COMBINATORIAL AUCTIONS WITH TRANSPORTATION COST

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Abstract

The ability to discover equilibrium prices efficiently makes auctions an effective way to trade goods. One of the recent trends in the development of auctions is combinatorial auctions. Combinatorial auctions allow the simultaneous sale of more than one item. In the existing literature, the factor of transportation cost has not been considered in combinatorial auctions. In this paper, we formulate the combinatorial double auction problem and propose an algorithm for finding approximate solutions. The algorithm is developed by applying the subgradient algorithm to iteratively adjust the shadow prices and proposing a heuristic algorithm for finding approximate solutions. A numerical example is used to illustrate the preliminary result.

Keywords: Auction, e-Commerce, Decision Support System.

1. INTRODUCTION

The ability to discover equilibrium prices efficiently makes auctions an effective way to trade goods (Abrache, et al. 2004; Ba et al. 2001; Block et al. 2008; Catalán et al. 2009), Choi (2008), Fan et al. (1999). One of the recent trends in the development of auctions is combinatorial auctions (Li et al. 2009;Li et al. 2009;Li et al. 2009; Meeus et al. 2009;Nicolaisen, et al. 2001;Özer et al. 2009;Perugini et al. 2005;Polyak 1969;Rothkopf et al. 1998; Schellhorn et al. 2009; Wang et al. 2004;Xia et al. 2005;Yang, et al. 2009. In a combinatorial auction, a bidder can place a bid on a bundle of items with a price. An important research subject in combinatorial auctions is the winner determination problem (WDP), which aims to determine the winners that maximize the seller's revenue based on the set of bids placed. An excellent survey on combinatorial auctions can be found in (de Vries & Vohra 2003; Pekeč & Rothkopf 2003). In the existing studies, the factor of transportation cost is rarely considered in combinatorial auctions with the exception of (Chen et al. 2005). In this paper, we will study the combinatorial auction problem with transportation cost. The problem we study in this paper considers the transportation cost for delivering the items to the winning buyers. The transportation cost depends on the item to be delivered.

Combinatorial auctions are notoriously difficult to solve from a computational point of view (Hsieh, & Tsai, 2008; Leskelä et al. 2007) due to the exponential growth of the number of combinations. The WDP can be modelled as a set packing problem (SPP) (Andersson et al. 2000; Fujishima et al. 1999;, Hoos & Boutilier 2007; Vemuganti 1998; Xia, et al. 2005). Sandholm et al. mentions that WDP for combinatorial auction is NP-complete (Sandholm, 1999, 2000, 2002). Many centralized algorithms have been developed for WDP (Sandholm, 2002; Andersson et al. 2000; Gonen & Lehmann 2000; Jones & Koehler 2002; Guo et al. 2005; Hsieh 2007; Hsieh & Tsai 2008; Hsieh 2010; Hsieh & Lin; Hsieh & Huang 2010; Yang et al. 2009). In this paper, we will propose a subgradient algorithm for solving WDP. We assume all the players tell truth. The WDP for combinatorial auction can be modelled as an integer programming problem.

Many problems in the real world that can be formulated as integer programming problems are notoriously difficult to solve. Motivated by the complexity and distributed nature of these problems, one approach is to model the overall system as a collection of simpler interacting components or agents (Gordon 2007). Under such a system architecture, the decision making process for any single component in the system is dictated by an optimization problem that is greatly simplified as compared to the centralized problem, but coupled to the decisions of other interconnected components. An important issue is the design, analysis and implementation of solution algorithms.

Instead of finding the exact solution, we will set up a fictitious market based on multi-agent system architecture and develop a subgradient algorithm to determine the winning bids in the fictitious market to reduce the computational complexity in solving WDP. In the fictitious market, each buyer and the seller is represented by an entity. The issue is to develop solution algorithms for all the buyers and the seller in the system to collectively solve the WDP for combinatorial auctions. In this paper, we adopt a Lagrangian relaxation approach (Fisher, 1981) in conjunction with a subgradient algorithm (Polyak, 1969) to develop a solution algorithm for finding approximate solutions. Based on the proposed algorithms, we demonstrate the effectiveness of our method by numerical examples.

The remainder of this paper is organized as follows. We first we describe and formulate the WDP for combinatorial auctions with transportation cost in Section 2 and Section 3, respectively. We develop a subgradient algorithm for solving WDP in Section 4. In Section 5, we present our numerical results. We conclude this paper in Section 6.

2. COMBINATORIAL AUCTION WITH TRANSPORTATION COST

In this paper, we first formulate the combinatorial auction problem as an integer programming problem. We assume that there are a set of buyers and a seller for trading the goods. In a combinatorial auction, buyers submit bids to the seller. The surplus of a combinatorial auction is the difference between buyers' total payment and sellers' total revenue.

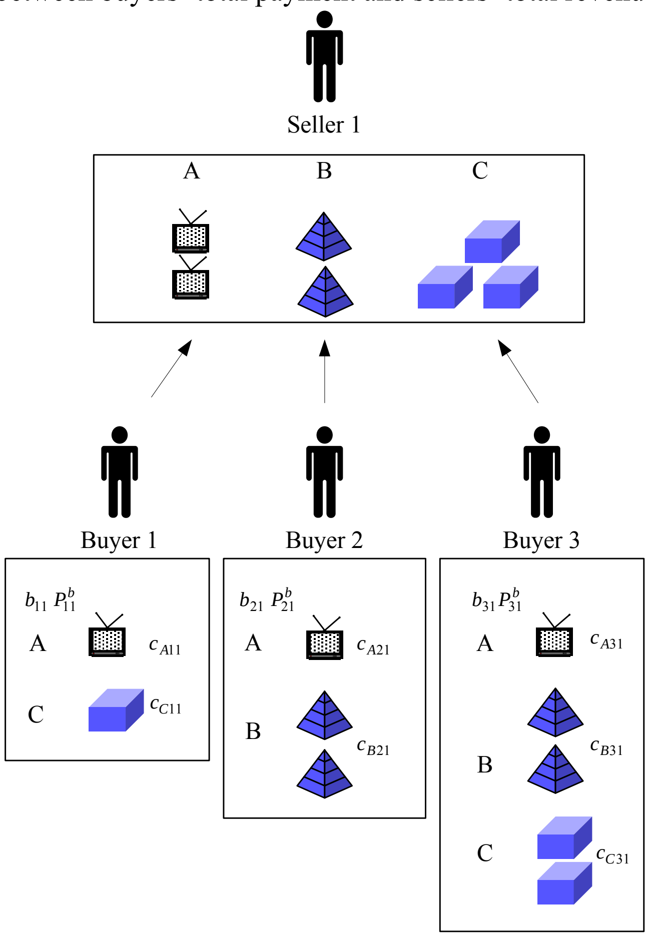

Figure 1. A combinatorial auction with transportation cost.

A combinatorial auction problem with transportation cost can be modeled as an optimization problem that maximizes the surplus of the seller. The surplus of a combinatorial auction is the difference between buyers' total payment minus the transportation cost and the price of the bundle corresponding to the bid of the seller. We assume that there are a set of buyers and a seller for trading the goods. Fig. 1 illustrates an application scenario in which Seller1 has a number of available items. Assume the available items of Seller 1 are 2A, 2B and 3C. But the total items in the winning bids of the buyers cannot exceed the available items of the seller. Buyer 1 requests to purchase at least a bundle of items 1A and 1C from the market at price P_{11}^b. Buyer 2 requests to purchase at least a bundle of items 1A and 2B from the market at price P_{21}^b. Buyer 3 requests to purchase at least a bundle of items 1A, 2B and 2C from the market at price P_{31}^b. The bids submitted by Buyer 1, Buyer 2 and Buyer 3 are represented by b_{11} =(1A, 0B, 1C, P_{11}^b), b_{21} =(1A, 2B, 0C, P_{21}^b) and b_{31} =(1A, 2B, 2C, P_{31}^b), respectively. Delivery of the items from the seller to the winning buyer(s) incurs transportation cost. Suppose the transportation cost to deliver one unit of item A and one unit of item C from Seller1 to Buyer 1 is c_{A11} and c_{C11}, respectively. Suppose the transportation cost to deliver one unit of item A and one unit of item B from

Seller1 to Buyer 2 is c_{A21} and c_{B21}, respectively. Similarly, suppose the transportation cost to deliver one unit of item A, one unit of item B and one unit of item C from Seller1 to Buyer 3 is c_{A31}, c_{B31} and c_{C31}, respectively. Suppose P_{11}^b=120, P_{21}^b=160, P_{31}^b=380, c_{A11}=10, c_{C11}=10, c_{A21}=10, c_{B21}=5, c_{A31}=30, c_{B31}=30, c_{C31}=30. For this example, the solution (winning bids) for this combinatorial auction problem with transportation cost includes Buyer1: b_{11}=(2A, 2B, 0C, P_{12}^s) and Buyer 2: b_{21}=(1A, 2B, 0C, P_{21}^b).

3. PROBLEM FORMULATION

To formulate the problem, let's define the notations in this paper.

Notations:

K : the number of items requested.

i : a seller.

N : the number of potential buyers in a combinatorial auction. Each $n \in \{1,2,3,....,N\}$ represents a buyer.

d_{nhk} : the buyer- n 's desired units of the $k-th$ items in the $h-th$ request for tender, where $k \in \{1,2,3,....,K\}$.

j : the $j-th$ bid submitted by the seller in a combinatorial auction.

h : the $h-th$ bid created by the buyer in a combinatorial auction to represent the requirement of the buyer i.

q_{ijk} is a nonnegative integer that denotes the quantity of the $k-th$ items in the bid submitted by the seller i.

$s_{ij} = (q_{ij1}, q_{ij2}, q_{ij3},..., q_{ijK})$: a vector to represent the bid submitted by seller i.

x_{ij} : the variable to indicate the bid j placed by seller i is a winning bid (x_{ij} =1) or not (x_{ij} =0).

p_{ij} is a real positive number that denotes the price of the bundle corresponding to the $j-th$ bid of the seller.

p_{nh} is a real positive number that denotes the price of the bundle corresponding to the bid h submitted by buyer n.

$b_{nh} = (d_{nh1}, d_{nh2}, d_{nh3},..., d_{nhK}, p_{nh})$: a vector to represent the bid h submitted by buyer n. The bid b_{nh} is actually an offer to deliver d_{nhk} units of items for each $k \in \{1,2,3,....,K\}$ a total price of p_{nh}.

y_{nh} : the variable to indicate the bid h placed by seller n is active (y_{nh} =1) or inactive (y_{nh} =0).

H_n : the number of bids placed by buyer $n \in \{1,2,3,....,N\}$.

c_{ijknh} : the transportation cost for delivering one unit of item $k \in \{1,2,3,....,K\}$ to fulfill the bid h of buyer n from the bid j of seller i

f_{ijknh} : the units of item $k \in \{1,2,3,....,K\}$ that flows from the bid j of seller i to bid h of buyer n

The winner determination problem is formulated as an Integer Programming problem as follows.

Winner Determination Problem (WDP):

$$\max\left[\left(\sum_{n=1}^{N}\sum_{h=1}^{H} y_{nh}p_{nh}\right) - x_{ij}p_{ij}\right] - \left[\sum_{n=1}^{N}\sum_{h=1}^{H} f_{ijknh}c_{ijknh}\right] \qquad (2-1)$$

s.t.

$$\left[\left(\sum_{n=1}^{N}\sum_{h=1}^{H} y_{nh}p_{nh}\right) - x_{ij}p_{ij}\right] - \sum_{n=1}^{N}\sum_{h=1}^{H} c_{ijknh}f_{ijknh} \geq 0 \qquad (2-2)$$

$$x_{ij}q_{ijk} \leq s_{ik} \quad \forall i \in \{1,...,I\}, k \in \{1,...,K\} \qquad (2-3)$$

$$\sum_{n=1}^{N}\sum_{h=1}^{H} f_{ijknh} \leq x_{ij}q_{ijk} \qquad (2-4)$$

$$\forall k \in \{1,...,K\}$$

$$f_{ijknh} \geq y_{nh}d_{nhk} \qquad (2-5)$$

$$\forall n \in \{1,...,N\}, h \in \{1,2,...,H\}, k \in \{1,...,K\}$$

$$f_{ijknh} \leq x_{ij}q_{ijk} \qquad (2-6)$$

$$\forall n \in \{1,...,N\}, h \in \{1,2,...,H\},$$

$$\forall k \in \{1,...,K\}$$

$$f_{ijknh} \leq y_{nh}d_{nhk} \qquad (2-7)$$

$$\forall n \in \{1,...,N\}, h \in \{1,2,...,H\}, k \in \{1,...,K\}$$

$$f_{ijknh} \in \mathrm{N} \cup \{0\}$$

$$x_{ij} \in \{0,1\}$$

$$y_{nh} \in \{0,1\} \quad \forall n, h$$

4. SOLVING WDP BASED ON A SUBGRADIENT ALGORITHM

The development of our subgradient algorithm is detailed in this section. The proposed subgradient algorithm consist of (i) an algorithm for each buyer to make decision according to the fictitious price announced by the seller and (ii) an algorithm for the seller to update the fictitious price iteratively based on the decisions of buyers.

In WDP problem, we observe that the coupling among the decision variables caused by the non-negative trade surplus constraints (2-2) and inventory constraints (2-3). Let λ denote the vector with λ_k representing the Lagrangian multiplier for the $k-th$ items, π be the vector with π_{ik} representing the Lagrangian multiplier for the $k-th$ items of the i 's seller and μ denote the Lagrangian multiplier for the non-negative trade surplus constraint. We define

$$L(\mu,\pi) =$$

$$\max\left[\left(\sum_{n=1}^{N}\sum_{h=1}^{H} y_{nh}p_{nh}\right) - x_{ij}p_{ij}\right] - \left[\sum_{n=1}^{N}\sum_{h=1}^{H}\sum_{k=1}^{K} c_{ijknh}f_{ijknh}\right] +$$

$$\mu\left\{\left[\left(\sum_{n=1}^{N}\sum_{h=1}^{H} y_{nh}p_{nh}\right) - x_{ij}p_{ij}\right] - \sum_{n=1}^{N}\sum_{h=1}^{H}\sum_{k=1}^{K} c_{ijknh}f_{ijknh} + \sum_{k=1}^{K} \pi_{ik}(x_{ij}q_{ijk} - s_{ik})\right.$$

$$s.t.\ (2-4), (2-5), (2-6), (2-7)$$

$$= \max\left\{\left(\sum_{n=1}^{N}\sum_{h=1}^{H} w_{nh}y_{nh}\right) - \left(z_{ij}x_{ij} + \sum_{k=1}^{K} \pi_{ik}q_{ijk}\right)\right.$$

$$\left. - \left[\sum_{n=1}^{N}\sum_{h=1}^{H} (1+\mu)c_{ijknh}f_{ijknh}\right] + \sum_{k=1}^{K} \pi_{ik}s_{ik}\right\}$$

$s.t.\ (2-4),(2-5),(2-6),(2-7),$

where $w_{nh} = (1+\mu)p_{nh}$ and $z_{ij} = (1+\mu)p_{ij} + \sum_{k=1}^{K} \pi_{ik} q_{ijk}$.

For given Lagrangian multipliers μ and π, it is relatively easy to find x, y and f by applying a generalized network flow algorithm such as the one supported in (CPLEX Optimizer 2014).

Let l be the iteration index. Let x^l, y^l and f^l denote the optimal solution to the dual subproblem for given Lagrange multipliers μ^l and π^l at iteration l. We define the subgradients of $\min_{\mu\ge 0,\pi\ge 0} L(\mu,\pi)$ with respect to Lagrangian multipliers μ and π_{ik} as follows.

$$g_1^l = 0 - \left[\left(\sum_{n=1}^{N}\sum_{h=1}^{H} y_{nh}p_{nh}\right) - x_{ij}p_{ij}\right] - \sum_{n=1}^{N}\sum_{h=1}^{H}\sum_{k=1}^{K} f_{ijknh}c_{ijknh}$$

$$g_2^l(i,k) = x_{ij}q_{ijk} - s_{ik},\ where\ k \in \{1,...,k\}$$

$$\mu^{l+1} = \begin{cases} \mu^l + \alpha_1^l g_1^l \ if\ \mu^l + \alpha_1^l g_1^l \ge 0 \\ 0\ otherwise \end{cases}$$

$$\pi^{l+1}(i,k) = \begin{cases} \pi^l(i,k) + \alpha_2^l g_1^l(i,k)\ if\ \pi^l(i,k) + \alpha_2^l g_2^l(i,k) \ge 0 \\ 0\ otherwise \end{cases}$$

$$\alpha_1^l = c\frac{L(\mu,\pi) - \overline{L}}{\left(g_1^l\right)^2 + \sum_{i=1}^{I}\sum_{k=1}^{K}(g_2^l(i,k))^2}, 0 \le c \le 2$$

$$\alpha_2^l = c\frac{L(\mu,\pi) - \overline{L}}{\left(g_1^l\right)^2 + \sum_{i=1}^{I}\sum_{k=1}^{K}(g_2^l(i,k))^2}, 0 \le c \le 2$$

5. NUMERICAL RESULTS

Based on the proposed algorithms for combinatorial auctions, we design and implement a software system based on J2EE platform to verify the effectiveness of our solution algorithm. We conduct a numerical experiment to illustrate the effectiveness of our method to demonstrate the effectiveness of the algorithm proposed in this paper.

Example: Suppose Seller 1 has a bundle of available items to be sold through combinatorial auctions. The number of available items is shown in Table 1 as follows. Suppose Seller 1 holds one combinatorial auction for his bundle available items. Six potential buyers place bids on the combinatorial auction. The bids placed by the potential buyers (Buyer 1~Buyer 6) are shown in Table 2. The transportation cost is shown in Table 3.

Seller	**Seller ID**	k				
i	j	1	2	3	4	5
1	1	5	2	9	1	8

Table 1. Available Items of Seller.

Buyer	Bid ID	k					price
n	h	1	2	3	4	5	p_{nh}
1	1	0	0	9	1	0	100
2	1	5	1	0	0	3	200
3	1	0	1	0	0	5	150
4	1	0	1	0	0	5	44
5	1	0	0	6	1	0	30
6	1	0	0	3	0	0	30

Table 2. Buyers' Bids.

Buyer	Bid ID	c_{ijknh}				
n	h	k=1	k=2	k=3	k=4	k=5
1	1	1	1	1	1	1
2	1	2	2	2	2	2
3	1	1	1	1	1	1
4	1	3	3	3	3	3
5	1	5	5	5	5	5
6	1	4	4	4	4	4

Table 3. Transportation Cost.

For this example, the values of the decision variables generated by applying our algorithms are listed in Table 4 and Table 5, respectively. The results are obtained by initializing all Lagrange multipliers μ^0 and π^0 with zero at iteration 0. The algorithm applied by the seller and the buyers converges to the solution with $x_{ij} = 1$ and y_{nh} as well as f_{ijknh} shown in Table 4 and Table 5, respectively. The value of the objective function is 397. Note that this solution is the optimal solution for example.

Buyer	Bid ID	
n	h	y_{nh}
1	1	1
2	1	1
3	1	1
4	1	0
5	1	0
6	1	0

Table 4. Decision Variables y_{nh}.

Buyer	Bid ID	f_{ijknh}				
n	h	k=1	k=2	k=3	k=4	k=5

1	1	0	0	9	1	0
2	1	5	1	0	0	3
3	1	0	1	0	0	5
4	1	0	0	0	0	0
5	1	0	0	0	0	0
6	1	0	0	0	0	0

Table 5. Decision Variables f_{ijknh} .

6. CONCLUSION

We formulate the winner determination optimization problem for combinatorial auctions with transportation cost as an integer programming problem. The surplus of the problem consists of the difference between the price offered by the wining buyers and the reserve price of the seller minus the transportation cost. The problem is to determine the winners to maximize the total surplus of the seller. Due to computational complexity, it is hard to develop a computationally efficient method to find an exact optimal solution for combinatorial auctions with transportation cost based on centralized computing architecture. To reduce computational complexity, an alternative way to find a solution for combinatorial auctions is to set up a fictitious market and develop an algorithm to determine the winning bids in the fictitious market. In this paper, we propose a solution algorithm by combining Lagrangian relaxation and a subgradient method to find a solution for the fictitious market. In our solution algorithm, we consider two kinds of fictitious prices, which are associated with the shadow prices corresponding to non-negative trade surplus constraint, and inventory constraints. We conduct experiments to study our proposed algorithm. Although our algorithm does not guarantee generation of optimal solutions, it often leads to optimal or near optimal solutions.

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