Probabilistic fatigue life estimates for riveted railway bridges

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ABSTRACT: A large percentage of the railway bridges in the UK rail network and around Europe are of riveted construction exceeding in many cases 100 years of age. The remaining fatigue life of these bridges is difficult to estimate due to the uncertainties regarding the fatigue behaviour of wrought-iron and older steel material, which was used for their construction, as well as the actual loading and the number of cycles that these bridges have experienced in the past and will be experiencing in the future. This paper presents probabilistic fatigue life estimates for a stringer-to-cross-girder connection of a typical riveted rail bridge. On the loading side, the problem is randomised through the frequency of train traffic, dynamic amplification and the ratio between actual and calculated stresses. On the response side, S-N curves pertaining to the various assumed classes for the connection details and the Miner sum are also treated as random.

1 INTRODUCTION

Many riveted bridges that were built during the second half of the 19th century and the beginning of the 20th century are still in service today. Given their age, it is important to ascertain whether these bridges can be used safely in the future. However, the fatigue assessment of these bridges is not a straightforward task due to the high uncertainties associated with the applied loading and the fatigue phenomenon in general. The type of loading and the number of cycles that these bridges have experienced in the past is not well known due to lack of data regarding the past loading history. The problem is further compounded by the fact that prediction of the loading in the future entails even greater uncertainty. Furthermore, on the fatigue side, it is well known that experimental scatter is higher at low stress amplitudes near the fatigue limit (Schijve 2005). Since, in general, the stress levels experienced by riveted bridges are low it is evident that the problem of fatigue assessment of such bridges will be highly uncertain.

The use of probabilistic methods for the fatigue assessment of railway bridges allows the previously mentioned uncertainties associated with the loading as well as the material response to be treated in a formal way. Not surprisingly, the majority of the methodologies developed for the fatigue assessment of railway bridges in the past few years have been based on reliability methods.

In general, two different methods can be used in order to carry out a fatigue assessment, namely, the S-N method and the fracture mechanics method. The former, which is based on the S-N curve of the fatigue detail in question, is used in conjunction with Miner's rule (Miner 1945). By contrast, the latter method considers explicitly the growth of fatigue cracks and for this reason, it is more appropriate in cases where a fatigue crack has been detected. Since crack detection is mostly case-specific, most of the fatigue assessment methodologies that have been developed for railway bridges are based on the S-N approach.

Kunz & Hirt (1993) demonstrated that the fatigue limit as well as the model uncertainty have considerable influence on the probability of fatigue failure. Comparisons between a deterministic and a probabilistic fatigue assessment carried out by Bruhwiler & Kunz (1993) indicated that the latter provides more conservative results than the former. Subsequently, Kunz et al. (1994) developed a methodology for the fatigue assessment of steel bridges based on a modification of the S-N curves using fracture mechanics methods. By performing a sensitivity analysis, the authors concluded that remaining fatigue life estimates are more sensitive to the stress ranges experienced by the detail, its assumed S-N curve and the axle loads than to the traffic composition and the number of stress cycles.

Tobias & Foutch (1997) developed a reliability-based method for the fatigue evaluation of riveted railway bridges. Both fatigue loadings and strengths were described by probability distributions and the remaining fatigue life was calculated using a modified form of the Miner sum. Parametric studies carried out on a short-span, riveted bridge demonstrated that fatigue life estimates were sensitive to fatigue detail classification, variance in the fatigue resistance of the investigated detail, axle loads and spacings.

More recently, global finite element analyses of a typical wrought-iron riveted railway bridge indicated that the fatigue critical details are the inner stringer-to-cross-girder connections (Imam et al. 2005). The analyses were carried out under a historical load model (Imam et al. 2005), which was developed to represent rail traffic in the period 1900-1970, and present day traffic for the period 1970 onwards (BS5400 1980). Deterministic remaining fatigue life estimates of the connections were found to be sensitive to the level of dynamic amplification as well as the fatigue classification of the details. In the present paper the authors have extended their earlier methodology in a probabilistic manner that takes into account uncertainties associated with (a) on the loading side, the frequency of rail traffic, dynamic amplification and differences between actual and calculated stresses and (b) on the response side, the cumulative damage model and the appropriate S-N curve. This investigation is carried out on the most highly damaged stringer-to-cross-girder connection (Fig. 1), as identified previously through a global FE analysis of the riveted bridge. Details about the characteristics of the connection model can be found in the corresponding paper (Imam et al. 2005). Since the fatigue assessment presented herein is S-N based, far-field stresses (i.e. without stress concentration factors) are extracted from the FE model. An alternative approach using local stresses in the vicinity of any notches is currently being developed.

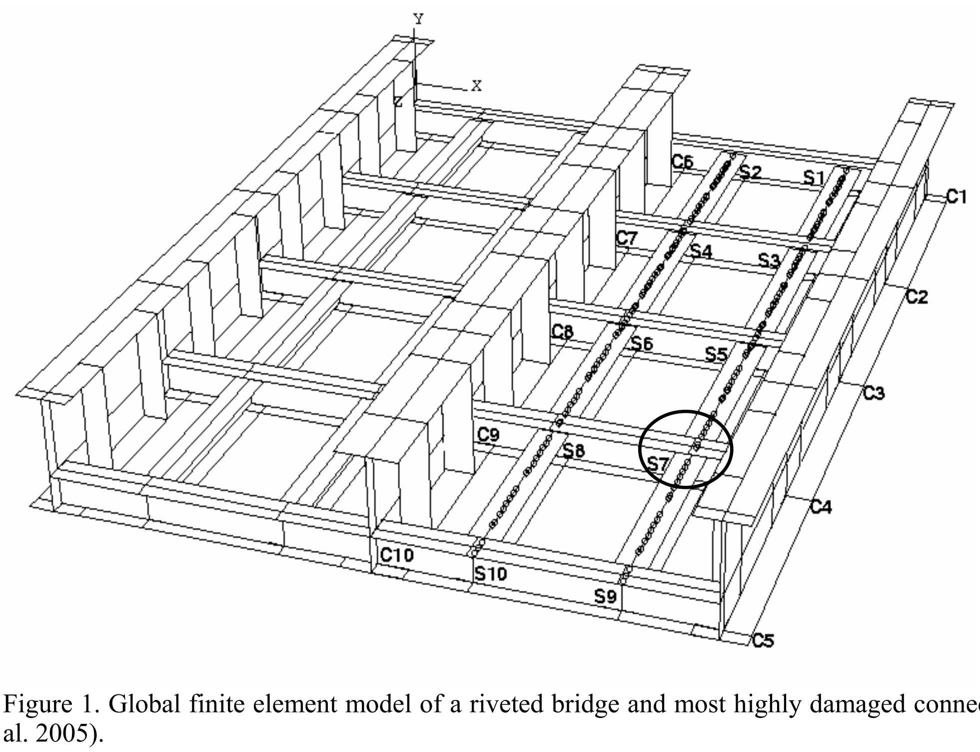

Figure 1. Global finite element model of a riveted bridge and most highly damaged connection (Imam et al. 2005).

2 PROBABILISTIC FRAMEWORK

2.1 *Random variables*

The S-N curve of a fatigue detail can be described by

$$NS^m = K \quad (1)$$

where N is the number of cycles to failure, S is the constant amplitude applied stress range and K and m are constants pertinent to the fatigue detail in question. Equation 1 is here randomised through K, which is assumed to be a lognormally-distributed random variable with a mean value equal to the BS5400 (1980) mean value and a coefficient of variation (CoV) of 0.3. The latter value is in accordance with the value proposed in BS5400, as is the endurance limit which is assumed to be deterministic and equal to 10^7 cycles. The variable m is also here taken as deterministic and equal to, depending on the fatigue class, the value proposed in BS5400. Note that the proposed random treatment of Equation 1 results in a parallel shift of the S-N line and hence in a random fatigue limit which is fully correlated with K. In the present treatment, the BS5400 change of slope from m to m+2 at 10^7 cycles is adopted.

For the case of variable amplitude loading, expressed through b distinct stress range blocks, Miner's rule (Miner 1945) is used to calculate fatigue damage according to

$$D = \sum_{i=1}^{i=b} \frac{n_i}{N_i} \quad (2)$$

where n_i is the number of applied stress cycles of stress range i and N_i is the constant amplitude number of cycles to failure at the same stress range. For a deterministic analysis, failure is typically assumed to occur when $D \geq \Delta$ where Δ is a damage limit taken equal to unity. However, the damage limit Δ has been found to exhibit considerable uncertainty, especially in the case of variable amplitude loading. Mean values ranging between 0.60 and 1.2 and CoVs ranging between 0.26 and 0.90 have been reported (Wirsching et al. 1995). Here, a lognormal distribution, which is typically used to describe Δ, with a mean value of 0.9 and a CoV of 0.30, is considered (Wirsching 1995). The number of applied stress cycles n_i, which depend on the annual frequency of trains passing over the bridge, is also treated as random. The uncertainty in the annual frequency of the trains is directly reflected in the variable n_i since both their CoVs are equal. The annual train frequencies for the period 1900-1970 (historical load model) were given by Imam et al. (2005), whereas the annual frequencies for the period from 1970 onwards are obtained from BS5400 (1980) considering medium traffic. For the purposes of subsequent analyses, lognormal distributions of n_i with means equal to the deterministic values corresponding to the annual frequencies previously discussed and CoVs of 0.137 are assumed (Ebrahimpour et al. 1993).

Annual response spectra (stress range histograms) at the location shown in Figure 1 for each period of the rail traffic model are produced by using the deterministic stress ranges obtained from the global finite element analysis of a short-span, wrought-iron riveted railway bridge (Imam et al. 2005). The deterministic stress ranges are calculated through rainflow counting from stress history results of the finite element analysis. The deterministic stress ranges are multiplied by a dynamic amplification factor (DAF) and a factor α, which accounts for the differences between stresses obtained from FE analysis and field measurements in riveted railway bridges. Measured stresses are, in most cases, lower than their calculated counterparts due to the longitudinal and transverse distribution of train axle loads through the rails, sleepers and ballast, the higher design axle loads given in codes as compared to the true axle loads obtained through field measurements, and, finally, the partial end fixity of the various bridge members. In relation to the last item, increase in the end fixity of stringers has been found to result in higher stresses at stringer-to-cross-girder connections leading to conservative fatigue life estimates (Imam et al. 2004). A normal distribution which is used here with a mean value of 0.80 for the α factor can be justified from previous studies on stresses obtained through field measurements and analytically/numerically (Byers 1976, Foutch et al. 1990, Adamson & Kulak 1995, Sweeney et al. 1997, DiBattista et al. 1998). Following Byers (1976), a CoV of 0.14 is assumed for α. The DAF is assumed to be normally distributed with a mean value of 1.10, which is typical for stringers in riveted railway bridges (Byers 1970, Szeliski & Elkholy 1984), and a standard deviation of 0.15 which is in agreement with field measurements on short-span railway bridges (Byers 1970).

2.2 *Problem formulation*

By combining Equations 1 and 2 and taking into account the four distinct loading periods (1900-20, 1920-40, 1940-70, 1970-), the limit state function can be formulated as follows

$$g = \Delta - \sum_{j=1}^{4}\sum_{i=1}^{a_j}\frac{n_{i,j}S_{i,j}^{\ \ m}}{K} \tag{3}$$

where $S_{i,j}$ are the stress ranges in the j[th] period including the effect of the DAF and the α factor and $n_{i,j}$ are the corresponding number of applied cycles, both obtained from the annual response spectra. The remaining fatigue life of the connection is estimated by extrapolating present-day traffic (BS5400, medium traffic) to the future. Based on the limit state function of Equation 3, the probability of failure can be defined as the probability of $g \le 0$. This probability, which is a function of time via $n_{i,j}$, is here calculated by using Monte Carlo simulation with 10^6 samples.

For the purposes of the probabilistic analysis, three different scenarios are considered as shown in Table 1. Three different fatigue classes are used to describe the constant amplitude fatigue behaviour of the stringer-to-cross-girder connection (see Fig. 1) under investigation. A wrought-iron detail class (Class WI) suggested by Network Rail (Railtrack 2001) used to classify wrought-iron riveted details and the BS5400 (1980) Class B and Class D details. The Class B S-N curve is here modified by a stress concentration factor of 2.4 (modified Class B) and can be used to represent the case of having low or no clamping force in the rivets. On the other had, Class D is used to represent riveted or bolted steel details with high clamping force. As can be seen in Table 1, differences between the three scenarios are introduced through the different detail classification and the use of different mean values of the random variables (Δ, DAF and α). The characteristics of the base model, which is thought to represent a realistic combination of the investigated variables, were discussed extensively in Section 2.1. The optimistic and pessimistic models can be viewed as upper and lower bounds for remaining fatigue life estimates.

Table 1. Mean value characteristics of various models used for remaining fatigue life estimates.

Variable	Pessimistic model	Base model	Optimistic model	CoV
Detail class	Modified Class B	Class WI	Class D	
K	2.34×10^{15}	9.33×10^{13}	3.99×10^{12}	0.30
m	4	4	3	Deterministic
Δ	0.80	0.90	1.00	0.30
DAF	1.20	1.10	1.05	0.14
α	0.90	0.80	0.70	0.14
$n_{i,j}$	*	*	*	0.14
$S_{i,j}$	*	*	*	Deterministic

*Obtained from corresponding annual response spectra.

3 RESULTS AND DISCUSSION

A typical annual response spectrum for the period from 1970 onwards for the connection of Figure 1 is shown in Figure 2. This histogram was obtained by using the base model's probabilistic DAF and α factor ($S_{i,4}$ = stress range×DAF×α) for each BS5400 train crossing the bridge. For this case, the annual frequency of the trains and consequently the applied number of cycles $n_{i,4}$ were assumed as deterministic. As shown in Figure 2, only a few stress cycles are above the fatigue limits of the three fatigue classes.

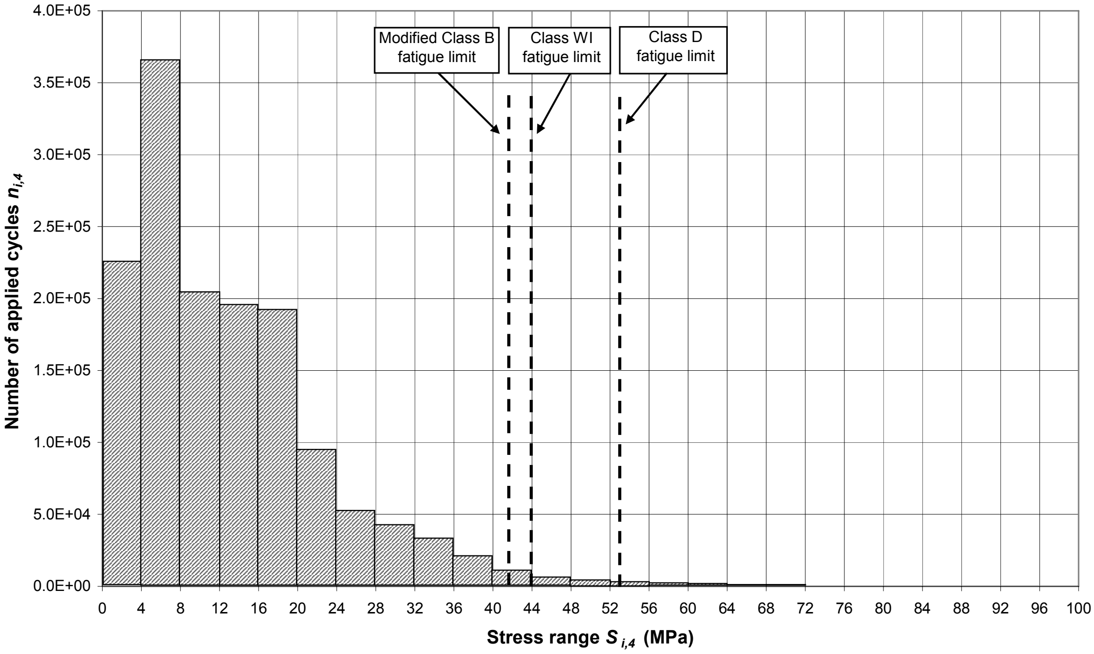

Figure 2. Annual response spectrum for period 1970 onwards.

Figure 3 depicts the probability of failure (P[$g \leq 0$]) versus time for the three models presented in Table 1. For presentation purposes, the vertical axis is shown on a logarithmic scale. The time scale begins from year 2004. The inset in the same figure depicts the probability density functions (PDFs) of the time to failure as well as the mean values and CoVs associated with each model. It can be seen that, for a 2.3% probability of failure and assuming a base model, the remaining fatigue life of the connection is approximately 480 years. For the same probability of failure and assuming a pessimistic model, the remaining fatigue life drops to 68 years. Finally, for the case of the optimistic model, the remaining fatigue life can be assumed to be virtually infinite. Mean remaining fatigue life estimates for the base, pessimistic and optimistic models are 671, 104 and 5079 years, respectively. The CoVs associated with each model can be seen to be quite similar ranging between 0.163 and 0.192. Even under pessimistic assumptions regarding the various variables which affect the problem are made, there is a considerable fatigue life reserve for the considered connection. However, it has to be mentioned that the results pertaining to the three models shown in Figure 3 are based on the assumption of a BS5400 future traffic and hence non-evolving loads. Given the number of connections present in the network it is also worth considering a limit lower than the 2.3% used here.

The effect of different variables on the time to attainment of a 2.3% probability of failure assuming a base model is shown in Figure 4. The results are obtained by varying each time a single variable (in the case of random variables, their mean value), while keeping the others fixed. It can be seen that the most substantial increase in fatigue life (180% assuming a Class D) is brought about through the detail classification which is in agreement with other observations (Kunz et al. 1994, Tobias & Foutch 1997, Imam et al. 2005). By contrast, the detrimental effect of loss of rivet clamping force, which is modelled here via the modified Class B, is manifest by reducing the time to attain the 2.3% failure probability by 40%.

The variable with the second greatest influence can be seen in Figure 4 as being the α factor. The reduction in the mean value of this factor from 0.80, which is the case in the base model (see Table 1), to 0.70 leads to an increase in the fatigue life at the 2.3% failure probability by about 110%. However, an increase in the mean value of α from 0.80 to 0.90 leads to a decrease in this fatigue life by about 50%. These effects may be attributed to the actual stress ranges the detail is experiencing and their relation to the fatigue limit. Clearly, field measurements can be of considerable help in determining the α factor thus leading to more accurate fatigue life estimates.

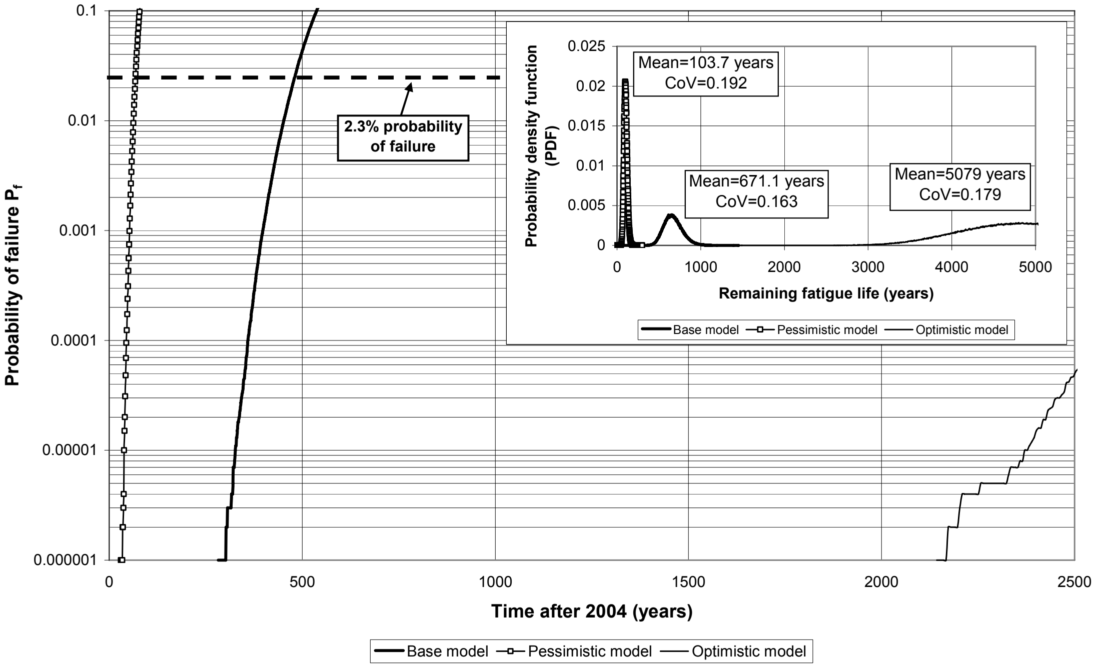

Figure 3. Probability of failure versus time for various models.

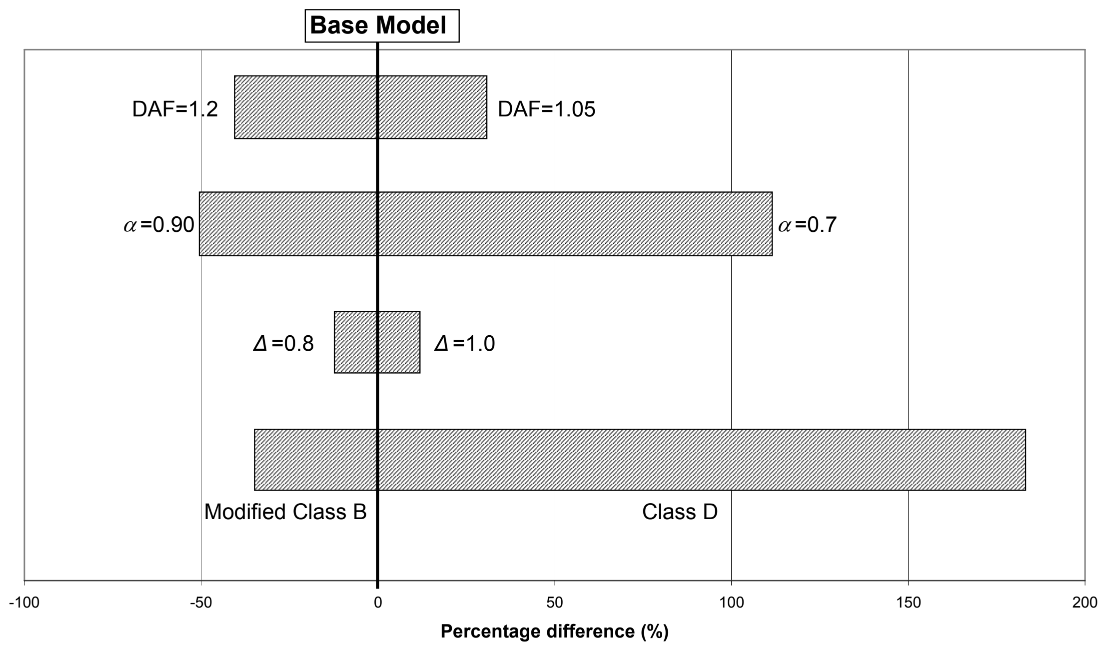

Figure 4. Effect of different variables on remaining fatigue life for a 2.3% probability of failure.

When compared to detail classification and α, the effect of the dynamic amplification factor and the damage index Δ can be seen to be less profound. The observation of the relative insensitivity of remaining fatigue life estimates brought about by the uncertainty in the cumulative damage model (Δ) was also made by Ebrahimpour et al. (1993).

The effect the uncertainty in the applied number of cycles $n_{i,j}$ has on the failure probability of the connection is shown in Figure 5. This uncertainty may be seen to represent the uncertainty in the annual train frequency. Results are presented for the original base model corresponding to a CoV of 0.137 (as shown in Table 1) and for two additional cases assuming CoVs of 0.3 and 0.5. It is evident that an increase in the CoV of $n_{i,j}$ is accompanied by an increase in the probability of failure of the connection. The differences between the CoV=0.137 (base model) and CoV=0.3 are small over the entire time period depicted in Figure 5. Although over time the differences for all three models tend to diminish, large differences by an order of magnitude may be observed early on between the base model and the CoV=0.5 model. Furthermore, for a 2.3% failure probability, the remaining fatigue life of the connection is estimated as 478, 474 and 457 years for CoVs of 0.137, 0.3 and 0.5, respectively. These observations are in broad agreement with Ebrahimpour et al. (1993) and Kunz et al. (1994) who showed that the probability of fatigue failure is not very sensitive to the rail traffic volume.

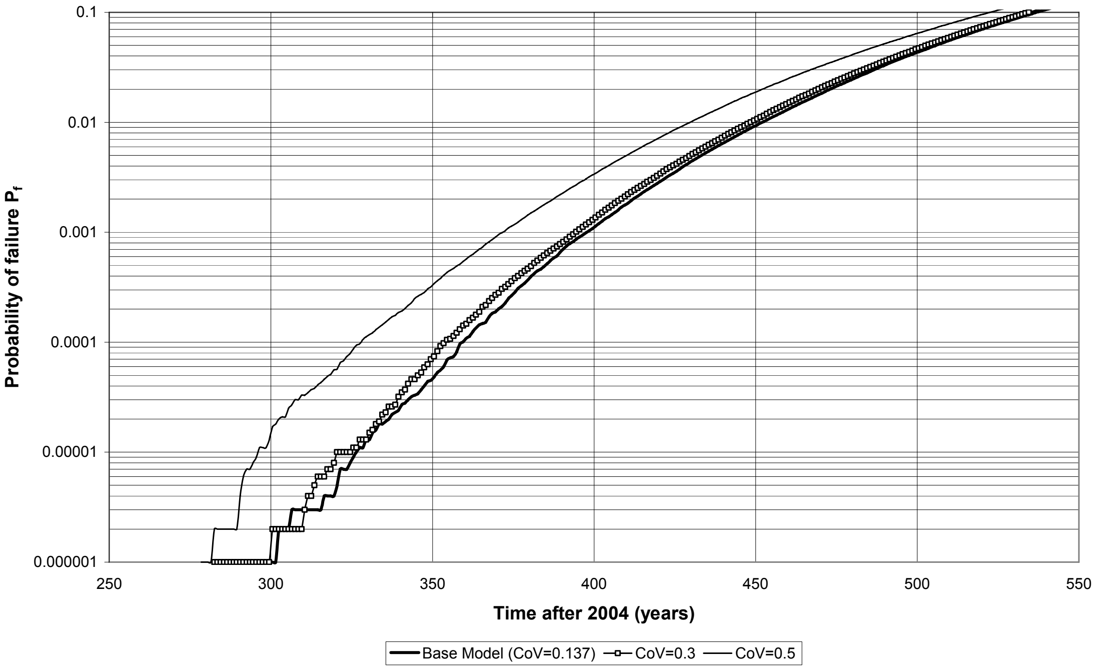

Figure 5. Probability of fatigue failure versus time for different levels of uncertainty in the applied cycles.

4 CONCLUSIONS

The probabilistic fatigue analysis of a riveted stringer-to-cross-girder connection of a typical short-span riveted railway bridge was presented in this paper. Uncertainties regarding the S-N curves of the assumed fatigue classes, the cumulative damage model (Miner sum), the applied number of cycles, the dynamic amplification and the differences between measured and calculated stresses were taken into account. It was found that the most heavily fatigue-loaded stringer-to-cross-girder connection has considerable characteristic fatigue life reserve even under pessimistic assumptions. Fatigue life estimates, which are always case-specific, were found to exhibit the highest sensitivity to detail classification, in other words the constant amplitude fatigue behaviour of the detail, and the factor α, which takes into account the difference between measured and calculated stresses. The cumulative damage model and the DAF were found to be of less importance. Uncertainties in the number of applied cycles were found to play an important role early on in the fatigue process. The results presented herein can be used to direct further research into the areas of maximum potential benefit: fatigue detail characterisation and stress range estimation through field observations. It should also be noted that this work has focused on S-N based assessment which is the first step in estimating remaining fatigue lives.

However, should cracks be present, the remaining fatigue life would have to be estimated using fracture mechanics approaches.

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