products have some (unknown) age. Inclusion of the age of all products in the system state is far from practical since the state space explodes. Therefore, we have to rely upon heuristics.

We propose a decomposition approach over products in which we derive the characteristics of the total demand for spares from the repair/replacement decisions for a single product. Based on the total demand characteristics, we can estimate the probability that a spare part will be available at a certain point in time and thus that replacement is still possible. If a spare is not available, we have to use another (more expensive) solution, such as additional minimal repairs. Next, it is an option to construct a feedback loop by including the spare part availability over time in the determination of near-optimal repair/replacement decisions for a single product. In this section, we show this principle for the simple case where we have $N\geq 1$ products that are as good as new at t=0 and we have to cover a warranty period W. In the next section, we extend our heuristic to a more realistic model where all products have a certain age and a limited remaining warranty period when the last time buy has to be taken.

5.1. Decomposition approach

We focus on a single tagged product and consider the total demand for spare parts, generated by replacements of all products, as an external process. In principle, the demand is not entirely an external process, since historic replacements of the tagged product itself is part of the total demand. We expect that this will have a minor impact if the total number of products is not very small.

We model the impact of the external demand process by a probability $p_T(s)$ that a spare part is still available when we still have time T to go until the end of the warranty period of the tagged product and the initial stock level is s. Note that time W-T has passed since the start of the warranty period if we still have time T to go. We estimate these probabilities as follows: We compute the mean and standard deviation of demand for spares in [0, W-T] for an individual product from its optimal repair/replacement policy, assuming that we have infinite supply of spares. We aggregate these demand characteristics over all products, fit an appropriate distribution function, and estimate the probabilities $p_T(s)$ (Section 5.2). Next, we can use these probabilities to evaluate or even re-optimize the value functions under limited spare supply (Section 5.3). Finally, we can evaluate the system in terms of total relevant costs and/or service levels (Section 5.4). We summarize the algorithm in Section 5.5, and give results from numerical experiments in Section 5.6.

5.2. Estimating the spare part availability probabilities $p_T(s)$

At first sight, we can estimate the demand characteristics using a similar recursion as (1). This will give us the demand for a single product when a warranty period $T\leq W$ is remaining and the product is as good as new, so in the time interval [W-T, W]. However, we need the demand in the preceding period, so in the interval [0, W-T], to estimate the probability $p_T(s)$ that a spare part is available. To this end, we define:

$R_T(w)$ = Number of replacements of a single product in the next T periods if the product is as good as new and w periods are remaining until the end of the warranty period, under the current policy $\{\tau(w)\}$

That is, for any $w \leq W$ and $T \leq w$, we consider the number of product replacements in the interval [W-w, W-w+T] (counting from the start of the warranty period). We define a recursion for the survival probabilities of $R_T(w)$ by discretizing time. Because the product is as good as new at the start of the interval, the first product replacement occurs at $t \in (\tau(w), T]$ with density $g(t; \tau(w))$. We have a renewal at time t, and thus we find:

$$P\{R_T(w) \geq n\} = \sum_{t=\tau(w)+1}^{T} g(t; \tau(w)) P\{R_{T-t}(w-t) \geq n-1\}, n \geq 1 \qquad (3)$$

In words, the probability of having at least n product replacements in [W-w, W-w+T] equals the probability of a product replacement in period $t \in (\tau(w), T]$ multiplied by the probability of having at least (n-1) replacements from time t on, so in the interval [W-w+t, W-w+T]. We start the recursions with the values $P\{R_T(w) \geq 0\} = 1$ $(0 \leq T \leq w \leq W)$. From these probabilities, we compute the first two moments using the standard expressions:

$$E[R_T(w)] = \sum_{n=1}^{\infty} Pr\{R_T(w) \geq n\} \qquad (4)$$

$$E[R_T^2(w)] = \sum_{n=1}^{\infty} (2n-1) * Pr\{R_T(w) \geq n\} \qquad (5)$$

We cut-off the summations at some value n when $\Pr\{R_T(w) \geq n\}$ drops below some small number $\varepsilon > 0$. As the number of replacements of a single product is typically very small, the number of terms in the sum is usually small as well. Next, we can easily compute the mean and standard deviation of the number of replacements of a single product. Because the failure behavior is independent between products, the mean and standard deviation of the total demand for spare parts in [W-T, W], denoted by $D_T(W)$, are given by

$$E[D_T(W)] = N * E[R_T(W)] \qquad (6)$$

$$\sigma[D_T(W)] = \sqrt{N} \sigma[R_T(W)] \qquad (7)$$

Next, we use a two-moment approximation for $D_T(W)$ to compute the spare part availability probabilities:

$$p_T(s) = Pr\{D_T(W) \leq s-1\} \qquad (8)$$

We propose to use a Normal distribution for $D_T(W)$ and to apply a continuity correction to estimate $p_T(s)$.

5.3. Value functions under limited spare part supply

Under limited spare part supply, we have to modify recursion (1) since a spare part may be unavailable. We include the spare part availability probabilities and have to decide what happens if a spare part is not available. We can assign some fixed penalty costs for finding an alternative solution (e.g. replacing the product by another, more expensive product), which is easy to include in the recursion. However, we assume that we will continue performing minimal repairs when we are lout of stock. If the stock level is low, the number of additional repairs and thus the corresponding costs will increase strongly, which is a trigger to stock sufficient spares. Under this assumption, we find the recursion

$$V(w;s) = \min_{0\le\tau(w)\le w} \left\{ C_m H(\tau(w)) + \sum_{t=\tau(w)+1}^{w} g_{t,\tau(w)} \{ p_{w-t+1}(s) [C_d + C_p + V(w-t;s)] + (1 - p_{w-t+1}(s)) C_m [1 + H(w) - H(t)] \} \right\} \quad (9)$$

So, if a spare part is not available when a replacement is required upon failure at time *t* according to the current policy $\tau(w)$, we perform at least one minimal repair. We expect further additional minimal repairs during the remaining time warranty period, which number equals $H(w)$-$H(t)$.

Remarks:

- In equation 6, we re-optimize the policy $\tau(w)$. Of course, we can also use the policy $\tau(w)$ that we already found for unrestricted spare part supply and use the recursion just to find the value functions $V(W;s)$ for all relevant stock levels under limited spare part supply.
- We can improve the value function recursion by replacing $V(w-t; s-1)$ in (16) by the average of two value functions as explained at the end of Section 4.1.

5.4. Total relevant costs and service levels

The total costs under stock level s and repair/replace policy $\tau(w)$ consist of the costs of (i) all minimal repairs, (ii) product replacements, (iii) spare part procurement, and (iv) scrap costs of spare parts remaining at the end of the warranty period (which may be negative if there is some positive salvage value). The value function (6) includes the cost components (i), (ii) and a part of (iii), namely all spare parts that have actually been used to replace failed products during the warranty period. This means that the total expected costs consists of $N * V(w;s)$ plus the procurement and scrap costs of spare parts remaining at the end of the warranty period. The latter number can easily be approximated from the demand characteristics as derived in Section 5.2. So we find for the total relevant costs *TRC*:

$$TRC(s) = N * V(W;s) + (C_p + C_s) * E[(s - D_T(W))^+] \quad (10)$$

where we use the shorthand notation X^+=Max$\{X, 0\}$. Under a Normal distribution for $D_T(W)$, it is straightforward to compute $E[(s - D_T(W))^+]$. Let us write the stock level as $s = E[D_T(W)] + k\sigma[D_T(W)]$, where *k* denotes the safety factor. Then $E[(s - D_T(W))^+] = \sigma[D_T(W)] * \{k + L(k)\}$, where $L(k)$ denotes the Normal loss function in *k*.

We may also choose for a service level approach instead of a cost approach. In that case, we use a target service level such as the probability that we do not run out of stock in the warranty period, or the fill rate, i.e., the fraction of demand that is satisfied from stock on shelf. For both service levels, we can use standard formulas from inventory management to find the appropriate spare part stock level based on a Normal distribution for the total demand $D_T(W)$ (cf. Silver, Pyke and Peterson [1998]).

5.5. Algorithm

We can construct the following algorithm based on the results from the previous sections:

1. Set $p_T(s)$=1 $\forall T,s$

2. Find the policy $\tau(W)$ and the corresponding value functions $V(w; s)$ from (9)
3. Find the first two moments of $R_T(w)$ from (3), (4) and (5)
4. Find estimates of the spare part probabilities $p_T(s)$ from (8) using a Normal approximation of $D_T(W)$, based on (6) and (7)
5. Find a near-optimal stock level: Evaluate the total relevant costs for s=0, 1, 2, 3, … from (10) until the costs increase. Denote by s^* the near-optimum value of s.
6. Optional iteration: Repeat Step 2-5 until the value of s^* does not change anymore.

This algorithm minimizes the total relevant costs. If we prefer a service level approach, we can compute the appropriate service level in Step 5 (see Section 5.4) and find the stock level s^* that satisfies the service level target.

5.6. Numerical experiments

We validated our method by comparison to results from discrete event simulation. As this is just a step towards the more realistic model that we discuss in Section 6, we skip details on the results and refer to Section 7.1 for comparison to simulation for the extended model.

A relevant question at this point is to which extent the optional iteration (Step 6 in our algorithm) has added value. To this end, we optimized the repair/replacement policy $\{\tau(w)\}$ and the stock level s with and without iterations for the following parameter settings:

- the time to failure has a Weibull distribution with mean 1 year and coefficient of variation 0.25, 0.5 or 0.75
- the costs of minimal repair equal $C_m = 1$
- The procurement costs of a spare product equal $C_p = 1.5$, 2.5 or 3.5
- The salvage value an operational spare product remaining at the end of the warranty period (C_s) equal 0, 25% or 50% of the procurement costs (so $C_s = 0$, $-0.25C_p$ or $0.5C_p$)
- The additional costs of replacing a failed product by a new one (C_d) equal 0
- The length of the warranty period equals $W = 3$ or 5
- The number of systems in the installed base equals $N = 10$ or 100

So, we have $3^3 * 2^2 = 108$ settings. We discretized time in 300 intervals.

We found that the feedback loop to improve the repair replacement policy $\{\tau(w)\}$ based on the spare part availability $p_t(s)$ has little impact. Even though the iterations may yield a little higher replacement frequency higher stock level, the impact on the total relevant costs is low. On average, the improvement is 0.06% only, with a maximum of 0.47%. In 33 out of the 108 cases we found no improvement at all. So we conclude that a single iteration is sufficient to find a near optimal repair/replacement policy and stock level. Therefore, we will only use a single iteration for our extended model in the next section.

6. Extension: partial warranty periods left at last time buy

In the previous section, we assumed that all products in the installed base are new and have a full warranty period with length W at the last time buy decision. Obviously, that will not be the case in practice: We shall have products in the installed base with each their own remaining warranty period $w \leq W$ and age $a \geq 0$ (products will generally not be as good as new when the last time buy occurs). At best, a company has this information available at the location where the last time buy decision needs to be made (usually a central stockpoint in the service supply chain). Alternatively, central information may only consist of (estimates of) the size of the installed base and the distribution of the remaining warranty period over these items. For example, a company may be able to give a frequency distribution of the remaining warranty period for the entire installed base. The presence of limited installed base information only is plausible given our observations in industry.

In this section, we use the same decomposition approach as before to analyse the multi-product model. We assume that the repair/replacement policy $\tau(w)$ corresponding to the infinite spare part supply is used throughout the remaining warranty period for all items. This is a simple and reasonable approach, since the numerical results in Section 5.6 revealed that inclusion of limited spare part supply in the optimization of the policy $\tau(W)$ hardly influences the structure and costs of the near-optimal policy.

We follow same optimization approach as in the previous section, but extend the formulas for the distribution of demand for spare parts as well as the value functions to the modified situation. We start with the mean and standard deviation of the number of product replacements for a single product with age a and remaining warranty period w in Section 6.1. Next, we derive the same characteristics for the case when we only know the probability distribution of the remaining warranty period, since we may be able to estimate a frequency distribution of the remaining warranty period over the entire installed base (Section 6.2). In Section 6.3, we show how to evaluate the value function and the total relevant costs over the installed base given a frequency distribution for the remaining warranty period. Based on these characteristics, we can apply the algorithm from Section 5.5 with our modified formulas. We validate our method in Section 7 by comparison to results from discrete event simulation.

6.1. Number of product replacements given age a and remaining warranty period w

As before, we focus on a single product, this time with age a and remaining warranty period w, where $0 \leq a \leq W\text{-}w$. As explained above, we may assume that the repair/replacement policy $\{\tau(w)\}$ is given. Let us define $\hat{R}_T(w, a)$ as the number of product replacements in the first T periods of the remaining service period w for a product with age a. In general, the product will not be as good as new at the start of the period. We can relate $\hat{R}_T(w, a)$ to the number of replacements when the product is as good as new, $R_T(w)$. For this, we use the random variable $Z(w,a)$, representing the time until the next replacement replenishment of a product with age a and remaining warranty period w. We derive an expression for $Z(w,a)$ below.

The previous product replacement (if any) occurred with time $w+a$ to go, so the current critical limit for product replacement equals $\tau(w + a)$. So no product replacements occur in the next $\tau(w + a) - a$ periods, insofar this number is positive. So replacement will occur in the next T periods if (i) our repair/replacement policy prescribes so ($\tau(w + a) - a \geq T$), or (ii) the first failure at which the product should be replaced occurs after T. Otherwise, we have at least one replacement after $Z(w, a)$ periods plus the number of replacements from that time on given that the product is as good as new. So we find:

$$\hat{R}_T(w, a) = \begin{cases} 0 & T \leq \tau(w + a) - a \ OR \ Z(w, a) > T \\ 1 + R_{T-Z(w,a)}(w - Z(w, a)) & \tau(w + a) - a < Z(w, a) \leq T \end{cases} \tag{11}$$

We find an expression for the distribution of $\Pr\{Z(w, a) = z\}$ as follows: The product should fail at the age $a + z$ ($\geq \tau(w + a)$) conditional on the fact that it survived until age a and until $\tau(w + a)$:

$$\Pr\{Z(w, a) = z\} = g(a + z; \max\{a, \tau(w + a)) \tag{12}$$

We can derive the mean and variance of $\hat{R}_T(w, a)$ from these two expressions by conditioning on $Z(w, a)$. We simply apply the two well-known rules for any correlated random variables X and Y:

$$E[X] = E\{E[X|Y]\} \tag{13}$$

$$Var[X] = E\{Var[X|Y]\} + Var\{E[X|Y]\} \qquad (14)$$

A drawback of this model is the computational effort: We have to compute the mean and variance of $\hat{R}_T(w, a)$ for all combinations (T, w, a). If we discretize the warranty period in many periods, the computational effort increases rapidly, namely with the order $(\text{number of periods})^3$. A solution is to keep the number of discrete periods limited (say up to 100). Because we focus on the situation with limited installed base information (see next subsection), we skip the detailed derivations for this specific model.

6.2. Number of product replacements for an arbitrary remaining warranty period

Suppose that we do not know the age and remaining warranty period of each product in the installed base at the last time buy. We just have information on the distribution of the remaining warranty period over all products in the installed base, for example a frequency distribution. We can use the formulas from the previous subsection to find the distribution of the number of products replacements $\hat{R}_T(w)$ if we only know that the remaining warranty period is w (and not the age a). We do this by defining age as a random variable A and finding the probability distribution of A given the remaining warranty period w. From $\hat{R}_T(w)$, we can find the probability distribution of the number of product replacements $\hat{R}_T$ by weighing with the frequency distribution of the remaining warranty period. In fact, we first uncondition over the distribution over the age a, and next over the remaining warranty period w.

a) Distribution of the age given the remaining service period, A|w

We define the age $A(w; W)$ as he age of the product if there is still a period w to go until the end of a warranty period with length W ($w \leq W$) and the product is as good as new at the start of the warranty period. We do not replace the product until time $\tau(W)$, so until the time that we still have W-$\tau(W)$ to go. So we find for $w \geq W$-$\tau(W)$:

$$Pr\{A(w; W) = a\} = \begin{cases} 1 & a = W - w \\ 0 & otherwise \end{cases} \quad if\ w \geq W - \tau(W) \qquad (14)$$

If $w <$ W-$\tau(W)$, one or more product replacements may occur in ($\tau(W)$, W-w]. The age is W-w with the probability $\bar{G}_{W-w;\tau(W)}$ that no product is replaced in the interval ($\tau(W)$, W-w]. With probability $g_{t;\tau(W)}$, we replace the product at time $t \in [\tau(W), W\text{-}w]$. Then the age of the product is W-w-t if there have not been any other replacements since then, otherwise the age can be less. So we find for $w < W$-$\tau(W)$:

$$Pr\{A(w; W) = a\} = \begin{cases} \bar{G}_{W-w;\tau(W)} & a = W - w \\ \sum_{t=\tau(W)+1}^{W-w} g_{t;\tau(W)} Pr\{A(w; W - t) = a\} & a \leq W - w - \tau(W) \\ 0 & otherwise \end{cases} \qquad (15)$$

Using equations (14) and (15), we can recursively compute the discrete probability distributions of all product ages $A(w; W)$.

b) Distribution of the time until the next product replacement Z(w)

Now it is straightforward to compute the probability distribution of the time until the next product replacement if the remaining warranty period is w, $Z(w)$, by conditioning on the product age $A(w; W)$: We numerically combine equation (12) for $Z(w; a)$ with the equations (14) and (15) for $A(w; W)$.

c) Distribution of the number of product replacements in remaining warranty period w, $\hat{R}_T(w)$

Now we can the rules of the conditional expectation and conditional variance (13) and (14) to compute the mean and variance of $\hat{R}_T(w)$:

$$E\left[\hat{R}_T(w)\right] = \sum_{z=1}^{T} \Pr\{Z(w) = z\} * \{1 + E[R_{T-z}(w-z)]\} \tag{16}$$

$$Var\left[\hat{R}_T(w)\right] = Var\left\{E[\hat{R}_T(w|Z)]\right\} + E\left\{Var[\hat{R}_T(w|Z)]\right\} \tag{17}$$

To elaborate equation (17), we first note that it is straightforward to compute the last term:

$$E\left\{Var[\hat{R}_T(w|Z)]\right\} = \sum_{z=1}^{T} \Pr\{Z(w) = z\} * Var[R_{T-z}(w-z)] \tag{18}$$

where $Var[R_T(w)]$ is found by combining (4) and (5) with (3), see Section 5.2.

To compute $Var\left\{E[\hat{R}_T(w|Z)]\right\}$, we note that the second moment of conditional expectation is given by:

$$E\left\{E[\hat{R}_T(w|Z)]^2\right\} = \sum_{z=1}^{T} \Pr\{Z(w) = z\} * \{1 + E[R_{T-z}(w-z)]\}^2 \tag{19}$$

Subtracting the square of $E\left\{E[\hat{R}_T(w|Z)]\right\} = E\left[\hat{R}_T(w)\right]$ gives us the variance of the conditional expectation.

d) Distribution of the number of product replacements of an arbitrary product, $\hat{R}_T$

Again, we can find the mean and variance of $\hat{R}_T$ using the conditioning rules (13) and (14). Let us define q_w as the fraction of products that has remaining service period *w*. Noting that we do not observe replacements after the service period (*T*>*w*), we can compute the mean and variance of $\hat{R}_T$ as:

$$E\left[\hat{R}_T\right] = \sum_{w=1}^{W} q_w E\left[\hat{R}_{\min\{T,w\}}(w)\right] \tag{20}$$

$$Var\left[\hat{R}_T\right] = Var\left\{E\left[\hat{R}_T(w)\right]\right\} + E\left\{Var\left[\hat{R}_T(w)\right]\right\} \tag{21}$$

The last term in (21) is found from

$$E\left\{Var[\hat{R}_T(w)]\right\} = \sum_{w=1}^{W} q_w Var\left[\hat{R}_{\min\{T,w\}}(w)\right] \tag{22}$$

To compute $Var\left\{E[\hat{R}_T(w)]\right\}$, we again start with the second moment of conditional expectation:

$$E\left\{E[\hat{R}_T(w)]^2\right\} = \sum_{w=1}^{W} q_w \left\{E\left[R_{\min\{T,w\}}(w)\right]\right\}^2 \tag{23}$$

Subtracting the square of $E\left\{E[\hat{R}_T(w)]\right\} = E\left[\hat{R}_T\right]$ yields the variance of the conditional expectation.

We find the mean and standard deviation of the total demand for spare parts in any time period *T* similar to (6) and (7). This provides us sufficient information to cover the service level approaches. However, we cannot compute the expected costs yet, and we cannot perform any cost minimization. We address this remaining issue in the next subsection.

6.3. Value function and total relevant costs

As before, we use conditioning to derive the value functions, taking into account that a product is <u>not</u> as good as new at the start of the remaining warranty period. We define:

$\hat{V}(w, a; s)$ = Value function if *w* periods are remaining, the product age is *a* and we have *s* spares

We can relate this value function to another value function where the product is as good as new at the start of the remaining warranty period *w*:

$\tilde{V}(w, W; s)$ = Value function if w periods are remaining of the warranty period for a single product *that is as good as new*, whereas the installed base still has a warranty period *W* to go and we have *s* spares.

Regarding the latter definition, we have to take into account that other products than the tagged product have a warranty period different from the tagged product, because this influences the demand for spare parts and thus the spare part availability for the tagged product.

We find the relation between $\hat{V}(w, a; s)$ and $\tilde{V}(w, W; s)$ as follows. For a product having age a, the repair/replacement limit $\tau(w + a)$ applies. That is, we will have minimal repairs until $\tau(w + a)$ has been reached, insofar that has not happened yet. So the expected number of minimal repairs is $H(\tau(w + a)) - H(a)$ if $a \le \tau(w + a)$ and zero otherwise. If $a \le \tau(w + a)$, the next product replacement occurs at time *t* with probability $g_{a+t,\tau(w+a)}$. Then, a spare part is available with probability $p_{W-t+1}(s)$. Note that we have to use the total warranty period *W* and not the remaining warranty period *w*. The other terms are the same as before, see equation (9). Putting it together, we find for $a \le \tau(w + a)$:

$$\hat{V}(w, a; s) = C_m\{H(\tau(w + a)) - H(a)\} + \sum_{t=\tau(w+a)-a+1}^{w} g_{a+t,\tau(w+a)}\{p_{W-t+1}(s)[C_d + C_p + \tilde{V}(w - t, W - t; s)] + (1 - p_{W-t+1}(s))C_m[1 + H(a + w) - H(a + t)]\} \tag{24}$$

If $a > \tau(w + a)$, there are no minimal repairs before the next planned product replacement. Then we find:

$$\hat{V}(w, a; s) = \sum_{t=1}^{w} g_{a+t,a}\{p_{W-t+1}(s)[C_d + C_p + \tilde{V}(w - t, W - t; s)] + (1 - p_{W-t+1}(s))C_m[1 + H(a + w) - H(a + t)]\} \tag{25}$$

As before, we can find the value function that we need by unconditioning over the age *A*|*w* first and over the remaining service period *w* next. This gives us the unconditional value function $\hat{V}(s)$. The remaining issue is the calculation of $\tilde{V}(w, W; s)$. This is very similar to equation (9), with the probability that a spare is available as only difference: We have to take into account the total warranty period *W* instead of the remaining warranty period *w*. For sake of completeness, we give the expression below:

$$\tilde{V}(w, W; s) = C_m H(\tau(w)) + \sum_{t=\tau(w)+1}^{w} g_{t,\tau(w)}\{p_{W-t+1}(s)[C_d + C_p + \tilde{V}(w - t, U - t; s)] + (1 - p_{W-t+1}(s))C_m[1 + H(w) - H(t)]\} \tag{26}$$

Finally, the total relevant costs *TRC(s)* are similar to (10):

$$TRC(s) = N * \hat{V}(s) + (C_p + C_s) * E\left[(s - \hat{D}_T)^+\right] \tag{27}$$

Where $\widehat{D}_T$ is the total demand in the first T periods after the last time buy decision under frequency distribution q_w for the remaining warranty period in the installed base as discussed before.

6.4. Algorithm

The algorithm is the same as in Section 5.5 under replacement of the equations by the corresponding equations that we derived in this section. In our numerical experiments, we will omit the iteration step 6 because it is more involved in this setting and we have seen before that there is hardly any impact on the repair/replacement policy $\tau(w)$.

7. Numerical results for the multi-product model

7.1. Validation by comparison to discrete event simulation

We constructed a discrete event simulation model in Plant Simulation 8.2 to validate our approximations step by step. In this section, we show the overall results of the method that we constructed in Section 6. For sake of comparison, we do not optimize the stock level, but we compare the key performance characteristics from our approximation to the simulated values for a range of stock levels to show the validity of our model.

We take the following input data. The time to failure has a Weibull distribution $F(t) = e^{-\left(\frac{x}{\alpha}\right)^{\beta}}$ with scale parameter α=1 and shape parameter β=2 (so the mean is 0.8862 and the standard deviation is 0.4633). As cost factors, we take C_m = 1, C_p = 1.5, and C_s = C_d = 0. The warranty period is W=3, which we discretize in 100 periods. The size of the installed base is N=10 or 100. We take a range of values for the spare part stock level around the optimal value. For N=10, we vary s between 0 and 20. For N=100, we vary s between 100 and 150. We assume a uniform distribution for the remaining warranty period, so q_w=1/W. In the Tables 4 and 5, we show the total relevant costs *TRC* as well as the probability of not running out of stock for both our approximations and the simulation. For the simulation, we used a very high number of replications in order to obtain accurate results (100,000 replications for N=10 and 10,000 replications for N=100, so the total number of warranty periods simulated equals 1,000,000 in all instances).

	TRC		*Pr{No stockout}*	
s	sim	appr	sim	appr
0	44.11	44.42	0.000	0.000
4	36.20	36.93	0.002	0.004
8	29.15	29.91	0.094	0.096
10	27.17	27.67	0.265	0.259
11	26.73	27.11	0.384	0.376
12	**26.71**	**26.98**	0.512	0.506
13	27.08	27.26	0.640	0.636
14	27.78	27.90	0.750	0.751
16	29.93	30.00	0.906	0.910
18	32.64	32.72	0.974	0.977
20	35.56	35.65	0.996	0.996

Table 4. Approximation versus simulation for installed base size N=10

s	*TRC* sim	*TRC* appr	*Pr{No stockout}* sim	*Pr{No stockout}* appr
100	260.62	262.16	0.005	0.006
105	255.60	256.86	0.022	0.023
110	251.80	252.62	0.068	0.071
118	248.19	249.12	0.267	0.264
119	**248.15**	**249.06**	0.307	0.300
120	248.32	249.08	0.335	0.337
125	249.84	250.64	0.538	0.541
130	253.70	254.47	0.727	0.734
140	266.02	266.77	0.949	0.953
150	280.64	281.43	0.996	0.997
180	325.60	326.41	1.000	1.000

Table 5. Approximation versus simulation for installed base size N=100

From the two tables we observe that:

1. The accuracy of the approximations is very good
2. We find the same optimum values for the spare part stock level using simulation and approximation
3. The approximations error decreases with the size of the installed base and the service level
4. The minimum cost solution has a high probability that we run out of stock before the end of the warranty period. The reason is that the cost penalty of running out of stock remains within reasonable limits, because the size of the shortage is usually small. Then we can compensate by performing additional minimal repairs at reasonable costs. For the cost optimal solutions in Table 4 and 5, we find fill rates of 0.884 and 0.942, respectively.

7.2. Impact of key parameters

We conducted an experiment with the same 108 combinations of parameter setting as described in Section 5.6. Again, we assumed a uniform distribution for the remaining service period. The optimum stock levels are on average 55% of the stock levels that we found for the theoretical instances in which all products are new when the last time buy decision has to be taken (excluding the cases with stock level 0). The percentages vary between 22% and 70%, where the highest percentages occur if the coefficient of variation of the time to failure is low.

We studied the impact of the key parameter settings on the following performance indicators:

- The expected total relevant costs per product per year remaining: $TRC/(N*W/2)$. As we have uniform distribution of the remaining warranty period, the average number of years remaining is $W/2$
- The service level, expressed as the fill rate P_2 (this provides a better indicator than the probability P_1 that no stock-out occurs during the remaining warranty period as we include the *size* of the shortage)
- The ratio of the expected number of product replacements to the expected number of minimal repairs

We summarize the impact of the experimental factors of these performance indicators in Table 6-8. This yields the following main findings:

1. The total relevant costs per product per year decrease in the coefficient of variation of the time to failure CV, the size of the installed base N and the salvage value C_s. The costs increase in the procurement costs C_p. Most impact has CV, followed by C_P, see Table 6. The impact of CV can be explained by the fact that it is costly to cover stock-outs by additional minimal repairs if the failure rate

function is strongly increasing. We observe in Table 7 that the fill rate is also very high if the failure rate function is strongly increasing (low CV).

2. The procurement costs of spares has a high impact on all performance indicators. The impact on the ratio of product replacements to minimal repairs is clearly nonlinear: Raising C_p from 1.5 to 2.5 reduces the number of replacements considerable, and a further increase has little impact. For high values of C_p, there are quite some cases in which only minimal repairs are performed because product replacement is simply too expensive.
3. The fill rate as well as the number of product replacements increases with the size of the installed base *N* and the length of the warranty period *W*. This is probably due to the risk-pooling effect which makes it somewhat more advantageous to stock additional spares.

Factor ↓ Level→	**low**	**medium**	**high**
CV	2.57	2.13	1.54
C_p	1.53	2.13	2.58
- C_s/C_p	2.11	1.72	2.41
W	2.00		2.16
N	2.16		2.00

Table 6. Average total relevant costs per product per year

Factor ↓ Level→	**low**	**medium**	**high**
CV	0.99	0.93	0.38
C_p	0.92	0.75	0.63
- C_s/C_p	0.75	0.87	0.68
W	0.72		0.81
N	0.73		0.79

Table 7. Average fill rate

Factor ↓ Level→	**low**	**medium**	**high**
CV	2.99	0.87	0.14
C_p	2.71	0.76	0.53
- C_s/C_p	1.18	2.18	0.64
W	1.03		1.63
N	1.12		1.54

Table 8. Average ratio product replacements : minimal repairs

8. Conclusions and further research

In this paper, we developed methods for the joint decision of repair/replacement of products sold under warranty and the procurement of products for replacement at a last time buy occasion. Our key results consists of approximations for the performance of a multi-product model in the variants (i) all products are as good as new at the last time buy, (ii) the age and the remaining warranty period of all products in the installed base are known (iii) we only know a frequency distribution for the remaining warranty period of all products in the installed base. The first model is purely theoretical. The third model is most useful for practical applications, since detailed installed base information is generally not available at the central location in the supply chain where the last time buy decision has to be taken.

Comparison to simulation results revealed that our approximations are very accurate. A sensitivity study on the most important parameter settings revealed insights in the effects of the key parameters on the system performance in terms of costs, service levels and number of product replacement versus minimal repairs. In particular, the coefficient of variation of the time t failure has a high impact. A low coefficient of variation (i.e., a strongly increasing failure rate function) yields high costs, the necessity of a high service level for spares and many product replacements, because the number of minimal repairs increases fast if a product is not replaced upon failure.

As a line of further research, we believe that an interesting topic is the interaction between the last time buy decision and the repair/replacement decision for modules of advanced capital goods. Because these capital goods tend to be downtime critical, system repair is often performed by replacement of a failed module. Because the modules can be very expensive, they are repaired if it is economically and technically feasible. An issue is when repair is economically justified, based on e.g. the age of the module, and when replacement is a better option. Repair of a module usually consists of replacing a failed component by a new one. We can see this as a minimal repair of the module, because the status of all other components is unaltered. A Under degradation, a question is when the module should still be repaired and when it should it be discarded and replaced by a new one. After manufacturing has been discontinued, replacement is only possible if a new module is available on stock. A critical difference compared to our current model is that the time required for model repair is typically significant and may not be ignored. Also, because usually a repair-by-replacement policy is applied on a system level, the repaired module is added to spare part stock after repair and not immediately used in the field. This means that the spare part stock consists of a heterogeneous set of new and used parts with different ages. The analysis of this system is far from trivial and an interesting topic for further research.

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