Color Television Market Penetration Dynamics

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A mathematical model describing the dynamic relationship between value, cost, and market penetration is formulated and tested against 1954 to 1971 color television historical data. The model contains three parameters whose values are estimated from color television ownership data available as a function of time and consumer income. Although the basic model constitutes a deterministic theory, stochastic effects are considered heuristically and forecasts to 1980 are presented together with uncertainty bounds.

Introduction

Market penetration forecasts are essential inputs to business planning. A wide variety of forecasting techniques have therefore been developed. For example, [1][1] and [2] are excellent references and contain bibliographies citing well over 200 sources. Much of forecasting methodology relies heavily on techniques developed for modeling stationary processes. For such processes, it is possible to draw useful inferences from analysis of historical data without direct reference to underlying, causative mechanisms. However, it becomes necessary to consider these causative mechanisms for

- Long-range forecasting where the assumption of stationarity becomes questionable
- Forecasts of demand for new goods or services
- Forecasting the effect of a change in design or cost.

Recently a theory relating market penetration to value and cost forcing functions has been proposed by Katolay [3]. In order to test this theory and to identify realistic value and cost functions, a study of the growth history of color television ownership in the United States was performed. Within the general framework of Katolay's approach, it was found possible to formulate a mathematical model for CTV (color television) market penetration dynamics. The structure of this model, its ability to explain available data, its application to forecasting, and the effect of external factors (e.g., advertising, black-and-white TV aging, etc.) are discussed herein.

Model Formulation

Consider a market, M, consisting of $M(t)$ members at time t. Let

x = income in \$/year

$X(t)$ = median income for M at t

ξ = normalized income, defined as $x/X(t)$

m = that subset of M having normalized income less than ξ at time t and containing $m(\xi, t)$ members

s = that subset of m with color television at t and numbering $s(\xi, t)$ members

We shall assume that specification of ξ is sufficient for purposes of determining the conditional probability of purchasing a color television given the state of the market and its environment at time t.

Within an infinitesimal income group of size $(\partial m/\partial \xi)d\xi$,[2] the size of that subset having color television is just $(\partial s/\partial \xi)d\xi$. Assuming that the rate at which color television ownership increases within this income group is proportional to the number of its members not already having color television

$$\frac{\partial}{\partial t}\left(\frac{\partial s}{\partial \xi}\right) d\xi = \lambda(\xi, t)\left(\frac{\partial m}{\partial \xi} - \frac{\partial s}{\partial \xi}\right)d\xi \tag{1}$$

where λ is the transition rate coefficient of proportionality. Introducing

$$p(\xi, t) = \frac{\partial s/\partial \xi}{\partial m/\partial \xi}$$

which is the fractional penetration of color television ownership within income group ξ at time t, equation (1) may be rewritten as

$$\frac{\partial p}{\partial t} = \lambda(\xi, t)(1 - p) - \frac{p\,\dfrac{\partial^2 m}{\partial t \partial \xi}}{(\partial m/\partial \xi)} \tag{2}$$

Analysis of U. S. income data [4] indicates that over the past twenty years the distribution of normalized income has been approximately time-invariant. (See Fig. 3.) This implies that

$$m(\xi, t) = \int_0^{\xi} M(t)D(\xi)d\xi \tag{3}$$

where $D(\xi)$ is the normalized income distribution density function. (See Fig. 4.) Therefore

[1]Numbers in brackets designate References at end of paper.
[2]It is assumed that $m(\xi,t)$ and $s(\xi,t)$ may be approximated by sufficiently differentiable functions of ξ and t.

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$$\left(\frac{\partial^2 m}{\partial t \partial \xi} \right) / (\partial m / \partial \xi) = \frac{M'(t)}{M(t)} . \tag{4}$$

Growth in the household population [5] of the United States has been very nearly exponential over the same period (see Fig. 5). Therefore, using the total U. S. household population as the potential market for color television

$$M(t) \sim M_0 e^{t/\tau}. \tag{5}$$

and

$$\frac{M'(t)}{M(t)} \simeq \frac{1}{\tau} = \frac{1}{57 \text{ yr}} . \tag{6}$$

Using these approximations, equation (2) becomes

$$\frac{\partial p}{\partial t} = \lambda(\xi, t)(1 - p) - \frac{p}{\tau} . \tag{7}$$

Once $p(\xi, t)$ has been determined, the fraction, $P(t)$, of the total market having color television is given by

$$P(t) = \int_0^\infty p(\xi, t) D(\xi) d\xi. \tag{8}$$

Transition Model. So far, a certain viewpoint has been developed and appropriate terminology has been defined. We are now ready to address the central issue of the model formulation—identification of a realistic model for the transition rate coefficient $\lambda(\xi, t)$. We shall assume that the transition rate is related to the value, $v(t)$, of the given product or service relative to its environment at time t and to the probability, $\Omega(\xi, t)$, that, given such a value, a member of income group ξ would be willing to pay for it. Since one would expect that $\lambda = 0$ if either $v = 0$ or $\Omega = 0$ we shall assume that

$$\lambda(\xi, t) = \kappa v(t) \Omega(\xi, t). \tag{9}$$

Here κ is a scale factor determined by external environmental factors—i.e., by those factors which influence the transition rate but are not explicitly included in v or Ω. At this point, we shall assume that κ is simply a constant. Later when the effect of stochastic environment is considered, it will be useful to think of κ as a random variable.

Value Function. The value function we shall use for color television is the fraction of network programming broadcast in color at time t. This value function represents a compromise between conflicting objectives of realism and simplicity. During the study, more elaborate functions modeling the effects of prestige value, peer group contagion, value recognition time lags, and an income-dependent value assessment process were considered. The slightly better agreement with available data thus produced did not, however, justify the presence of additional model parameters.

Cost Function. Consumer willingness to pay a given price for a product of specified value is here postulated to vary lognormally with consumer income. Specifically, we shall assume that the cost function, $\Omega(\xi, t)$, varies lognormally with the ratio of normalized income ξ to normalized cost $c(t)/A(t)$ where

$c(t)$ = the lowest[3] retail price of a color set at time t

$A(t)$ = the median annual household expenditure for durable goods. Furthermore it is assumed that Ω will be zero if available household funds are less than the cost of a color set. That is

$$\Omega(\xi, t) = \begin{cases} 0 & \xi \leq \dfrac{c(t)}{A(t)} \\ \displaystyle\int_0^{\frac{\xi A(t)}{c(t)} - 1} \frac{\exp\left\{\dfrac{-[\ln(\eta) - \mu]^2}{2\sigma^2}\right\}}{\eta \sigma \sqrt{2\pi}} d\eta & \xi > \dfrac{c(t)}{A(t)} \end{cases} \tag{10}$$

Here μ and σ^2 are the mean and variance of the normal (Gaussian) distribution associated with $\ln \left(\dfrac{\xi}{c(t)/A(t)} - 1 \right)$.

Model Recapitulation. A moderately realistic model describing the color television market penetration process has now been formulated. With the total household population of the United States as the potential market the following notation has been introduced:

$M(t)$ = number of U. S. households at time t

x = annual household income

$X(t)$ = median value of x for year t

ξ = $x/X(t)$ = normalized income

$D(\xi)$ = normalized income distribution density function

$p(\xi, t)$ = fractional penetration of color television ownership for income group ξ at time t.

$P(t)$ = fraction of U. S. households with color television at time t

$v(t)$ = fraction of network programming broadcast in color at time t

$A(t)$ = median annual household expenditure for durable goods at time t

$c(t)$ = lowest retail price of a color set a time t

It is then argued that color television penetration of homogeneous income group segments of this market is described by

$$\frac{\partial p}{\partial t} = \kappa(1 - p) v(t) \left[\int_0^{\frac{\xi A(t)}{c(t)} - 1} \frac{\exp\left\{\dfrac{-(\ln\eta - \mu)^2}{2\sigma^2}\right\} d\eta}{\eta \sigma \sqrt{2\pi}} \right] - p/\tau \tag{11}$$

$$\text{for } \xi > \frac{c(t)}{A(t)}$$

and

$$p \equiv 0 \text{ for } \xi \leq \frac{c(t)}{A(t)}$$

Penetration of the total market is given by

$$P(t) = \int_0^\infty p(\xi, t) D(\xi) d\xi. \tag{8}$$

Solution

Equation (11) may be regarded as a first order, linear, ordinary differential equation with ξ as a parameter. Its solution may be written as

$$p(\xi, t) = p(\xi, 0) e^{-\int_0^t \left[\lambda(\xi, \eta) + \frac{1}{\tau} \right] d\eta} + \int_0^t \lambda e^{-\int_\eta^t \left[\lambda(\xi, z) + \frac{1}{\tau} \right] dz} d\eta \tag{12}$$

[3] It is asserted that lowest price should be a better measure than "average price actually paid" for two reasons. First, the lowest-price color sets of different years tend to have more nearly constant screen size than the average-price sets, and second, the initial decision to visit a color television dealer may be more closely related to the lowest advertised price than to the price finally paid.

The behavior of $p(\xi, t)$ for selected values of ξ and parameter and forcing function values corresponding to color television history is illustrated in Fig. 8.

Equations (8) and (12) together with specification of three parameters (κ, μ, σ); an initial condition $p(\xi, 0)$; and data concerning costs, household income, expenditures for durable goods, and hours of color programming define a unique realization of the theoretical CTV market penetration process. We now turn our attention to an examination of available data as a test for the validity of the model and as a means of estimating its parameters.

Historical Data

The first weekly network program broadcast in color was the Colgate Comedy Hour which went on the air in November, 1953. To compare theory versus reality for the period following that date, information concerning the forcing functions and response functions of the model is required. These data are shown in Figs. 1 through 7.

Fig. 1 shows the suggested retail price of the lowest-cost color sets available during each year since 1954 [6].

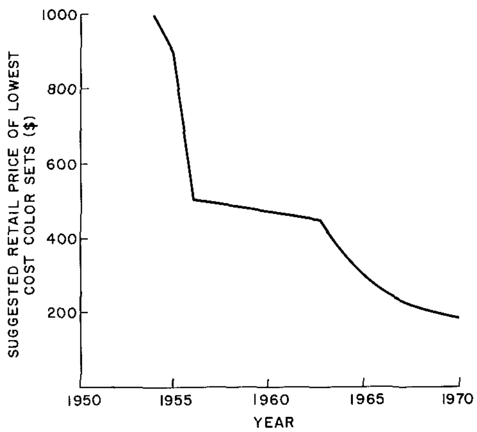

Fig. 1 Color television price history

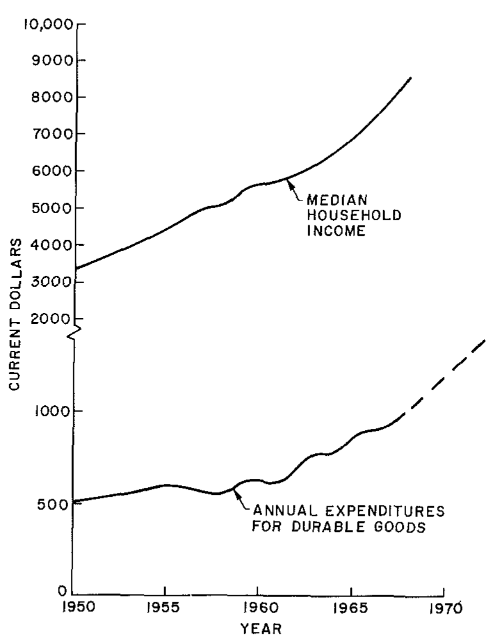

Fig. 2 Annual household income and expenditures for durable goods

Fig. 2 shows U. S. Department of Commerce statistics for median household income and household expenditures for durable goods in current dollars for the period 1950 through 1970 [5]. These data were used for $X(t)$ and $A(t)$ in the model. The unit of measure is appropriate since set costs have also been given in current dollars.

Fig. 3 shows normalized family income distribution data for the years 1950, 1960 and 1968 [4]. These data have been plotted using lognormal probability scales. The coordinates are such that a lognormal distribution is represented by a straight line. These data indicate that the distribution of normalized family income has been approximately independent of time for the period under study. Furthermore this distribution is reasonably well described by a piecewise lognormal function. To obtain a continuous income distribution density function, $D(\xi)$, as shown in Fig. 4, the piecewise lognormal segments have been jointed in the intervals indicated in Fig. 3 by vertical dashed lines using a geometrically weighted average of neighboring lognormal density functions.

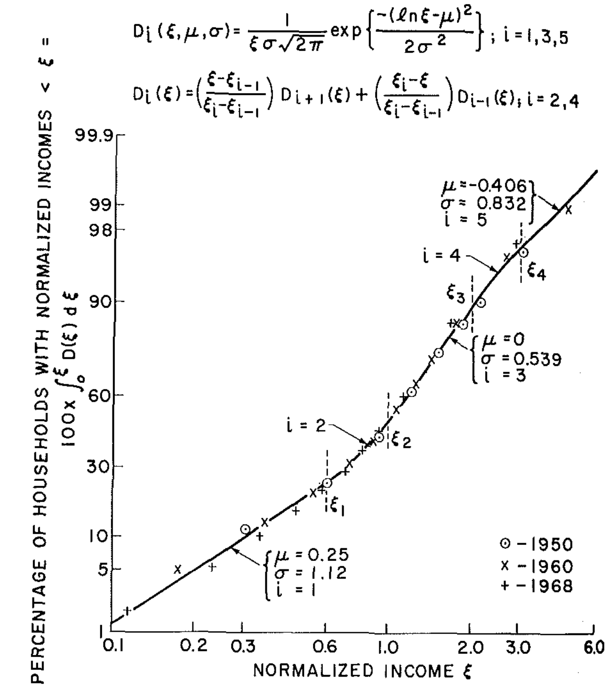

Fig. 3 The cummulative normalized income distribution function

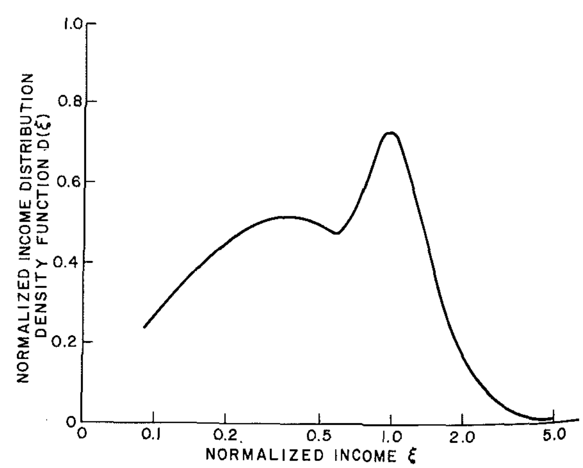

Fig. 4 The normalized income distribution density function

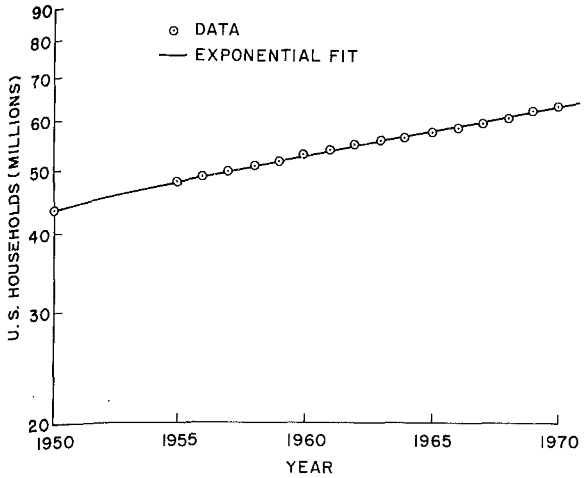

Fig. 5 Household population of the United States

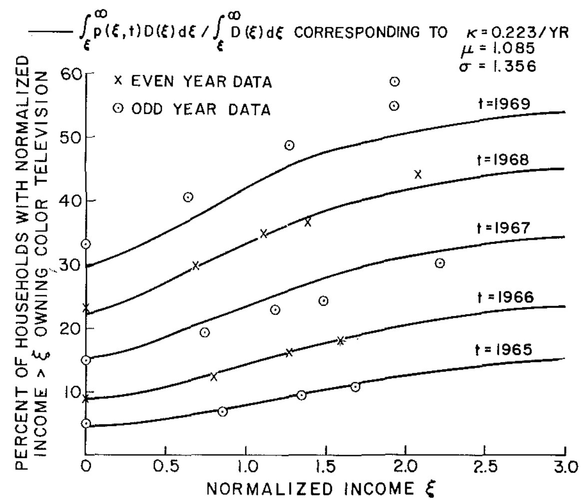

Fig. 7 Theory versus data for color television ownership as a function of household income

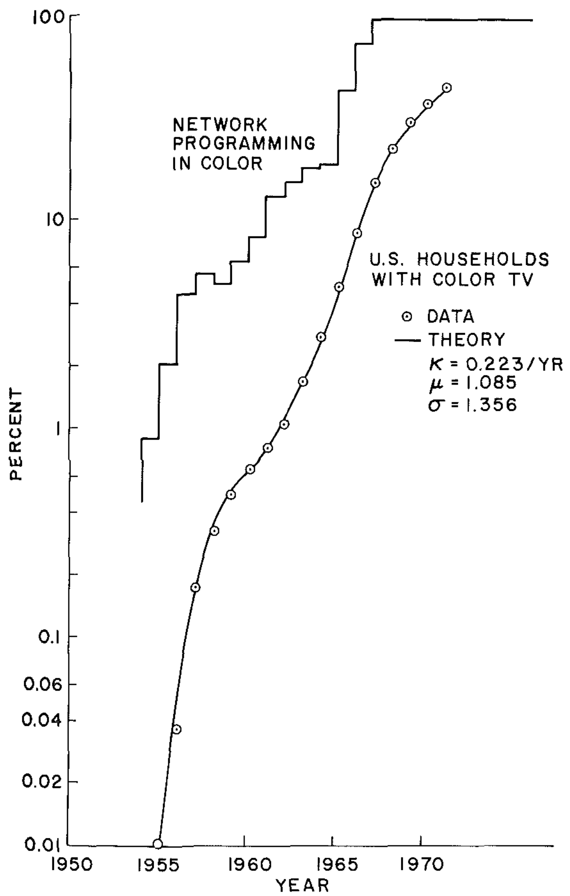

Fig. 6 Color television programming and market penetration

Fig. 5 shows the growth in household population [5] of the United States during the last twenty years. The individual points shown correspond to beginning of the year data and the solid line corresponds to the exponential approximation given by equation (6).

Fig. 6 represents data concerning the fraction of network programming in color [7] and the fraction of U. S. households with color sets [8]. These data show that

1 The percentage rate of growth of CTV ownerhsip was highest in its earliest years

2 The rate of growth slowed significantly during the 1959–1963 period

3 A resurgence of growth occurred after 1963

4 Recently, growth has once again been slowing down.

It is interesting to note that the revival of vigorous growth in 1963–1964 coincided with price reductions beginning at about the same time, as shown in Fig. 1. Furthermore the most recent slowdown coincides with color programming having reached saturation and with approaching market saturation.

Information concerning the income characteristics of color television owners is available from surveys conducted late in the years 1964–1968 [9, 10]. These data are shown as individual points in Fig. 7 where the fraction of those households with normalized incomes greater than ξ and owning color television on January 1 of the year indicated is plotted versus ξ. Color television penetration is clearly shown to increase with increasing household income.

Model Validity and Parameter Estimation

The solid curves shown in Figs. 6 and 7 correspond to the theory associated with equations (8) and (12). Parameter values have been chosen to minimize the sum of the square percentage deviations from available data. Percentage deviations were used since it is more reasonable to associate percentage confidence limits rather than absolute limits with these data. The individual points shown represent the data.

Given the forcing functions of the theory, it is seen that the market penetration response function exhibits the right qualitative behavior. Alternating periods of rapid and slow growth faithfully mirror actual behavior. Although market penetration data varies over a range exceeding three orders of magnitude, the maximum percentage difference between theory and data is less than 30 percent. Furthermore, Fig. 7 indicates that the relationship between household income and CTV ownership is realistically portrayed. These comparisons demonstrate the ability of the theory to explain past history. Before attempting to forecast future CTV ownership levels however, certain stochastic elements in the process should be considered.

Forecasting Considerations

In the "real world," consumer behavior involves many additional factors beyond the purely rational and objective considerations of value and cost. Random environmental phenomena, information dissemination, competition, technological innovation, and subjective factors can have major roles in influencing demand. Variations of these factors over time may be considered within the framework of present theory by adding stochastic elements to the transition function, λ. In the absence of adequate theoretical understanding of the stochastic nature of λ at the present time, the following heuristic considerations provide some estimate of forecast uncertainty.

Since the relative value function has saturated for color television, and since, at present price and income levels, the relative willingness-to-pay function is already at the 70 percent level for median income households, the greatest source of uncertainty in forecasts of color television demand would appear to be associated with stochastic variation in the environmental coefficient, κ. An analysis of available data is presented in the Appendix which suggests that forecasts generated by assuming ± 50 percent discontinuities in κ relative to its estimated value at the time of the forecast would provide reasonable limits for the uncertainty associated with future CTV market penetration.

To test both this hypothesis and the forecasting ability of the CTV model, κ, μ, and σ were recomputed using data through 1965 only. Forecasts were then made through 1971 using these 1965 parameter estimates together with the actual forcing functions for 1965–1971. The forecast, uncertainty bounds gen-

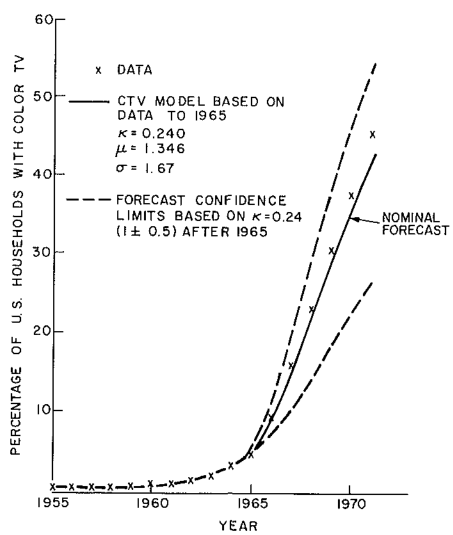

Fig. 8 Forecasts based on data through 1965 in retrospect

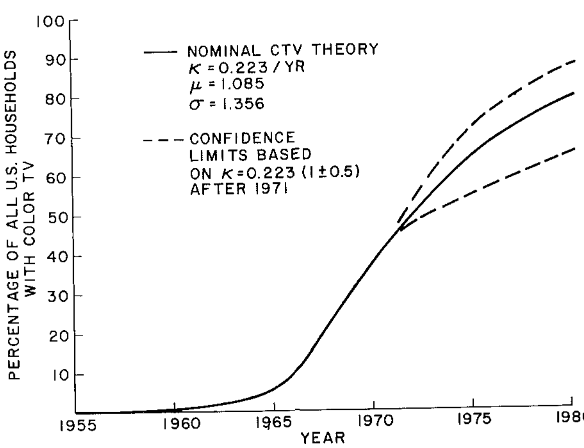

Fig. 9 Market penetration forecast

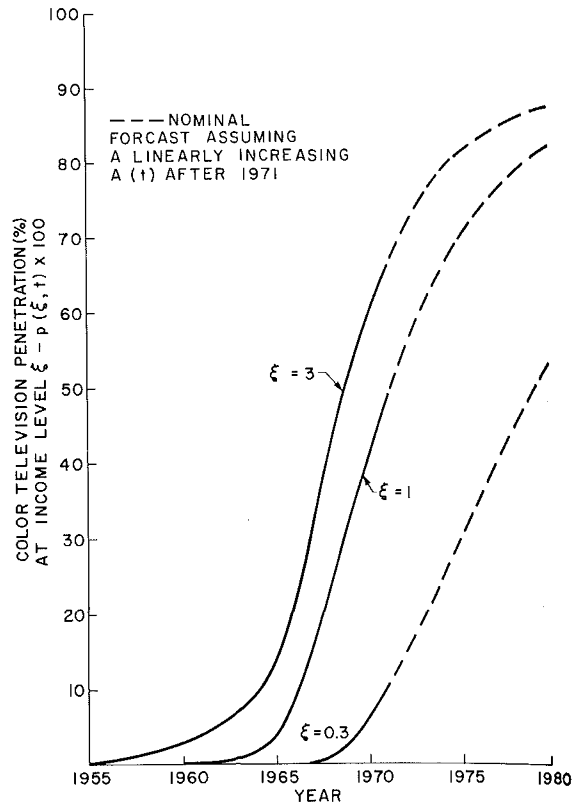

Fig. 10 Color television ownership as a function of time at selected income levels

erated by the ± 50 percent κ discontinuity hypothesis, and actual CTV ownership data through 1971 are shown in Fig. 8. Since, according to the theory, the rate of growth of CTV ownership depends on current levels of ownership, successive errors are highly correlated. However, all the actual data lies within the confidence limits of the forecast.

CTV ownership forecasts to 1980 are presented in Figs. 9 and 10. The solid curve in Fig. 9 represents a forecast assuming no further reductions in the price of color television, linearly increasing median household expenditures for durable goods (as illustrated by the dashed line in Fig. 2), and 1971 estimates for κ, μ, and σ. The dashed curves shown in Fig. 9 represent confidence limits computed assuming ± 50 discontinuities in κ as of January 1, 1971. Fig. 10 presents color television ownerhsip forecasts for selected household income levels.

Summary and Concluding Remarks

The main objective of this study has been to determine to what extent market penetration dynamics can be modeled in terms of simple, basic considerations of value and cost. For the case of CTV, such an approach has been demonstrated to be suprisingly successful. Although the CTV model contains only three parameters and the market penetration data exhibits alternating stages of rapid and slow variation over a range exceeding three orders of magnitude, the maximum percentage difference between the best least-square-error-fit theoretical curve and survey data is less than 30 percent. The forecasting ability of the model has also been demonstrated. The success of this approach implies that either (1) other factors which could have influenced market penetration were essentially invariant over the 1954–1971 time period or (2) variations in such factors were highly correlated with the value cost function variations included in the model.

Direct applicability of this model is obviously limited to the color television situation. It is believed, however, that this is sufficiently representative so that somewhat more general conclusions may be drawn.

It has been found that the transition rate associated with the market penetration process can be modeled as a product of a value function and a cost function. The value function for color television was found to be adequately represented by a rather simple measure of current functional utility—the fraction of network programming broadcast in color. Similarly a reasonable representation for the cost function was provided by a lognormal function of the ratio of available consumer income to cost.

One shortcoming of the theory as presently formulated is that it lacks an adequate theoretical description of stochastic variation in the transition function, λ. For the present situation, it was nevertheless possible to obtain bounds for color television ownership forecast uncertainty. In general, however, questions concerning confidence limits and optimum parameter estimation procedures remain to be answered by future studies.

Acknowledgments

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Appendix

Forecast Uncertainty Bounds

A reasonable estimate of forecast uncertainty due to a stochastic environment may be obtained as follows. Multiplying equation (7) by $D(\xi)$ and integrating with respect to ξ we have

$$\dot{P}(t) = \kappa f(t) - \frac{P(t)}{\tau} \tag{A1}$$

We assume that κ is a stochastic variable so that the value of $\dot{P}$ which is actually realized is

$$\dot{P}_R(t) = \bar{\kappa}(1 + \epsilon)f(t) - \frac{P_R(t)}{\tau} \tag{A2}$$

where the expected value of ϵ is zero. The theoretical forecast for $\dot{P}$ then is

$$\dot{P}_T(t) = \bar{\kappa}f(t) - \frac{P_R(t)}{\tau} \tag{A3}$$

Therefore

$$\epsilon(t) = \frac{\dot{P}_R - \dot{P}_T}{\dot{P}_T + P_R/\tau} \tag{A4}$$

Inspection of Fig. 6 suggests that $P(t)$ can be approximated using a moving exponential fit to local segments of data. Therefore

$$\dot{P}(t) \simeq P(t)\,\frac{\ln[P(t + \Delta t_1)/P(t - \Delta t_2)]}{(\Delta t_1 + \Delta t_2)} \tag{A5}$$

Letting $\Delta t_1 = 0$ for $t = 1971$, $\Delta t_2 = 0$ for 1955 and $\Delta t_i = 1$ everywhere else, equations (A4) and (A5) may be combined to obtain estimates for $\epsilon(t)$. The results of such computations are given in Table 1.

Table 1 Variance computations

Year	$100 \times \dot{P}_{theory}$	$100 \times \dot{P}_{data}$	$100 \times \epsilon$
1955	0.00973	0.0105	−14.9
1956	0.0482	0.037	−24.2
1957	0.167	0.172	28.9
1958	0.274	0.328	50.2
1959	0.377	0.488	28.5
1960	0.538	0.637	−16.9
1961	0.829	0.837	−34.8
1962	1.22	1.09	−14.5
1963	1.76	1.72	25.4
1964	2.6	2.88	23.8
1965	4.5	4.94	3.1
1966	8.77	9.03	−2.7
1967	15.2	15.6	1.6
1968	22.3	22.8	3.1
1969	29.4	30.4	4.2
1970	36.4	37.4	7.8
1971	42.9	45.3	20.6

The standard deviation for ϵ may be estimated by

$$\sigma_\epsilon \simeq \sqrt{\frac{\sum_{1955}^{1971} \epsilon(t)^2}{14}} = 0.245$$

In reference [11] it is shown that assuming κ to undergo a single random variation at the time of the forecast leads to wider confidence limits than assuming κ to undergo a continuous stochastic process with the same distribution. We conclude that "reasonable bounds" for color television forecasts may be obtained by assuming ±50 discontinuities in κ relative to its estimated value.