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ON NUMERICAL SIMULATIONS OF A NONLINEAR SELF-EXCITED SYSTEM WITH TWO NON-IDEAL SOURCES

J.L Palacios
Universidade Regional Integrada do Alto Uruguai e das Missões, Departamento de Ciências Exatas e da Terra-URI, CEP 98802-470, Santo Ângelo, RS Tel: (0xx55) 33137984, fax: (0xx55) 33137902.
jorgelpfelix@yahoo.com.br

J.M. Balthazar
Universidade Estadual Paulista, Instituto de Geociências e Ciências Exatas, Departamento de Estatística, Matemática Aplicada e Computação-UNESP, CP 178, CEP 13500-230, Rio Claro, SP. Tel: (0xx19) 35340220R233. Prof. visitante do DPM -FEM –UNICAMP. jmbaltha@rc.unesp.br

R.M.L.R.F. Brasil
Universidade de São Paulo, Departamento de Estrutura e Fundações – USP, CEP 05508-900, São Paulo, SP. e-mail: rmlrdfbr@usp.br

J. Warminski
Department of Applied Mechanics, Technical University of Lubin. Nadbystrzycka 36, 20-618 Lubin, Poland. Tel.: +48-81-5381197, fax: +48-81-5250808. e-mail:
jwar@archimedes.pol.lublin.pl

ABSTRACT

In this work, the dynamic behavior of self-synchronization and synchronization through mechanical interactions between the nonlinear self-excited oscillating system and two non-ideal sources are examined by numerical simulations. The physical model of the system vibrating consists of a non-linear spring of Duffing type and a nonlinear damping described by Rayleigh's term. This system is additional forced by two unbalanced identical direct current motors with limited power (non-ideal excitations). The present work mathematically implements the parametric excitation described by two periodically changing stiffness of Mathieu type that are switched on/off.

INTRODUCTION

In engineering practice, we can distinguish systems where oscillations are caused by different reasons. The well-known oscillations are: a) Self-excited systems in which, roughly speaking, a constant input produces periodic output; b) Parametrically excited systems characterized by periodically changing in time parameters and c) Systems excited by an external force.

All these systems were comprehensively analyzed in literature separately. However, we can find some papers that are devoted to interaction between different kinds of vibrations, for example: self and external excited vibrations, self and parametric excited vibrations, as well as between self-parametric and external excited vibrations [15, 16].

We notice that when a forcing function is independent of the system it acts on, then the problem is called ideal. In such case, the excitation may be formally expressed as a pure function of time. If in a certain model its ideal source is replaced by a non-ideal source, the excitation can be put in the form, where it is a function, which depends on the response of the system. Therefore, non-ideal source cannot be expressed as a pure function of time but rather as an equation that relates the source to the system of equations that describes the model. Hence, non-ideal models always have one additional degree of freedom when compared with similar ideal models [1, 2, 3, 4, 5, 10].

In current literature, the name "self-synchronization" is used for synchronous rotation in the absence of any "direct" Kinematic coupling between rotating components. The behavior of the phenomenon of self-synchronization has been studied by a number of authors. Among them we mention the works of [6, 7, 9]. Recently [11, 12, 13] studied self-synchronization of the two identical DC motors, with a limited power supply and with masses attached eccentrically to their rotating shafts supported by a structural frame.

When synchronization is achieved by proper interconnections in the systems, i. e. without any artificially introduced external action, then the system is referred to as self-synchronized. A classical example of self-synchronization is the pair of pendulum clocks hanging from a light weight beam that was reported by Huygens. He observed that both pendulums oscillated with the same frequency [6].

In this paper, we are interested in analyzing the influence of the response of the nonlinear and self-excited systems on the two unbalanced identical direct current motors, which is being an extension of the work of [16].

MODEL

Let us consider a nonlinear and self-excited model, which includes two identical direct current (DC) motors with limited power, operating on a structure (Fig. 1).

The excitation of the system is limited by the characteristics of the energy source. Vibration of the system depends on the motion of the motors, and the energy sources motion depends on vibration of the system, as well. Then, coupling of the vibrating oscillator and the two DC motors takes place.

Hence, it is important to analyses what will happen to the motors, as the response of the system changes.

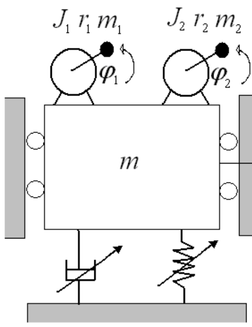

Figure 1. Non-ideal nonlinear and self-excited system model

Taking into account the differential equation of the complete electro-mechanical system presented in [16] and Fig. 1, we can write for $(s = 1,2)$:

$$
\begin{aligned}
&J_s\ddot{\varphi}_s = L_s(\dot{\varphi}_s) - H_s(\dot{\varphi}_s) + m_s r_s \ddot{x}\cos\varphi_s - m_s g r_s \cos\varphi_s \\
&m\ddot{x} + (-c_1 + \hat{c}_1\dot{x}^2)\dot{x} + (kx + k_1x^3) \\
&\qquad = \sum_{j=1}^{2} m_j r_j (\dot{\varphi}_j^2 \sin\varphi_j - \ddot{\varphi}_j \cos\varphi_j)
\end{aligned} \tag{1}
$$

where x is oscillatory coordinate of vibrating body; φ_s is rotational coordinate of each DC motor; $\dot{\varphi}_s$ is rotational speed of rotors; J_s is a moment of inertia of each motor; m_s is unbalanced mass of each motor; r_s is eccentricity of each unbalanced mass; $L_s(\dot{\varphi}_s)$ is a controlled torque of each DC motor; $H_s(\dot{\varphi}_s)$ is a resistance torque of each DC motor.

In order, to obtain the governing equations of the system in their dimensionless form, we define the non-dimensional time $\tau = \omega t$ and the non-dimensional displacement $X = \frac{M}{m_1 r_1}x$, where $\omega = \sqrt{\frac{k}{M}}$ is natural frequency of the system and $M = m + m_1 + m_2$.

Hence, we will obtain:

$$
\begin{aligned}
&\varphi_s'' = \hat{a}_s - \hat{b}_s\varphi_s' + \beta_s R_s X''\cos\varphi_s \\
&X'' + (-\mu + qX'^2)X' + (X + pX^3) \\
&\qquad = \sum_{j=1}^{2} R_j (\varphi_j'^2 \sin\varphi_j - \varphi_j'' \cos\varphi_j)
\end{aligned} \tag{2}
$$

The dimensionless parameters in Eq. (2) are defined as:

$$
\hat{a}_s = \frac{a_s}{\omega^2 m_1 r_1^2 I_s},\quad \hat{b}_s = \frac{b_s}{\omega m_1 r_1^2 I_s},\quad R_s = \frac{m_s r_s}{m_1 r_1},\quad \beta_s = \frac{m_1}{M I_s},
$$

$$
\mu = \frac{c_1}{M\omega},\ q = \frac{\hat{c}_1 \omega m_1 r_1^2}{M^3},\ p = \frac{k_1 m_1 r_1^2}{M^2},\ I_s = \frac{J_s}{m_1 r_1^2}.
$$

From the mathematical point of view two different parametric excitations were placed in the equation of motion in their dimensionless form. From second equation of Eq. (2) we will obtain:

$$
\begin{aligned}
&X'' + (-\mu + qX'^2)X' + (1 - \sum_{j=1}^{2}\alpha_j \cos 2\varphi_j)(1 + pX^2)X \\
&\qquad = \sum_{j=1}^{2} R_j (\varphi_j'^2 \sin\varphi_j - \varphi_j'' \cos\varphi_j)
\end{aligned} \tag{3}
$$

NUMERICAL RESULT

Next, we carried out, a number of numerical simulations, in order to observe the interaction between the two identical non-ideal DC motors and nonlinear and self-excited structural system (as regular and irregular motions). Furthermore, we observe the self-synchronization and synchronization phenomena in pre-resonance, resonance and post-resonance regions.

Equation 2 was simulated by using the block Diagrams in SIMULINK that models the non-ideal nonlinear (parametrically) self-excited system. To obtain different regimes in the system we varied the torques of each DC motor and the initial rotation of second motor.

The first set of numerical results, shown in Fig. 2, illustrates the development of self-synchronization by intervals when the torques of each DC motors are approximately equal $\hat{a}_1 = 1$ and $\hat{a}_2 = 0.9$.

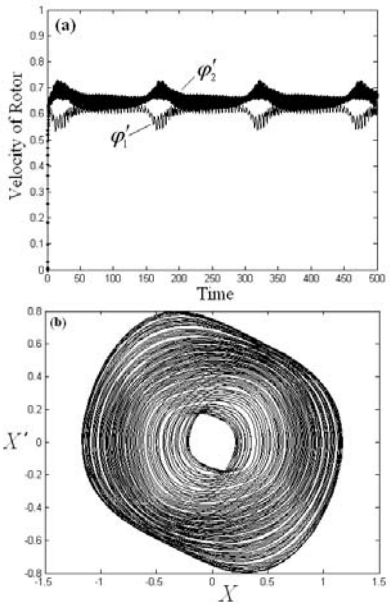

Figure 2. a) Velocity of rotors, b) phase plane for $\hat{a}_1 = 1$ and $\hat{a}_2 = 0.9$: self-synchronization by intervals.

Figure 2(a), shows that the rotors rotate in the same direction and arrive at some average angular velocity in steady state motion where the angular velocities are in phase and out of phase by intervals (the rotors synchronize phase and anti-phase). The frequencies of the rotors are below resonance (pre-resonance). Figures 2(b) shows the phase plane of the dynamical behavior of system, in this case, beat phenomenon.

The second set of numerical results, shown in Fig. 3, we illustrates the development of self-synchronization when the torques of each DC motor are equal $\hat{a}_1 = 1.0$ and $\hat{a}_2 = 0.93$.

Figure 3(a), shows that the rotors turn in the same direction and arrive it at some synchronous velocity in phase and in steady state motion, the velocities of rotors are below of resonance region (pre-resonance). Fig. 3(b) shows the dynamical behavior of system on phase plane of periodic motion.

In the fourth set of numerical results, shown in Fig. 4, we illustrate the absence of self-synchronization when the torques of each DC motor are different $\hat{a}_1 = 3.0$ and $\hat{a}_2 = 2.2$.

Figure 4(a), shows that the rotors turn in the same direction and arrive it at some average angular velocity in steady state motion, the velocities of rotors are in the post resonance region and synchronization anti-phase. Figures 4(b) shows the dynamical behavior of system on the phase plane of motion p-periodic.

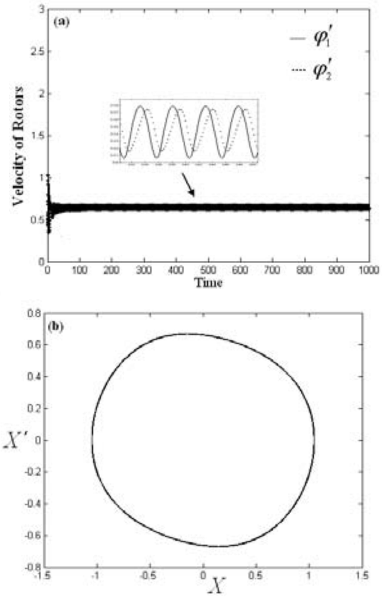

Figure 3. a) Velocity of rotors and b) phase plane for $\hat{a}_1 = 1.0$ and $\hat{a}_2 = 0.93$: self-synchronization.

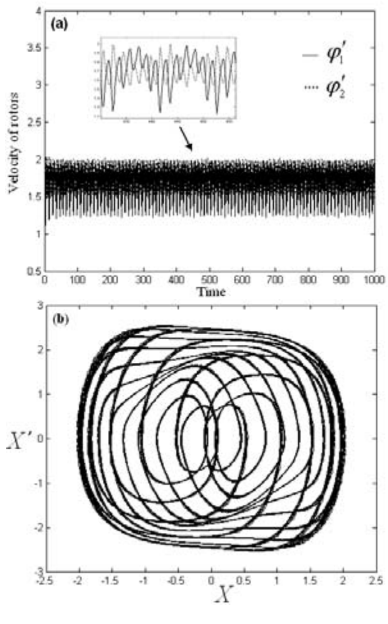

Figure 4. a) Velocity of rotors and b) phase plane for $\hat{a}_1 = 3.0$ and $\hat{a}_2 = 2.2$: absence of self-synchronization.

Figure 5, shows different regimes in the phase plane for the case of a non-ideal parametrically and self excited system when was considered the Eq. (3) in the motion equation and we varied the parameter α_2 of the parametric excitation of the second DC motor. Other parameters are fixed. For α_2=1, we see a development chaotic, α_2=1.5 we see a development periodic with p-period, α_2=2 we see that is tending to a limit cycle with 1-period, α_2=3 we see a development periodic with 1-period.

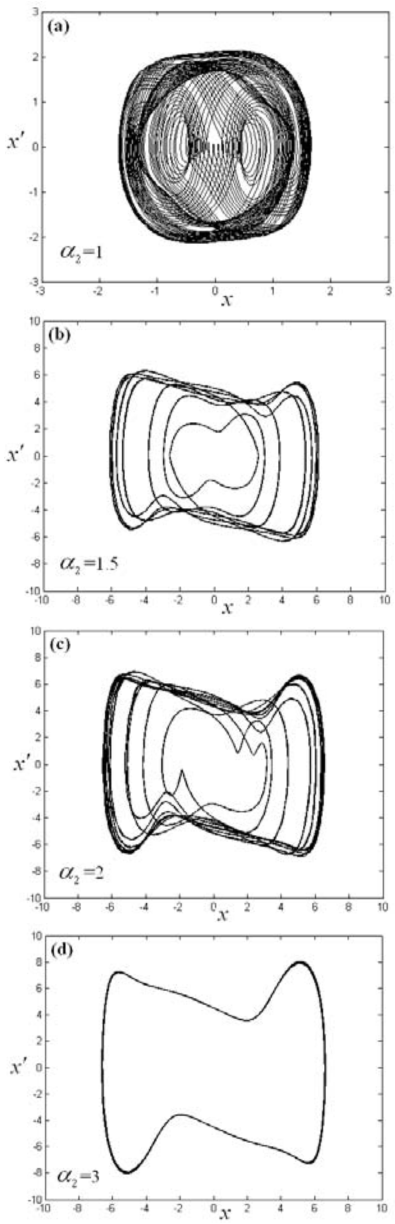

Figure 5. For values different of the parameter α_2 of the parametric excitation of second DC motor: (a) α_2 = 1, (b) α_2 = 1.5, (c) α_2 = 2 and (d) α_2 = 3.

Finally, Figure 6 shows the results of the influence self-excited coefficient q on the cubic nonlinear of the system. In this case, the interaction is examined when the two rotors arrive it at some synchronous velocity, with the cubic nonlinear coefficient fixed p and for values different self-excited coefficient q.

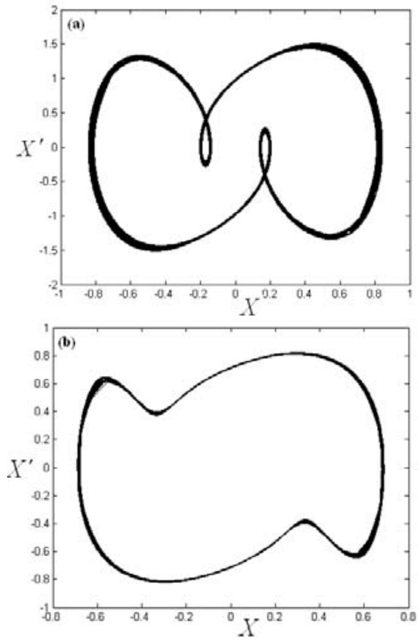

Figure 6. Phase plane for $\hat{a}_1$ = 2.0 and $\hat{a}_2$ =2.01: (a) q =0.05, p =16 and (b) q =4.0, p =16.

CONCLUSIONS

In this paper, we analyzed a type of non-linear phenomenon of self-synchronization and synchronization in non-ideal self-excited mechanical systems, i. e. we observe the behavior of the self-synchronization when is considered only mechanical interactions between the self-excited oscillating system and two unbalanced energy source with limited power.

The numerical simulation shows a bifurcation sequences on phase plane for slow transition through resonance.

We remarked that with control parameters (the constants torques) we might control the presence and absence of self-synchronization as synchronization in phase and anti-phase; in this case, we observe motions of regular (periodic, quasi-periodic) and irregular (chaotic).

From a mathematical point of view not yet applicable in the mechanical model, two different parametric excitations were placed in the equation of motion. We observe a bifurcation sequences from development chaotic until periodic for each variation of the parametric excitation.

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