Fluid Inertia Effects in a Squeeze Film Between Two Plane Annuli

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An analysis is presented for the laminar squeezing flow of a Newtonian incompressible fluid between parallel plane annuli. Both local and convective inertia of the flow are considered in the analysis. Power series expansions in terms of pertinent flow parameters are used to obtain a solution of the equations of motion. Expressions for the pressure and load capacity are presented, and compared with those based on the assumption of inertialess flow.

Introduction

An accurate knowledge of the forces generated by the squeezing action of a lubricant film between solid boundaries is essential in the analysis of the dynamic behavior of bearings. Due to the increase in operating speeds at the film boundaries, and the use of lubricants of low viscosity and relatively high density, it has become important to study the effect of fluid inertia. Fluid inertia effects in Newtonian squeeze films have been examined theoretically and experimentally by many investigators. Due to the large number of studies published, only those considered relevant to this study are mentioned.

Theoretical results for laminar squeezing flow between parallel disks were obtained by Ishizawa [1] and Grimm [2]. Ishizawa obtained a solution for the Navier-Stokes equations by using series expansions in terms of an infinite number of dimensionless time-dependent parameters, while Grimm used a numerical technique. Theoretical and experimental studies of laminar squeeze films between parallel disks were also reported by Kuzma [3] and Tichy and Winer [4].

Analysis of laminar flow in a squeeze film between plane annular disks has been obtained by Archibald [5]. Archibald's analysis does not include the effect of fluid inertia. Theoretical and experimental study of Newtonian squeeze films between fixed and rotating annuli was carried out by Allen and McKillop [6]. Allen and McKillop considered only the centrifugal inertia, and neglected both local and convective fluid inertia.

In this study the laminar squeezing flow of an incompressible Newtonian fluid between nonrotating annular surfaces is analyzed. Both local and convective inertia of the fluid are considered in the analysis. A solution of the equations of motion is obtained by using perturbation series expansions in terms of an infinite number of dimensionless flow parameter, and is valid for small values of these parameters. First-order expressions for the pressure and load capacity are presented, and compared with results based on the assumption of inertialess flow.

Although perturbation solutions of the type presented in this paper are not valid during the initial stages of the squeezing motion [7], previous agreements of first-order perturbation solution for circular squeeze films with Kuzma's experiments [3] and with Grimm's numerical solution [2] give confidence in the applicability of the expressions presented for the annular squeeze film.

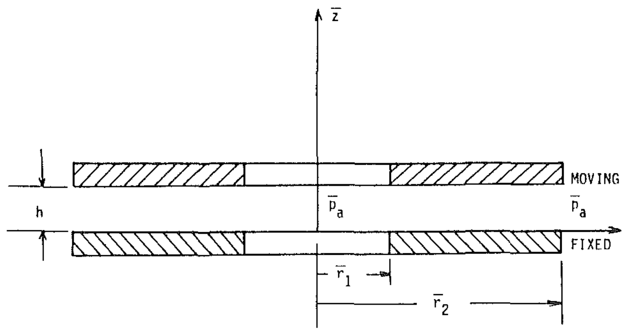

Fig. 1 Schematic representation of squeeze film

Analysis

We consider the axially symmetric flow of a Newtonian fluid with constant properties between two plane annular surfaces, Fig. 1. Applying the hydrodynamic lubrication assumptions for thin films, and retaining both local and convective inertia terms, one obtains the following approximate equation of motion in the radial direction:

$$\frac{\partial \bar{v}_r}{\partial t} + \bar{v}_r \frac{\partial \bar{v}_r}{\partial \bar{r}} + \bar{v}_z \frac{\partial \bar{v}_r}{\partial \bar{z}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{r}} + \nu\left(\frac{\partial^2 \bar{v}_r}{\partial \bar{z}^2}\right). \tag{1}$$

The equation of continuity is

$$\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{v}_r) + \frac{\partial \bar{v}_z}{\partial \bar{z}} = 0. \tag{2}$$

Neglecting the edge effects at the inner and outer boundaries, the boundary conditions are:

$$\bar{v}_r(t,\bar{r},0) = \bar{v}_r(t,\bar{r},h) = \bar{v}_z(t,\bar{r},0) = 0$$

$$\bar{v}_z(t,\bar{r},h) = \frac{dh}{dt}, \tag{3}$$

and

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$$\bar{p}(t,\bar{r}_1)=\bar{p}(t,\bar{r}_2)=\bar{p}_a.$$

The following dimensionless variables are introduced:

$$r=(\bar{r}/h) \qquad z=(\bar{z}/h)$$
$$v_r=(\bar{v}_r h/\nu), \qquad v_z=(\bar{v}_z h/\nu) \tag{4}$$

and

$$p=(\bar{p}h^2/\rho\nu^2).$$

In terms of these dimensionless variables the equation of motion is

$$\left(\frac{h^2}{\nu}\right)\frac{\partial v_r}{\partial t}-\left(\frac{h}{\nu}\frac{dh}{dt}\right)v_r+v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}=-\frac{\partial p}{\partial r}+\frac{\partial^2 v_r}{\partial z^2}, \tag{5}$$

and the equation of continuity is

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{\partial v_z}{\partial z}=0, \tag{6}$$

while the boundary conditions are given by

$$v_r(t,r,1)=v_r(t,r,0)=v_z(t,r,0)=0$$
$$v_z(t,r,1)=\text{Re}, \tag{7}$$

and

$$p(t,r_1)=p(t,r_2)=p_a,$$

where $\text{Re}=(h/\nu\, dh/dt)$ is the instantaneous Reynolds number, and $p_a=(\bar{p}_a h^2/\rho\nu^2)$ is the dimensionless ambient pressure.

In order to obtain a solution for equations (5) and (6), we consider the following power series expansions in r:

$$v_r=-\frac{1}{2}r\phi'_{-1}(z,t)+\left\{\frac{A(t)}{r}\right\}\left[\phi'_0(z,t)+\left\{\frac{A(t)}{r^2}\right\}\phi'_1(z,t)+\left\{\frac{A(t)}{r^2}\right\}^2\phi'_2(z,t)+\ldots\right]$$

$$v_z=\phi_{-1}(z,t)+\left[2\left\{\frac{A(t)}{r^2}\right\}^2\phi_1(z,t)+4\left\{\frac{A(t)}{r^2}\right\}^3\phi_2(z,t)+\ldots\right] \tag{8}$$

and

$$p=\frac{1}{2}r^2H_{-1}(t)+\left\{A(t)\right\}\left[H_0(t)\ln r+\left\{\frac{A(t)}{r^2}\right\}H_1(t)+\left\{\frac{A(t)}{r^2}\right\}^2H_2(t)+\ldots\right]+F(t),$$

where A and F are unknown functions of time to be determined from the boundary conditions on p, and $'\equiv\partial/\partial z$. With the above expressions for v_r and v_z the equation of continuity, equation (6), is satisfied. The boundary conditions on the functions ϕ_n and their derivatives are

$$\phi'_n(0,t)=\phi'_n(1,t)=0 \qquad n=-1,0,1,2,\ldots$$
$$\phi_{-1}(1,t)=\text{Re}$$
$$\phi_n(1,t)=0 \qquad n=1,2,3,\ldots \tag{9}$$
$$\phi_n(0,t)=0 \qquad n=-1,1,2,3,\ldots$$

and

$$\phi_0(1,t)-\phi_0(0,t)=\gamma=\text{constant}$$

which upon choosing $\gamma=1$, and

$$\phi_0(0,t)=0,$$

gives

$$\phi_0(1,t)=1.$$

The choice of γ is arbitrary at this point since A is still an unknown function of time that will be determined later by using the boundary conditions on p.

Substituting for v_r, v_z, and p from equation (8) into equation (5) and equating coefficients of equal powers in r, one obtains an infinite set of simultaneous partial differential equations. The first three differential equations are:

$$\phi'''_{-1}-\{\phi_{-1}-(\text{Re})Z\}\phi''_{-1}+\frac{1}{2}(\phi'_{-1})^2+2(\text{Re})\phi'_{-1}=-H_{-1}+\left(\frac{h^2}{\nu}\right)\frac{\partial\phi'_{-1}}{\partial t} \tag{10}$$

$$A\left[\phi'''_0-\{\phi_{-1}-(\text{Re})Z\}\phi''_0\right]=A\left\{H_0+\left(\frac{h^2}{\nu}\right)\frac{\partial\phi'_0}{\partial t}\right\}+\left(\frac{h^2}{\nu}\right)\frac{dA}{dt}\phi'_0 \tag{11}$$

and

$$A\left[\phi'''_1-\{\phi_{-1}-(\text{Re})Z\}\phi''_1-(\phi''_1+2\text{Re})\phi'_1+(\phi'_0)^2+\phi_1\phi''_{-1}\right]=-A\left\{2H_1-\left(\frac{h^2}{\nu}\right)\frac{\partial\phi'_1}{\partial t}\right\}+2\left(\frac{h^2}{\nu}\right)\frac{dA}{dt}\phi'_1 \tag{12}$$

Solution of equation (10), subject to the boundary conditions given in equation (9), corresponds to the solution for

Nomenclature

A = function of time
A_n = function of time
B = width of rectangular plate
C = correction coefficient for load capacity
F = function of time
h = instantaneous film thickness
H_n = function of t
$H_{n,m}$ = constant
K = r_1/r_2 = ratio of inner to outer radius
L = length of rectangular plate
$\bar{p}$ = pressure
p = $(\bar{p}h^2/\rho\nu^2)$ = dimensionless pressure
$\bar{p}_a$ = ambient pressure
p_a = $(\bar{p}_a h^2/\rho\nu^2)$ = dimensionless ambient pressure
$\overset{*}{p}$ = $(p-p_a)/(-3\text{R}_e r_2^2)$ = dimensionless pressure drop
$\bar{r}$ = radial coordinate
r = $\bar{r}/h$ = dimensionless radial coordinate
$\bar{r}_1$ = inner radius
r_1 = $\bar{r}_1/h$ = dimensionless inner radius
$\bar{r}_2$ = outer radius
r_2 = $\bar{r}_2/h$ = dimensionless outer radius
R_e = $(h/\nu)(dh/dt)$ = instantaneous film Reynolds Number
t = time
$\bar{v}_r$ = radial component of velocity
v_r = $\bar{v}_r h/\nu$ = dimensionless radial component of velocity
$\bar{v}_z$ = axial component of velocity
v_z = $\bar{v}_z h/\nu$ = dimensionless axial component of velocity
W = load capacity
$\bar{z}$ = axial coordinate
z = $\bar{z}/h$ = dimensionless axial coordinate
β_n = instantaneous flow parameter
ν = kinematic viscosity
ρ = density
σ = $(\overset{*}{r}-k)/(1-k)$ = dimensionless distance from inner radius
ϕ_n = function of z and t
$\phi_{n,m}$ = function of z only

flow in a squeeze film between parallel disks given by Ishizawa [1]. To obtain a solution for equations (10), (11), and (12), we expand ϕ_n, H_n, and A in power series of the dimensionless flow parameters.

$$\mathrm{Re}=\frac{h}{\nu}\frac{dh}{dt},\qquad \beta_1=\frac{h^3}{\nu^2}\frac{d^2h}{dt^2},$$

$$\beta_2=\frac{h^5}{\nu^3}\frac{d^3h}{dt^3},\qquad \beta_n=\frac{h^{(2n+1)}}{\nu^{(n+1)}}\frac{d^{(n+1)}h}{dt^{(n+1)}},\ .\ .\ .\ ,\tag{13}$$

where Re is the instantaneous Reynolds number, and the parameters β_n are introduced in order to account for the variation of the boundary velocity with time. The parameters β_n may be viewed as a measure of the promptness with which the fluid film responds to impressed changes at the boundary. In the limit as $\beta_n\rightarrow 0$ the fluid responds instantly, and the flow is classified as quasi-steady. For small values of the flow parameters in equation (13), the expansions for ϕ_n, H_n, and A are

$$\phi_n=\frac{1}{\mathrm{Re}^\alpha}[\mathrm{Re}\phi_{n,0}(z)+\beta_1\phi_{n,1}(z)+\beta_2\phi_{n,2}(z)+\ .\ .\ .$$
$$+\mathrm{Re}^2\phi_{n,00}(z)+\mathrm{Re}\beta_1\phi_{n,01}(z)+\ .\ .\ .$$
$$+\mathrm{Re}^3\phi_{n,000}(z)+\ .\ .\ .\ ]\tag{14}$$

$$H_n=\frac{1}{\mathrm{Re}^\alpha}[\mathrm{Re}H_{n,0}+\beta_1H_{n,1}+\beta_2H_{n,2}+\ .\ .\ .$$
$$+\mathrm{Re}^2H_{n,00}+\mathrm{Re}\beta_1H_{n,01}+\ .\ .\ .$$
$$+\mathrm{Re}^3H_{n,000}+\ .\ .\ .\ ]\tag{15}$$

$$A_n=[\mathrm{Re}A_0(t)+\beta_1A_1(t)+\beta_2A_2(t)+\ .\ .\ .$$
$$+\mathrm{Re}^2A_{00}(t)+\mathrm{Re}\beta_1A_{01}(t)+\ .\ .\ .$$
$$+\mathrm{Re}^3A_{000}(t)+\ .\ .\ .\ ]\tag{16}$$

where $\alpha=0$ for $n=-1$, and $\alpha=1$ for $n\neq-1$. Terms in the first columns between brackets in the above expansions represent the inertialess solution, while terms in each of the successive columns constitute a fluid inertia correction of a certain order.

The boundary conditions on the functions $\phi_{n,m}$ and their derivatives are:

$$\phi'_{n,m}(0)=\phi'_{n,m}(1)=\phi_{n,m}(0)=0\quad\text{for all } n \text{ and } m$$
$$\phi_{-1,0}(1)=\phi_{0,0}(1)=1\tag{17}$$
$$\phi_{n,m}(1)=0\qquad n=-1,0 \text{ and } m\neq 0$$
$$\phi_{n,m}(1)=0\qquad n=1,2,3,\ .\ .\ .\ \text{and all } m.$$

Substituting for H_n and A by their expansion from equations (15) and (16) into the expression for p in equation (8), using the condition $p(r_1,t)=p(r_2,t)$ and equating coefficients of like forms of the flow parameters, we are able to express the coefficients A_n in the first two columns of equation (16) as:

$$A_0=\left(\frac{1}{4}\right)\left(\frac{H_{-1,0}}{H_{0,0}}\right)(r_2^2)\left(\frac{1-k^2}{\ln k}\right)$$
$$A_1=A_0\left[\left(\frac{H_{-1,1}}{H_{-1,0}}\right)-\left(\frac{H_{0,1}}{H_{0,0}}\right)\right]\tag{18}$$

and

$$A_{00}=A_0\left[\left(\frac{H_{-1,00}}{H_{-1,0}}\right)-\left(\frac{H_{0,000}}{H_{0,0}}\right)-\left(\frac{1}{4}\right)\left(\frac{H_{-1,0}}{H_{0,0}}\right)\right.$$
$$\left.\left(\frac{H_{1,0}}{H_{0,0}}\right)\left(\frac{1}{K^2}\right)\left(\frac{1-K^2}{\ln K}\right)^2\right]$$

Substituting the expansions given by equations (14), (15), and (16), and equating coefficients of like forms of the parameters, we obtain the following systems of ordinary linear differential equations for equations (10), (11), and (12), respectively:

System 1

$$\phi'''_{-1,0}=-H_{-1,0}$$
$$\phi'''_{-1,1}=-H_{-1,1}+\phi'_{-1,0}\tag{19}$$
$$\phi'''_{-1,00}=-H_{-1,00}+\phi_{-1,0}\phi''_{-1,0}-\frac{1}{2}(\phi'_{-1,0})^2$$
$$-Z\phi''_{-1,0}-\phi'_{-1,0}\tag{19}$$

System 2

$$\phi'''_{0,0}=H_{0,0}$$
$$\phi'''_{0,1}=H_{0,1}+\phi'_{0,0}$$
$$\phi'''_{0,00}=H_{0,00}+\phi_{-1,0}\phi''_{0,0}-Z\phi''_{0,0}-\phi'_{0,0}\tag{20}$$

System 3

$$\phi'''_{1,0}=-2H_{1,0}-(\phi'_{0,0})^2$$
$$\phi'''_{1,1}=-2H_{1,1}+2\phi'_{1,0}-2\phi'_{0,0}\phi'_{0,1}\tag{21}$$
$$\phi'''_{1,00}=-2H_{1,00}+\phi_{1,0}\phi''_{1,0}-\phi_{1,0}\phi''_{-1,0}$$
$$-2\phi'_{0,0}\phi'_{0,00}+\phi'_{-1,0}\phi'_{1,0}-Z\phi''_{1,0}$$

Solutions of Systems 1, 2, and 3 subject to the boundary conditions given by equation (17) are

$$\phi_{-1,0}=Z^2(3-2Z)$$
$$\phi_{-1,1}=\frac{1}{20}(-2Z^5+5Z^4-4Z^3+Z^2)$$
$$\phi_{-1,00}=\frac{1}{70}(2Z^7-7Z^6+21Z^5-35Z^4+25Z^3-6Z^2)\tag{22}$$

with

$$H_{-1,0}=12,\quad H_{-1,1}=\frac{6}{5},\quad H_{-1,00}=-\frac{15}{7}.\tag{23}$$

$$\phi_{0,0}=Z^2(3-2Z)$$
$$\phi_{0,1}=\frac{1}{20}(-2Z^5+5Z^4-4Z^3+Z^2)\tag{24}$$
$$\phi_{0,00}=\frac{1}{70}(8Z^7-28Z^6+42Z^5-35Z^4+16Z^3-3Z^2)$$

with

$$H_{0,0}=-12,\quad H_{0,1}=-\frac{6}{5},\quad H_{0,00}=\frac{48}{35},\tag{25}$$

and

$$\phi_{1,0}=\frac{1}{35}(-6Z^7+21Z^6-21Z^5+9Z^3-3Z^2)$$
$$\phi_{1,1}=\frac{1}{4200}(-70Z^9+315Z^8-504Z^7+294Z^6+24Z^5$$
$$-60Z^4-8Z^3+9Z^2)\tag{26}$$
$$\phi_{1,00}=\frac{1}{46,300}(1360Z^{11}-7480Z^{10}+16,390Z^9$$
$$-17,655Z^8+8712Z^7-462Z^6-1188Z^5+330Z^4$$
$$-8Z^3+Z^2)$$

with

$$H_{1,0}=-\frac{27}{35},\quad H_{1,1}=\frac{1}{175},\quad H_{1,00}=\frac{1}{1925}.\tag{27}$$

Substituting for $H_{m,n}$ from equations (23), (25), and (27) into the expressions for A_n given by equation (18), one obtains

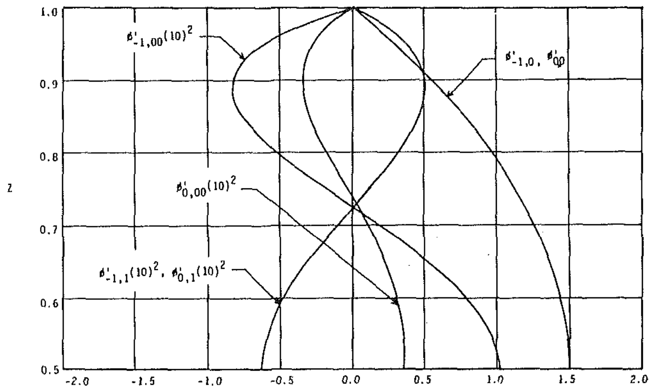

Fig. 2 Distribution of velocity functions, $\phi'_{n,m}$

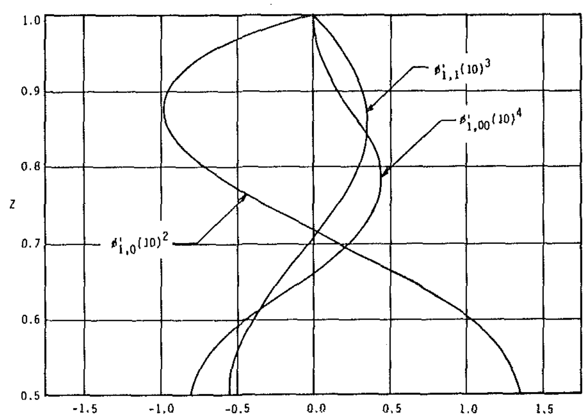

Fig. 3 Distribution of velocity functions, $\phi'_{n,m}$

$$A_0 = -\left(\frac{1}{4}\right)(r_2^2)\left(\frac{1-K^2}{\ln K}\right)$$

$$A_1 = 0 \tag{28}$$

and

$$A_{00} = A_0\left[-\left(\frac{9}{140}\right)+\left(\frac{9}{560}\right)\left(\frac{1}{K^2}\right)\left(\frac{1-K^2}{\ln K}\right)^2\right]$$

The first-order expressions for H_n and A, obtained by substituting for $H_{m,n}$ and A_n into equations (15) and (16) and neglecting higher order inertia corrections, are

$$H_{-1} = \left[12\text{Re}+\left(\frac{6}{5}\right)\beta_1-\left(\frac{15}{7}\right)\text{Re}^2\right] \tag{29}$$

$$H_0 = \frac{1}{\text{Re}}\left[-12\text{Re}-\left(\frac{6}{5}\right)\beta_1+\left(\frac{48}{35}\right)\text{Re}^2\right] \tag{30}$$

$$H_1 = \frac{1}{\text{Re}}\left[-\left(\frac{27}{35}\right)\text{Re}+\left(\frac{1}{175}\right)\beta_1+\left(\frac{1}{1925}\right)\text{Re}^2\right] \tag{31}$$

and

$$A = (\text{Re}A_0)\left[1+\text{Re}\left\{-\left(\frac{9}{140}\right)+\left(\frac{9}{560}\right)\left(\frac{1}{K^2}\right)\left(\frac{1-K^2}{\ln K}\right)^2\right\}\right] \tag{32}$$

Results and Discussion

The velocity functions $\phi'_{n,m}$, obtained by differentiating equations (22), (24), and (26), are presented in Figs. 2 and 3. These figures show that the functions $\phi'_{n,m}$ are symmetric about $Z=1/2$, and that their magnitudes decrease rapidly,

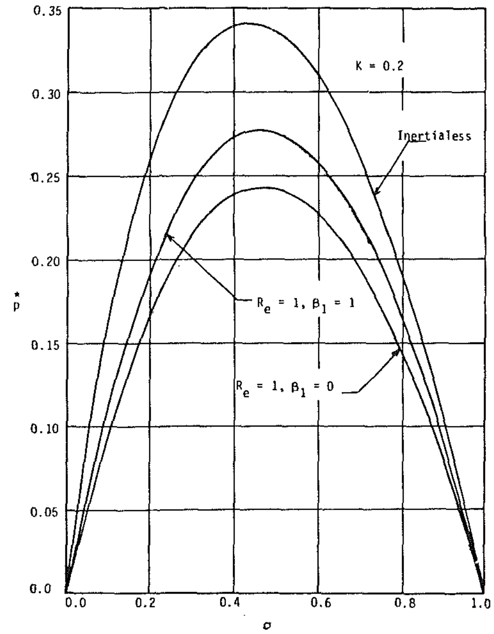

Fig. 4 Dimensionless pressure drop $\mathring{p}$ versus dimensionless radial distance from inner radius, σ

indicating the convergence of the power series expansions used in the analysis.

Substituting for H_n and A from equations (29), (30), (31), and (32) into equation (8) and neglecting higher order inertia corrections, we obtain the first-order expression for the pressure distribution as

$$p(r,t)-p_a = \frac{1}{4}\text{Re}\left[12+\frac{6}{5}\left(\frac{\beta_1}{\text{Re}}\right)-\frac{15}{7}(\text{Re})\right](r^2-r_2^2)$$

$$+\left\{\left(\frac{1}{4}\right)(r_2^2)\left(\frac{1-K^2}{\ln K}\right)(\text{Re})\right\}\left[\left\{12+\frac{6}{5}\left(\frac{\beta_1}{\text{Re}}\right)-\frac{15}{7}(\text{Re}) \quad (33)\right.\right.$$

$$\left.+\frac{27}{140}\left(\frac{\text{Re}}{K^2}\right)\left(\frac{1-K^2}{\ln K}\right)^2\right\}\ln\frac{r}{r_2}$$

$$\left.-\text{Re}\left(\frac{r_2}{r}\right)^2\left(\frac{1-K^2}{\ln K}\right)\left(\frac{27}{140}\right)\left(1-\frac{r^2}{r_2^2}\right)\right], \tag{33}$$

where the boundary condition $p(r_2,t)=p_a$ has been applied.

The expression for the inertialess pressure distribution is given by

$$p(r,t)-p_a = 3\text{Re}r_2^2\left[\left(\frac{r^2}{r_2^2}-1\right)+\left(\frac{1-K^2}{\ln K}\right)\ln\frac{r}{r_2}\right] \tag{34}$$

The dimensionless pressure drop is defined as

$$\mathring{p} = \frac{p-p_a}{(-3\text{Re}r_2^2)},$$

and the dimensionless radial distance measured from the inner radius is given by

$$\sigma = \frac{\mathring{r}-K}{1-K},$$

where $\mathring{r}=r/r_2$.

Distributions of the dimensionless pressure drop $\mathring{p}$ versus σ for Re = 1 and $\beta_1=0$, and for Re = 1 and $\beta_1=1$ are presented in Fig. 4 for $K=0.2$. A comparison of the distribution of $\mathring{p}$ for Re = 1 and $\beta_1=0$ with the inertialess distribution in Fig. 4 show a decrease in $\mathring{p}$ due to Re; while a comparison of the distribution of $\mathring{p}$ for Re = 1 and $\beta_1=1$ with that for Re = 1 and $\beta_1=0$ shows an increase in $\mathring{p}$ due to β_1.

The load capacity is obtained by integrating the pressure difference $(\bar{p}-\bar{p}_a)$ over the area of the plane annulus, that is

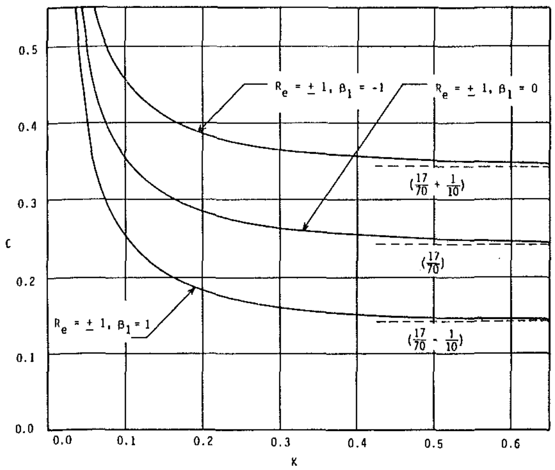

Fig. 5 Dependence of correction factor for load capacity C on K, Re, and β_1

$$W=\int_{\bar{r}_1}^{\bar{r}_2}(\bar{p}-\bar{p}_a)2\pi\bar{r}d\bar{r}=2\pi\rho\nu^2\int_{r_1}^{r_2}(p-p_a)rdr.$$

Substituting the first-order expression for $(p-p_a)$ from equation (33),

$$W=\left(\frac{3}{2}\pi r_2^4\rho\nu^2\right)\left[\left\{\mathrm{Re}+\frac{1}{10}\beta_1-\frac{5}{28}\mathrm{Re}^2\right\}\left\{(K^4-1)-\frac{(1-K^2)^2}{\ln K}\right\}+\left(\frac{9}{560}\right)\left(\frac{1-K^2}{\ln K}\right)^2\left\{4\ln K-\frac{(1-K^2)^2}{K^2\ln K}\right\}\mathrm{Re}^2\right] \quad (35)$$

The expression for the inertialess load capacity, obtained by using equation (34), is

$$W_{\text{inertialness}}=\left(\frac{3}{2}\pi r_2^4\rho\nu^2\right)(\mathrm{Re})\left[(K^4-1)-\frac{(1-K^2)^2}{\ln K}\right], \quad (36)$$

which corresponds to that obtained by Archibald [5].

Expressions for the load capacity for a squeeze film between rectangular plates of width B and length L can be obtained by substituting $(1-\epsilon)$ for K into equations (35) and (36), where ϵ is a small positive number. By letting $\epsilon\to 0$, and setting $2\pi\bar{r}_2=B$ and $\epsilon\bar{r}_2=L$, one obtains the following expressions of the load capacity for a rectangular plate of length L and width B:

$$W_p=-\left(\frac{BL^3}{h^4}\right)(\rho\nu^2)\left[\mathrm{Re}+\frac{1}{10}\beta_1-\frac{17}{70}\mathrm{Re}^2\right] \quad (37)$$

and

$$(W_p)_{\text{inertialess}}=-\left(\frac{BL^3}{h^4}\right)(\rho\nu^2)(\mathrm{Re}). \quad (38)$$

In order to study the effect of fluid inertia on the load capacity, we define a correction coefficient C as

$$C=\frac{W-W_{\text{inertialess}}}{|W|_{\text{inertialess}}}.$$

The inertia correction coefficient C, obtained by substituting for W and $W_{\text{inertialess}}$ from equations (35) and (36), is a function of K, Re, and β_1. By substituting for Wp and $(Wp)_{\text{inertialess}}$ from equations (37) and (38) into the definition for C, we obtain the limiting expression for C as $K\to 1$:

$$(C)_{\text{plate}}=\frac{1}{\mathrm{Re}}\left[-\frac{1}{10}\beta_1+\frac{17}{70}\mathrm{Re}^2\right]. \quad (39)$$

Plots showing the variation of C with K for $\mathrm{Re}=\pm 1$ and $\beta_1=1$, and for $\mathrm{Re}=\pm 1$ and $\beta_1=-1$ are presented in Fig. 5. From Fig. 5 it is shown that C decreases with increase in K, and approaches $(C)_{\text{plate}}$ as $K\to 1$. From Fig. 5 it is also shown that a positive β_1 causes a decrease in C, while a negative β_1 results in an increase.

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