The Dynamics of a Flexible Beam With a Lubricated Prismatic Kinematic Pair

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In this article we consider the influence of the prismatic kinematic pair lubrication film on the planar vibrations of a constant cross-sectional straight link that is attached to the joint. The main objective is to develop an analytical model that incorporates the effect of the lubricant film on the vibration of elastic links in mechanisms. It has been assumed that the beam on which the prismatic kinematic pair translates is a linear elastic body. Equations for the translational and rotational motions of the link are developed by applying Hamilton's principle. Kinetic energy that is required for the application of this principle has been derived by utilizing a generalized velocity field theory for elastic solids. This approach provides means to include the inertia terms directly in the equations of motion. The pressure field exerted through the viscous, incompressible, lubricant film is obtained from the solution of the Reynolds equation of lubrication. We introduce a scheme to solve the resulting two sets of equations for the vibrations of the link and the motion of the fluid. The pressure field is used to compute the external force exerted by the fluid on the link. The utility of the method is demonstrated by considering a planar mechanism that includes an elastic element with a prismatic joint.

1 Introduction

Structural flexibility is an important factor in the study of dynamics of high performance mechanical systems such as mechanisms, manipulator robots, drilling machines, antennas, etc. In general, flexible mechanical systems have several desirable features relative to stiff mechanisms and manipulators such as lower cost, higher speed, reduced power consumption, improved mobility, and safer operation. Yet, in such systems positioning accuracy depends on effective elimination of oscillations that occur due to flexible effects. Accurate modeling of such effects can greatly facilitate the development of control algorithms that can be used in positioning and trajectory tracking applications. Dynamic analysis of flexible links, however, is complicated by the coupling between the nonlinear rigid body motion and the linear elastic displacements of the link.

The importance of modeling mechanisms systems taking into account the elasticity of the links has long been realized and reviews of the work in this area are presented in Park et al. (1986), Turcic and Midha (1984), Nagarajan and Turcic (1990). The nonlinear differential equations of motion derived, consider the rigid body motion and the elastic motion to influence each other and sophisticated numerical algorithms are used to solve the problem.

The dynamics of elastic manipulators with prismatic joints has been investigated by Buffinton and Kane (1985), Kim (1988), Buffinton (1992), and Guran et al (1996). They examine the motion of a single beam as it moves longitudinally over two distinct points. Equations of motion are formulated by treating the beam's supports as kinematical constraints imposed on an unrestrained beam. Gordaninejad et al. (1989) examine the motion of a planar robot arm consisting of one revolute and one prismatic joint. All coupling terms between the rigid and the flexible motions of the robot arm are included in the model based on the Timoshenko beam theory. Wicker and Mote (1988) studied the vibration and stability of axially moving materials. Yuh and Young (1991) derived a time varying partial differential equation with boundary conditions for an axially moving beam in rotation.

One shortcoming of these studies is that they do not examine the load from oil pressure in the revolute and prismatic joints. In general, a fluid is introduced between two rubbing surfaces in order to separate them and thus reduce their frictional forces. The lubrication or friction reduction of two bodies in near contact is generally accomplished by a viscous fluid moving through a narrow but variable gap between the two bodies. The theory was developed by Reynolds (1886).

Benson and Talke (1987) investigate the dynamics of a magnetic recording slider of a rigid disk during its transition between sliding and flying. The slider is modeled as a three degree-of-freedom system, capable of lift, pitch, and roll. In addition to considering loads from air pressure, inertia, and the suspension arm, they also consider impulsive load arising from slider/disk collisions.

Our model is different from these models. We study the effect of lubricant film in prismatic joints of mechanism on the vibration of elastic members. We derive the equations of motion by using Hamilton's principle starting from a generalized form of kinetic energy. The equation of motion for a single elastic member attached to a lubricated prismatic joint is presented. The fluid pressure is computed by using a simplified form of Reynolds equation of lubrication. The resulting partial differential equation is transformed to an ordinary integro-differential equation by using the finite Fourier sine transform.

To our best knowledge this class of problems has never been fully discussed nor considered in any previous study in the area of elastodynamics.

Finally, the vibration problem of an elastic link of a slider-crank mechanism is considered. The equations of motion for the first mode of vibration are solved numerically using a Bulirsh-Stoer algorithm with adaptive step-size control. Several plots representing various aspects of the motion are presented.

The first step of dynamic analysis is the consideration of the relations that govern the kinematics of the linear elastic system. Indeed, effective formulation of equations of motion depends primarily on the ability to formulate proper mathematical expressions for kinematic quantities such as the linear and angular

Contributed by the Technical Committee on Vibration and Sound for publication in the Journal of Vibration and Acoustics. Manuscript received Nov. 1996. Associate Technical Editor: J. Q. Sun.

velocities and accelerations. Figure 1 depicts the linear-elastic body in motion in the three-dimensional space. The fixed coordinate frame is shown at O_1 with the unit vectors $[\mathbf{i}_1, \mathbf{j}_1, \mathbf{k}_1]$. A second moving frame which is attached to the border Σ of the body is shown at O with the unit vectors $[\mathbf{i}, \mathbf{j}, \mathbf{k}]$. For any given time instant, the linear-elastic body occupies the domain $D(t)$. The point C is the center of mass and $\mathbf{r}_C$ is the corresponding position vector. Here we track the motion of a point $P \in D(t)$ on the body. The position vector of this point with respect to the moving frame is expressed in terms of the sum of two vectors $\mathbf{r}$ and $\mathbf{u}$. The vector $\mathbf{r}$ marks the position of the particle subject to the rigid body motion only, whereas, the vector $\mathbf{u}$ represents the displacement as a result of elastic deformations. Furthermore, we attach a local vortex vector $\mathbf{\Omega}(r^*, t)$ to the point P. This vector represents the rotational motion of the medium in the neighborhood of the point as a result of rigid body motion and elastic deformations.

In the case of small deformations we assume uniform vorticity and drop the functional dependence on position ($\mathbf{\Omega}(r^*, t) = \boldsymbol{\omega}(t)$). As a result of this simplification the vortex vector becomes the angular velocity vector that is usually defined for rigid bodies. Now we use Poisson's formula to derive the kinematic relations for velocities and accelerations. Accordingly, the position vector for any particle on the linear-elastic solid $D(t)$ can be written as

$$\mathbf{r}^* = \mathbf{r}_0 + \mathbf{r} + \mathbf{u}, \tag{1}$$

where $\mathbf{r}^*$ is the position vector of P in the $[\mathbf{i}_1, \mathbf{j}_1, \mathbf{k}_1]$ frame, $\mathbf{r}_0$ is the position vector of O in the $[\mathbf{i}_1, \mathbf{j}_1, \mathbf{k}_1]$ frame, and $\mathbf{r}$ is the position vector of P in the frame attached to the linear-elastic body in motion.

Differentiating Eq. (1) with respect to time yields

$$\mathbf{v} = \mathbf{v}_0 + \boldsymbol{\omega} \times \mathbf{r} + \frac{\partial \mathbf{u}}{\partial t} + \boldsymbol{\omega} \times \mathbf{u}, \tag{2}$$

where the terms

$$\mathbf{v}_0 + \boldsymbol{\omega} \times \mathbf{r}$$

corresponds to the rigid body velocity field, and

$$\frac{\partial \mathbf{u}}{\partial t} + \boldsymbol{\omega} \times \mathbf{u}$$

represents the additional component that arises from the deformation of the linear-elastic body.

The kinetic energy of the linear-elastic body can be written as

$$T = \tfrac{1}{2} \iiint_{(D)} \mathbf{v} \cdot \mathbf{v} \rho d\tau = T_{rg} + T_{el} + T_s, \tag{3}$$

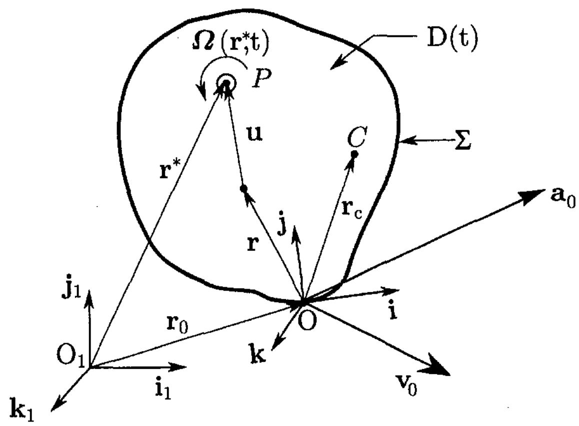

Fig. 1 Linear-elastic solid in motion

where

$$T_{rg} = \frac{M\mathbf{v}_0 \cdot \mathbf{v}_0}{2} + M\mathbf{v}_0 \cdot (\boldsymbol{\omega} \times \mathbf{r}_C) + \tfrac{1}{2}\boldsymbol{\omega} \cdot (\mathbf{J}_0 \boldsymbol{\omega}) \tag{4}$$

corresponds to a rigid body motion,

$$T_{el} = \tfrac{1}{2} \iiint_{(D)} \left(\frac{\partial \mathbf{u}}{\partial t}\right) \cdot \left(\frac{\partial \mathbf{u}}{\partial t}\right) \rho d\tau \tag{5}$$

corresponds to the elastic solid motion without rotation or translation, and

$$T_s = \tfrac{1}{2} \iiint_{(D)} \left[(\boldsymbol{\omega} \times \mathbf{u}) \cdot (\boldsymbol{\omega} \times \mathbf{u}) + 2(\mathbf{v}_0 + \boldsymbol{\omega} \times \mathbf{r}) \times \left(\frac{\partial \mathbf{u}}{\partial \mathbf{t}} + \boldsymbol{\omega} \times \mathbf{u}\right) + 2\frac{\partial \mathbf{u}}{\partial t} \cdot (\boldsymbol{\omega} \times \mathbf{u}) \right] \rho d\tau \tag{6}$$

is the coupling term arising from deformation and the rigid body motion.

2 Equations of Motion for Rectilinear Elastic Links

Figure 2 depicts a rectilinear kinematic link with variable cross-sectional area $A(x)$. We define

$$\mathbf{r} = x\mathbf{i}$$

as the position vector of any particle on the axis of the link,

$$\boldsymbol{\alpha}(t) = \dot{\boldsymbol{\omega}}(t) = \alpha_1(t)\mathbf{i} + \alpha_2(t)\mathbf{j} + \alpha_3(t)\mathbf{k}$$

as the rigid body angular acceleration of the link,

$$\mathbf{v}_O(t) = v_{01}(t)\mathbf{i} + v_{02}(t)\mathbf{j} + v_{03}(t)\mathbf{k}$$

as the translational velocity of the end O of the link,

$$\mathbf{a}_O(t) = a_{01}(t)\mathbf{i} + a_{02}(t)\mathbf{j} + a_{03}(t)\mathbf{k}$$

as the translational acceleration of the end O of the link, and

$$\mathbf{f}(x, t) = f_1(x, t)\mathbf{i} + f_2(x, t)\mathbf{j} + f_3(x, t)\mathbf{k}$$

is the external force per unit length of the link. Also, E is the Young modulus, $I_y(x)$ and $I_z(x)$ are the central moments of inertia of the bar cross-section at the axes Oy and Oz, respectively.

The equations of motions for the elastic link are developed by applying Hamilton's principle that can be written as

$$\delta \int_{t_0}^{t_1} (V - T - W) dt = 0, \tag{7}$$

where V is the strain energy, T is the kinetic energy, and W is the work function.

The strain energy can be written as

$$V = \frac{E}{2} \left[\int_0^L A(x) \left(\frac{\partial u_1}{\partial x}\right)^2 dx + \int_0^L I_z(x) \left(\frac{\partial^2 u_2}{\partial x^2}\right)^2 dx + \int_0^L I_y(x) \left(\frac{\partial^2 u_3}{\partial x^2}\right)^2 dx \right], \tag{8}$$

and the work function

$$W = \mathbf{f} \cdot \mathbf{u}. \tag{9}$$

The kinetic energy T is given by Eqs. (3), (4), (5), and (6). We note that the variational integral of the rigid body kinetic energy is

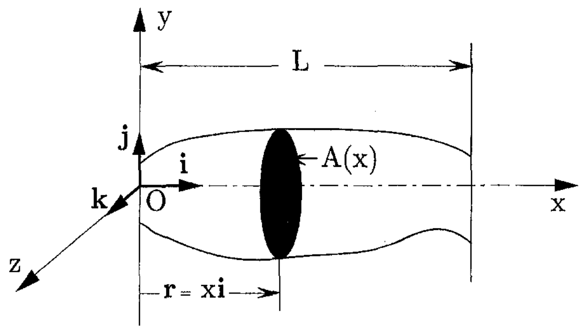

Fig. 2 Rectilinear kinematic link

$$\delta \int_{t_0}^{t_1} T_{rg} dt = 0,$$

for a prescribed rigid body motion. Then, using Eqs. (8), and (9) along with the expression for kinetic energy Eq. (3) yield the equations of motion of linear-elastic displacements $u_1(x, t)$, $u_2(x, t)$ and $u_3(x, t)$. These equations are given by

$$EA' \partial u_1/\partial x - EA \partial^2 u_1/\partial x^2 + \rho A \partial^2 u_1/\partial t^2 - \rho A u_1(\omega_2^2 + \omega_3^2) + \rho A \omega_1 \omega_2 u_2 + \rho A \omega_1 \omega_2 u_3 + \rho A a_{01} - \rho A(\omega_3 v_{02} - \omega_2 v_{03}) + \rho A \alpha_2 u_3 + \rho A \alpha_3 u_2 - 2\rho A \omega_3 \partial u_2/\partial t - 2\rho A \omega_2 \partial u_3/\partial t - \rho A x(\omega_2^2 + \omega_3^2) - f_1(x, t) = 0, \quad (10)$$

$$EI_z'' \partial^2 u_2/\partial x^2 + 2EI_z' \partial^3 u_2/\partial x^3 + EI_z \partial^4 u_2/\partial x^4 + \rho A \partial^2 u_2/\partial t^2 - \rho A u_2(\omega_1^2 + \omega_3^2) + \rho A \omega_2 \omega_3 u_3 + \rho A \omega_1 \omega_2 u_1 + \rho A a_{02} - \rho A(\omega_1 v_{03} + \omega_3 v_{01}) + \rho A \alpha_3 x - \rho A \alpha_1 u_3 - 2\rho A \omega_1 \partial u_3/\partial t + 2\rho A \omega_3 \partial u_1/\partial t + \rho A x(\omega_1 \omega_2 + \omega_3^2) - f_2(x, t) = 0, \quad (11)$$

$$EI_y'' \partial^2 u_3/\partial x^2 + 2EI_y' \partial^3 u_3/\partial x^3 + EI_y \partial^4 u_3/\partial x^4 + \rho A \partial^2 u_3/\partial t^2 - \rho A u_3(\omega_1^2 + \omega_2^2) + \rho A \omega_1 \omega_3 u_1 + \rho A \omega_2 \omega_3 u_2 + \rho A a_{03} - \rho A(\omega_2 v_{01} - \omega_1 v_{02}) + \rho A \alpha_1 u_2 - \rho A \alpha_2 u_1 - 2\rho A \omega_2 \partial u_1/\partial t + 2\rho A \omega_1 \partial u_2/\partial t + \rho A x(\omega_1 \omega_3 + \omega_2^2) - f_3(x, t) = 0, \quad (12)$$

where primes denote differentiation with respect to x. The effect of rotational inertia of the cross section and the influence of shear forces are neglected in Eqs. (11) and (12). We neglect these terms because their effects on the primary modes of vibration are small. Also, if we consider uniform cross-sectional links, then $A'(x) = I_y'(x) = I_y''(x) = I_z'(x) = I_z''(x) = 0$.

3 Effect of Lubrication Pressure Field on Prismatic Kinematic Pairs

As shown in Fig. 3 we consider a linear-elastic link on which a prismatic kinematic pair moves along the of axis the member. A lubrication film is located in the lower interface of the link and the slider joint. Our main focus here, is the effect of the pressure at the fluid interface on the fundamental vibration modes of the elastic member in the transverse direction ($u_2(x, t)$). Therefore, we further simplify the equations of motion by considering only the displacement $u_2(x, t)$. Then, carrying out the derivations in the previous section for this variable only yields

$$EI_z \partial^4 u_2/\partial x^4 + \rho A \partial^2 u_2/\partial t^2 + \omega^2 \rho A u_2 + \rho A a_{02} + \omega \rho A v_{01} + \alpha \rho A x - f_2(x, t) = 0, \quad (13)$$

where $\omega = \omega_3$. The term $f_2(x, t)$ represents the external force per unit length, which includes the fluid pressure and other external effects.

3.1 Reynolds Equation of Lubrication.

Now to develop the equations that represent the effect of fluid pressure, we define $\xi(X)$, $X \in [0, l]$ as the film thickness in the transverse direction. Using a simplified form of Reynolds equation for incompressible fluids, the differential equation for the pressure $p(X, Z, t)$ is given by (White, 1991)

$$\frac{\partial}{\partial X}\left(\xi^3 \frac{\partial p}{\partial X}\right) + \frac{\partial}{\partial Z}\left(\xi^3 \frac{\partial p}{\partial Z}\right) = 6\mu V_r(t) \frac{\partial \xi}{\partial X}, \quad (14)$$

where μ is the dynamic viscosity and $V_r(t)$ is the relative velocity between the link and the slider joint. Choosing the thickness of the translational joint as B (i.e. $Z \in [0, B]$) and assuming symmetry between the link and the joint in the cross-sectional plane, as shown in Fig. 3, we obtain the following relations:

$$p(X, B, t) = p(X, 0, t)$$

$$\frac{\partial p}{\partial Z}(X, B, t) = \frac{\partial p}{\partial Z}(X, 0, t).$$

We average the pressure in the transverse direction so the lubricant pressure distribution is reduced to a one dimensional problem introducing a variable $P(X, t)$ given as

$$P(X, t) = \frac{1}{B} \int_0^B p(X, Z, t) dZ. \quad (15)$$

Substituting Eq. (15) in (14) yields

$$\xi^3 \frac{\partial^2 P}{\partial X^2} + 3\xi^2 \frac{\partial \xi}{\partial X} \frac{\partial P}{\partial X} = 6\mu V_r(t) \frac{\partial \xi}{\partial X}. \quad (16)$$

Furthermore, using geometry we obtain

$$\xi = \frac{H}{2} - u_2(x^* + X, t).$$

The boundary conditions for $P(X, t)$ are given as

$$X = 0 \Rightarrow P(0, t) = p_{atm},$$

$$X = l \Rightarrow P(l, t) = p_{atm},$$

where p_{atm} is the atmospheric pressure.

The general solution for the Eq. (16) is

$$P(X, t) = 6\mu V_r(t) I_1(X, t) + C I_2(X, t) + C_1, \quad (17)$$

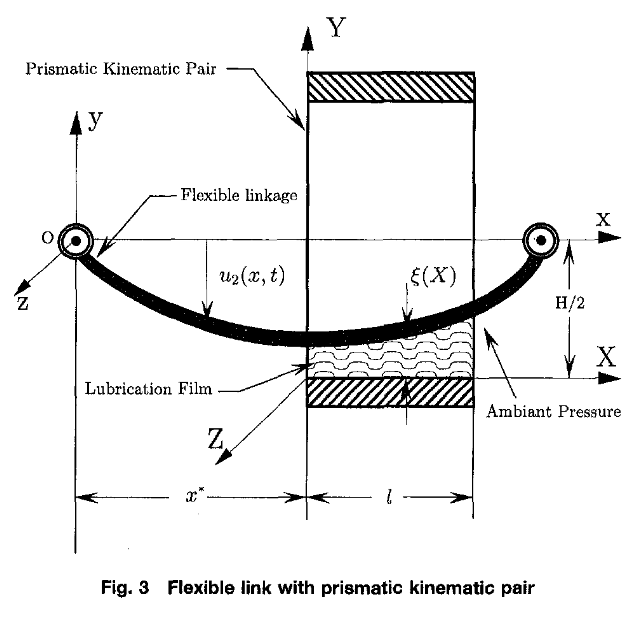

Fig. 3 Flexible link with prismatic kinematic pair

where

$$I_1(X, t) = \int dX/[H/2 - u_2(X + x^*)]]^2,$$

$$I_2(X, t) = \int dX/[H/2 - u_2(X + x^*)]]^3.$$

The constants C and C_1 can be computed from the boundary conditions. The integrals I_1 and I_2 are computed later by using Fourier transform methods. Here they are presented only in general forms.

4 Solution Method for an Elastic Link in a Kinematic Chain

The method of solution presented below assumes that the time profiles of rigid body accelerations, velocities, and the dynamic reaction forces at the joints are given. More specifically, the time functions in Eq. (13) v_{01}, a_{02}, ω, α, and f_2 are completely known at the onset of the computations.

The procedure followed to solve the equations of motion take the following steps:

(1) Use the rigid body joint velocities and accelerations, geometry of the mechanism, and the kinematic relations to compute the position $x^*(t)$ of the translational link on the elastic member, force and moment reactions in the translational joint.

(2) Transform Eq. (13) by applying finite Fourier sine transform which can be described by the following definition.
Consider a function $g(x)$ which satisfies Dirichlet's conditions in the interval $[0, L]$ where it has a finite number of maxima, minima, and discontinuities. The finite Fourier sine transform of the function $g(x)$ which is denoted by $g^*(n)$ is defined by the relation

$$g^*(n) = \int_0^L g(x) \sin (\beta_n x) dx,$$

where $\beta_n = n\pi/L$, $n = 1, 2, 3 \ldots$
Using the boundary conditions

$$\frac{\partial^2 u_2(0, t)}{\partial x^2} = \frac{\partial^2 u_2(L, t)}{\partial x^2} = u_2(0, t) = u_2(L, t) = 0,$$

and applying finite Fourier sine transform to Eq. (13) yields the following ordinary differential equation

$$\frac{d^2 u_2^*(n, t)}{dt^2} + \left[\frac{EI_z}{\rho A}\beta_n^4 + \omega^2(t)\right] u_2^*(n, t)$$

$$- \frac{f_2^*(u_2^*(n, t), n, t)}{\rho A} = - \frac{1}{\beta_n}\{[a_{02}(t)$$

$$+ \omega(t) v_{01}(t)][1 + (-1)^{(n+1)}]$$

$$+ \alpha(t)(-1)^{(n+1)} L\}. \quad (18)$$

The term $f_2^*(n, t)$ in Eq. (18) can be written as

$$f_2^*(u_2^*(n, t), n, t)$$

$$= R^*(n, t) + P^*(u_2^*(n, t), n, t), \quad (19)$$

where $R^*(n, t)$ is the finite Fourier sine transform of the joint reaction forces, which is a known function of time and the transform of the force per unit length in lubricant film $P^*(u_2^*(n, t), n, t)$. The pressure response to the nth harmonic of deflection can be approximated by the following expression

$$p^{(n)}(X, t) = 6\mu V_r(t) I_1^{(n)} + C I_2^{(n)} + C_1,$$

where

$$I_1^{(n)} = -\frac{L}{n\pi}(H\gamma_3 - \gamma_4),$$

$$I_2^{(n)} = -\frac{L}{n\pi}\left\{\gamma_3\left[\frac{H^2}{2} + \phi^2(n, t)\right] + \left[\frac{1}{\gamma_2} + \frac{3H}{2\gamma_1}\right]\right\},$$

$$\gamma_1 = \frac{H^2}{4} - \phi(n, t)^2,$$

$$\gamma_2 = -\frac{H}{2} + \phi(n, t) \sin\left(\frac{n\pi}{L} x\right),$$

$$\gamma_3 = \gamma_1^{-3/2} \tan^{-1}\left[\frac{\phi(1, t) - \frac{H}{2}\tan\left(\frac{n\pi}{L} x\right)}{\sqrt{\gamma_1}}\right],$$

$$\gamma_4 = \frac{\phi(n, t)}{2\gamma_1\gamma_2}\cos\left(\frac{n\pi}{L} x\right),$$

$$\phi(n, t) = \frac{2}{L} u_2^*(n, t).$$

Accordingly, the finite Fourier sine transform of the force per unit length can be written as

$$P^*(u_2^*(n, t), n, t) = B \int_{x^*}^{x^*+l} [6\mu V_r(t) I_1^{(n)}$$

$$+ C I_2^{(n)} + C_1] \sin (\beta_n x) dx. \quad (20)$$

(3) Obtain the numerical solution of the nonlinear ordinary differential equation given by Eq. (18) by using an IMSL library. Application of the differential solver also requires the numerical integration of Eq. (20) for given values of t and $u_2^*(n, t)$. This is realized by using an algorithm that is based on the trapezoidal rule with automatic step adjustment for accuracy.

(4) The final solution for the equation of motion is obtained by inverting finite Fourier sine transform and applying the superposition principle for the harmonics.

$$u_2(x, t) = \sum_{n=1}^{n=\infty} \phi(n, t) \sin (\beta_n x).$$

5 Impact Equations

Two possible scenarios for the prismatic kinematic pair and the flexible link are

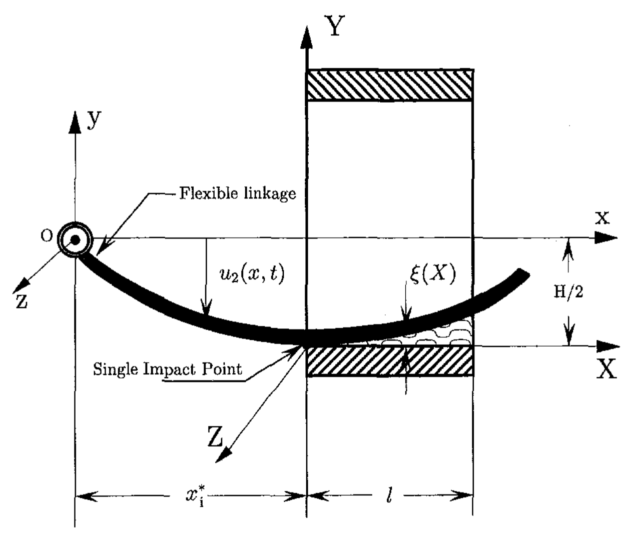

Fig. 4 The impact between the elastic link and the prismatic kinematic pair

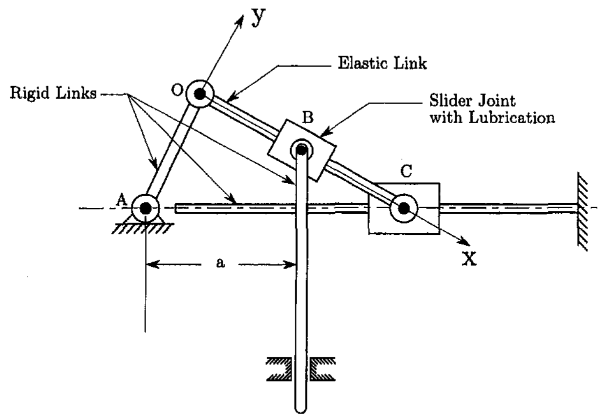

Fig. 5 Planar, slider-crank mechanism

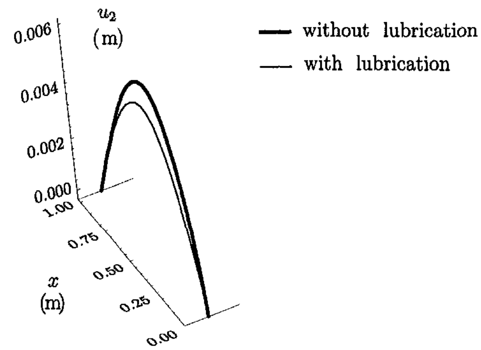

Fig. 7 Deflection of the flexible link with and without lubrication

(a) No contact (Fig. 3).
(b) Impact on a single point (Fig. 4).

The conditions for switching from one case to the other depend on the dynamics. There were additional difficulties because of the small space and time scales in which losses of contact, impacts, and so forth take place. It was necessary, therefore, to use very small time steps to capture very precisely the time of occurrence of such events.

A basic assumption was that the configuration of the link and the slider was held constant in the analysis of the collision process, with no significant change in mass and moments of inertia.

Let $\mathbf{v}_l^-$ and $\mathbf{v}_l^+$ be the velocities of the contact point of the elastic link before and after impact, respectively. Let $\mathbf{v}_s^-$ and $\mathbf{v}_s^+$ be the velocities of the contact point of the rigid slider before and after impact, respectively. With these notations, one can write

$$\mathbf{v}_s^+ - \mathbf{v}_l^+ = e(\mathbf{v}_l^- - \mathbf{v}_s^-), \tag{21}$$

where e is the coefficient of restitution. Solving the impact equations (Marghitu and Hurmuzlu, 1996) the unknown velocities after impact are determined.

6 Application

In this section we apply the method proposed above to investigate the vibration of the link OC of the slider mechanism presented in Fig. 5. To simplify our presentation, we consider the flexible effects in member OC only.

We consider the motion of the system when the link OA has a constant angular velocity ω_0. Then the kinematic variables of link OC can be written as

$$v_{O1} = -OA\omega_0 \sin(\omega_0 t)\cos\theta - OA\omega_0\cos(\omega_0 t)\sin\theta,$$

$$a_{O2} = -OA\omega_0^2 \sin\theta\cos(\omega_0 t) - OA\omega_0^2\cos\theta\sin(\omega_0 t),$$

$$\omega = -\frac{\lambda\omega_0\cos(\omega_0 t)}{[1-\lambda^2\sin(\omega_0 t)^2]^{1/2}},$$

$$\alpha = -\frac{\lambda^3\omega_0^2\cos(\omega_0 t)^2\sin(\omega_0 t)}{[1-\lambda^2\sin(\omega_0 t)^2)]^{3/2}} + \frac{\lambda\omega_0^2\sin(\omega_0 t)}{[1-\lambda^2\sin(\omega_0 t)^2]^{1/2}},$$

where

$$\lambda = OA/L,$$

$$\theta = \sin^{-1}[\lambda\sin(\omega_0 t)],$$

where $OA = 0.07$ m and $OC = 1.0$ m are the lengths of the members OA and OC respectively, and the distance between joint A and the vertical sliding member is selected as $a = 0.4$ m (see Fig. 4). The elastic link is made of carbon steel ($E = 207 \times 10^9$ N/m^2, $\rho = 7650$ kg/m^3) with equal cross-sectional width and height, i.e. $B = h$.

Throughout the succeeding analysis, only the fundamental mode of transverse oscillations (i.e. $n = 1$) is taken into consideration for our presentation. Simulations involving three, five and seven modes were performed and no perceptible difference was found with respect to the dynamic behavior. Thus one mode is considered adequate for accurately describing the elastic motion for the simulation reported in this paper.

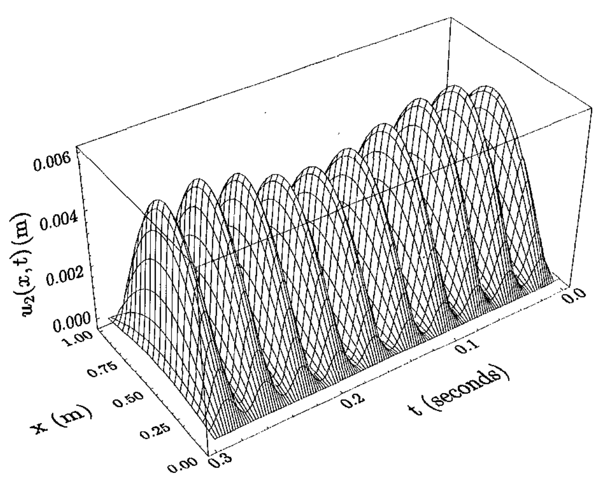

Fig. 6 Deflection of the flexible link in time and space

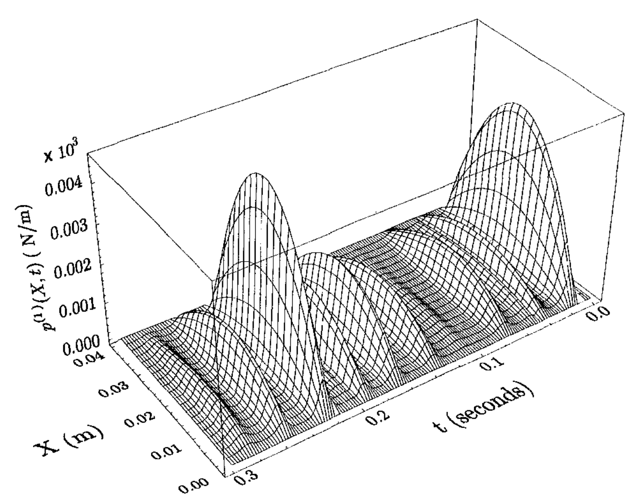

Fig. 8 Pressure response in the lubrication film

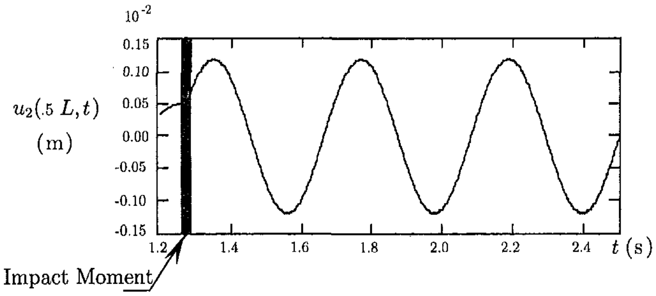

Fig. 9 The midpoint deflection of the link for the impact with the prismatic kinematic pair

Figure 6 depicts the time-space curves of the beam deflection for h = 0.01 m, ω_0 = 10π rad/s and zero initial conditions. The deflection surface depicted in Fig. 6 demonstrates a recurring pattern which seems to be synchronous with the motion of the crank OA. Actually, the overall motion is periodic. The transverse oscillation of the elastic link is influenced by the uniform motion of link OA, by the joint reaction and by the lubricant film pressure. Next, we focused on the difference among computer simulated data, with and without the effect of the lubricated joint. We observed that, for the same moment the deflection of the flexible link without lubrication is greater than the case with lubrication (Fig. 7).

Figure 8 depicts the time-space curves of the pressure response for the same characteristics of the planar chain. The pressure field exerted through the oil, which is considered as viscous and Newtonian, is obtained from the solution of the Reynolds equation of lubrication. The boundary conditions result in ambient pressure at the edges of the prismatic joint. In order to support the load exerted by the beam, the bearing-film pressure must rise above the ambient. The pressure in the prismatic joint (i.e. plane slider bearing) is a function of the local film thickness, which in our case is dependent on the deflection of the beam.

An interesting phenomena occurs during the motion of the present system when the deflection of the beam is as large as the height of the prismatic joint. The lubricant film breaks down, leading to direct collision between the beam and the prismatic joint. Figure 9 represents the deflection of the midpoint of the beam. The impact moment is at time t = 1.273 s and the elastic displacement $u(L/2, t)$ has a jump for e = 0.7.

7 Conclusions

Vibrations of deformable bodies undergoing general rigid body motions can be studied by continuous or discrete models. Continuous models are accurate and provide an effective way to predict the system response.

The approach has been applied to modal vibrations of an elastic link in a slider-crank mechanism subject to the effect of pressure in the lubricant film of a prismatic joint attached to the member. A numerical example that considers the first mode of vibration is presented.

Furthermore, experimental tests are needed in order to further generalize the results reported in the present article.

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