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A Quantitative Scoring Technique For Panel Tests of Color Vision

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Panel tests of color vision (eg FM100-Hue test) lack a common quantitative method for the scoring of cap arrangements. We describe a scoring method applicable to all panel tests that makes use of a novel technique to analyze test cap data, namely the calculation of a moment of inertia from the Color Difference Vectors (CDVs) of any arrangement pattern. Using the Farnsworth D-15 panel, as an example, we specify how to determine CDVs and demonstrate the benefits of calculating a moment of inertia for the analysis of these vectors. Moment of inertia analysis yields three factors which quantify cap arrangements: the first is the confusion angle which identifies the *type* of color defect; the second is the Confusion index (C-index) which quantifies the *degree* of color loss relative to a perfect arrangement of caps; and the third is the Selectivity index (S-index) which quantifies the amount of *polarity* or lack of randomness in a cap arrangement. A retrospective study on the results of 53 normal and 66 congenitally color defective observers is reported and provides normative data. We show that the technique differentiates between different types of color defect and provides useful clinical information regarding a loss of color vision. Likewise, a similar observation is made on a smaller sample of FM100-Hue results. A BASIC computer program is provided for anyone wishing to use the technique. Invest Ophthalmol Vis Sci 29:50–63, 1988

Panel or arrangement tests can be used for the evaluation of color vision defects[1] and perhaps the best known and most widely reported panel test is the Farnsworth-Munsell 100-Hue (FM100-Hue) test. Its popularity can be attributed to the fact that the result can be quantitatively scored[2–5] and compared to statistical norms[5,6] which makes it suited to clinical and scientific research. The Farnsworth Dichotomous test or D-15 panel is another arrangement test often used to differentiate between observers who have severe losses of color vision from milder color defectives and normals[7]; it may also be used to evaluate acquired losses of color vision.[1] The Desaturated D-15 (D-15DS) panel can be used to supplement standard D-15 testing for the diagnosis of milder congenital or acquired color defectives.[8] The main difficulty adopting panel tests other than the FM100-Hue for research purposes is that they lack a quantitative method for scoring their axis and total error score making them less suited to longitudinal or comparative studies.

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The usual scoring technique recommended for the D-15 and D-15DS panels[7,8] requires recording the number of major "crossings" made by a subject, with two or more "crossings" needed for a failure, as shown in Figure 1. However, the definition of a crossing is rather imprecise, especially along unusual axes or between more proximal caps; this can frustrate the diagnosis or monitoring of acquired losses of color vision because acquired defects may fail to produce errors typical of dichromatic observers (compare Fig. 1D–F with Fig. 5). Several investigators have suggested various modified scoring techniques in order to overcome this problem and to improve the predictive ability of these tests.[9–11] Bowman proposed a quantitative method for estimating a total error score by summing color differences between adjacent caps;[11] however, this method does not calculate an axis of confusion and is therefore limited in its application. Nevertheless, scoring color differences can enhance the clinical interpretation of a result and Bowman and Cameron have suggested that using both the D-15 and D-15DS panels together provides a viable alternative to the more lengthy and complicated FM100-Hue test if results are scored in this manner.[12] Data provided by Bowman et al support this claim by demonstrating that color difference scores are sensitive enough to detect losses of color vision due to aging effects[13] similar to those reported with the FM100-Hue test.[6]

What appears to be needed for all arrangement

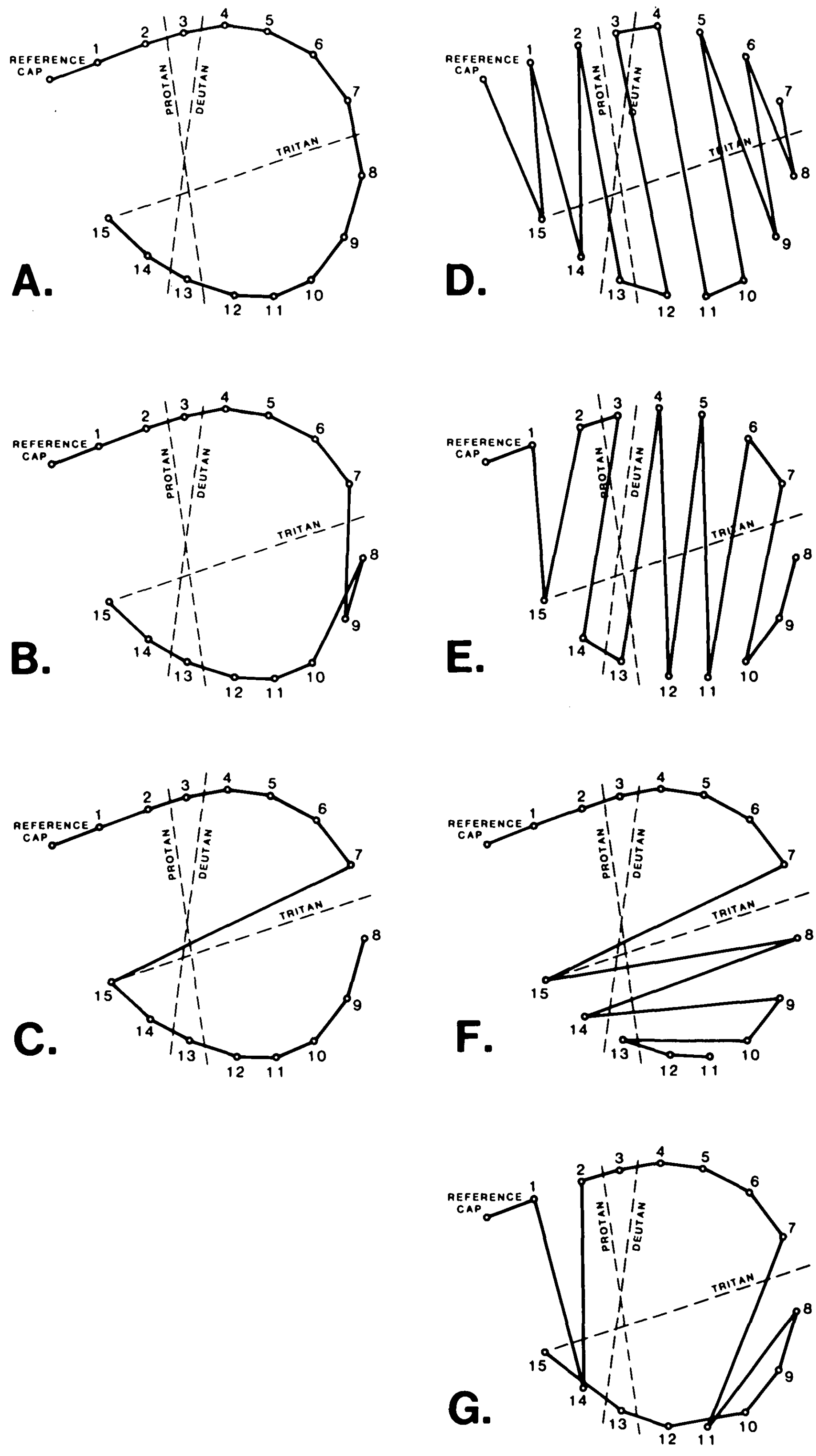

Fig. 1. Typical D-15 cap arrangements for various types of observers (modified after Farnsworth, 1947): (**A**) Normal-perfect arrangement, (**B**) Normal-minor transpositional error, (**C**) Normal-1 Tritan crossing, (**D**) Protanope, (**E**) Deuteranope, (**F**) Tritanope, (**G**) Deuteranomal.

tests is a common, quantitative scoring method that will provide both an estimate of the axis of confusion as well as an error score. This will allow mathematical and statistical analysis of the data and provide a vehicle for interpanel comparisons; it may also enhance a clinician's ability to make probabilistic statements

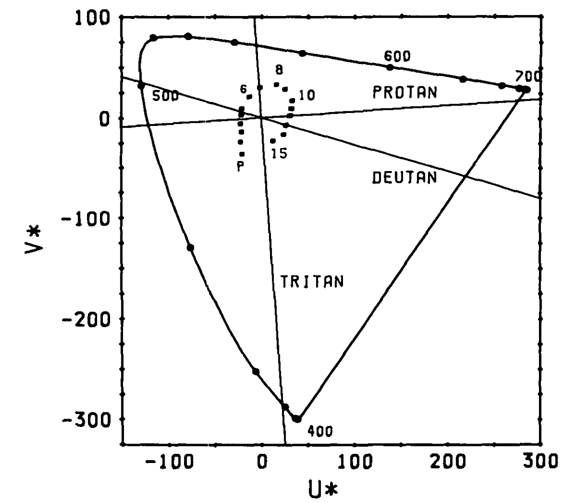

Fig. 2. The appropriate 1976 CIELUV space-plane showing the positions of the D-15 caps (filled symbols) relative to the spectral locus and the confusion loci of deutan, protan and tritan observers.

regarding the presence of subtle color vision defects. This paper describes such a quantitative procedure for scoring panel tests of color vision making use of a novel technique for analysing cap data. The method of analysis is based on Farnsworth's[14] original concept of transposing the caps into a uniform color space and calculating a difference in chroma for the adjacent caps of a given arrangement. Our method differs from Farnsworth's proposal because we use the actual calculated chroma difference for scoring (not a value of 1 as with the FM100-Hue) and we also estimate a hue angle, thereby generating a color difference vector (CDV) for any cap arrangement. The procedure may be used with any panel test and has the benefit of determining the following three factors: (1) an axis of confusion; (2) a measure of selectivity or randomness in the cap arrangement; and (3) an error score or estimate of the severity of color defect. We will demonstrate the principles involved using the D-15 panel as an example; however, we have also successfully applied the method to the FM100-Hue and D-15DS tests.

Materials and Methods

General Description of Scoring Technique

Our proposed calculation requires transforming the 1931 CIE tristimulus values of each cap into a uniform chromaticity space and determining the CDVs that exist between adjacent caps of a given arrangement. If the vectors are plotted in a manner that displays *relative* color differences between caps, ie originating at a common point, then they should align along a common confusion axis just as the confusion lines do in Figures 1D–F. Therefore vector direction will be a function of the type of color confusion made by the observer whereas its length will be a function of the degree of color confusion between adjacent caps.

Bowman suggested using the 1976 CIELAB color transform to calculate color differences[11] because it is recommended by the CIE for use with pigmentary colors.[15] He did note that the 1976 CIELUV space could also be used and it is our opinion that the CIELUV transform provides a better vehicle for the determination of vector resultants because it retains the linear relationships of the 1931 CIE color space[15,16] and of the dichromatic loci.

Principle of Calculation

Test caps are transposed into the 1976 CIELUV space (p. 165 of Wyszecki and Stiles[15]) and Figure 2 shows the result for the D-15 panel indicating the color confusion axes of dichromatic observers crossing at the reference illuminant (D65). Since the quantity L* (the correlate of lightness or Munsell value) has been chosen at a fixed level for all these tests,[8,14,17] the mathematical calculations can be simplified by considering a fixed plane in the three-dimensional LUV space (see Fig. 1(3.3.9) of Wyszecki and Stiles[16]) as shown in Figure 2. In Figure 2 it is apparent that dichromatic confusion lines tend to orient themselves at different angles, thus providing a diagnostic capability to determine type of color vision defect. A horizontal line in this color plane (Fig. 2) tends to lie along a red-green axis with the average protan locus being about +5°; the average deutan locus is some 12° below the right horizontal (−12°). On the other hand, a vertical line approximates the blue-yellow axis and the average tritan locus lies close to the vertical, being −85°.

Figure 3 shows the D-15 caps in the same plane of 1976 CIELUV space given in Figure 2 but replotted on a different scale indicating the CDVs seen with the normal (Fig. 3A) and protan (Fig. 3B) arrangements given in Figure 1A and 1D respectively. The length of each vector can be expressed as a *difference in chroma* and its direction as a difference in *hue angle* between the caps (p. 168 of Wyszecki and Stiles[16]).

Vector Analysis

Having created color difference vectors the problem becomes how to analyze this data. Figure 3 demonstrates some of the difficulties in applying normal vector addition concepts to this analysis. First, Figure 3A shows the resultant vector (R_n) obtained by adding all the normal CDVs together. It should be apparent from this diagram that the same resultant would

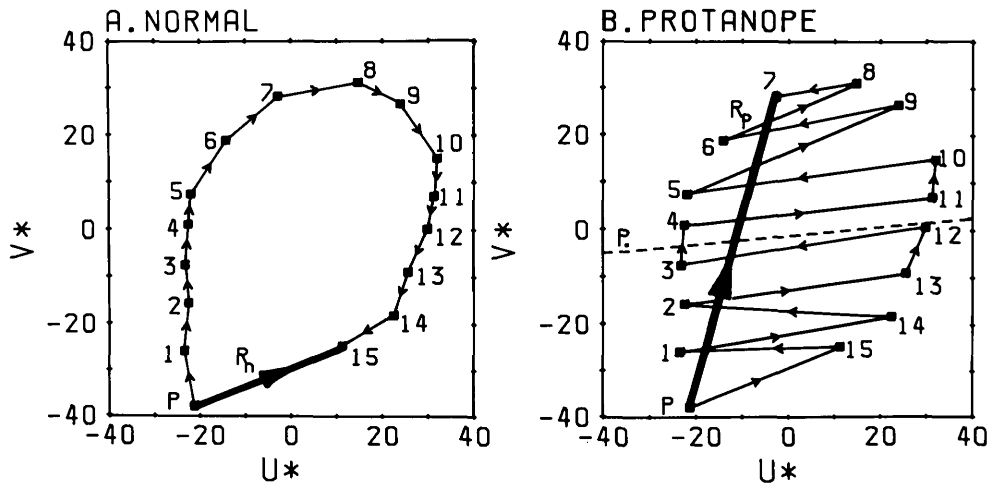

Fig. 3. The appropriate 1976 CIELUV space-plane showing the arrangement pattern and color difference vectors for: (A) Normal and (B) Protanopic observers. These plots are of the arrangements shown in Figure 1A, D respectively; protan confusion locus is given by a dashed line in Figure 3B.

be obtained for any cap series ending at cap 15 regardless of the arrangement of the intermediate caps. Thus a resultant obtained in this fashion would fail to indicate errors occurring before any cap ending a series.

A further problem is demonstrated in Figure 3B where it is seen that standard vector addition yields a resultant (R_p) that fails to reflect the correct confusion angle of a dichromatic observer. The protan confusion axis lies at some 5° above the right horizontal whereas the resultant obtained by vector addition is found to be +74° (Fig. 3B). Because vector addition fails to describe adequately the confusion locus of color defective observers and to reflect all errors in cap placements, we considered other methods of analyzing this data, adopting a technique that averages results by estimating a moment of inertia for these vectors.

Moment of Inertia Method

The problem of estimating an axis of confusion is illustrated further in Figure 4. In each of these five plots, *relative* color difference vectors have been replotted from diagrams like Figure 3, so that the "tail" of each vector is plotted at the origin and the "head" is marked with a square; cap numbers corresponding to the head of each vector are indicated. Figure 4A and 4B give data for normal and protanopic subjects replotted from Figure 3A and 3B; Figure 4C, 4D and 4E correspond to the deuteranope, tritanope and deuteranomalous data of Figure 1E, 1F and 1G. Vectors tend to align along a common axis for dichromats (Fig. 4B, C, D) whereas normal vectors (Fig. 4A) show greater angular scatter. What is needed is a method for quantifying the angle of alignment and severity of confusion from plots such as those similar to Figure 4. Our solution is to calculate *moments of inertia* for these plots and is performed in the following manner.

Imagine these plots as rigid figures; each square (head of the vector) has unit mass and is connected to the origin by a weightless, rigid bar (the "stem" of the vector). A moment of inertia may be calculated for this entire mass system about any axis passing through the origin and lying in the plane of the diagram. For example, in the case of the protanope (Fig. 4B) where most of the vectors lie close to the horizontal, the moment of inertia will be relatively large for a vertical axis because most of the squares (mass) are displaced a long way from this axis. Similarly the moment of inertia will be small for a horizontal axis. Note that a moment of inertia calculation considers only the general alignment of color difference vectors (eg horizontal, vertical, etc.) meaning that contributions from vectors with opposing angles (eg 0° and 180°) are additive, in contrast to the subtractive result obtained with vector addition; this ability is needed for the analysis of confusion axes because such vectors represent equivalent color confusions.

The technique requires solving for the "principal axes" which yield the maximum and minimum moments of inertia (these axes are at right angles to each other). The axis angle producing the minimum moment of inertia is our estimate of the *confusion angle;* for the protanope, Figure 4B, this angle is calculated as +9.7°. "Principal moments of inertia" can now be calculated for these two principal axes—eg the principal moment of inertia about the protanopic confusion angle will be small (we will call this the minor moment of inertia) whereas the moment of inertia about the axis at right angles (−80.3°) will be large (the major moment of inertia).

These major and minor moments of inertia could be used to quantify the severity of the defect; how-

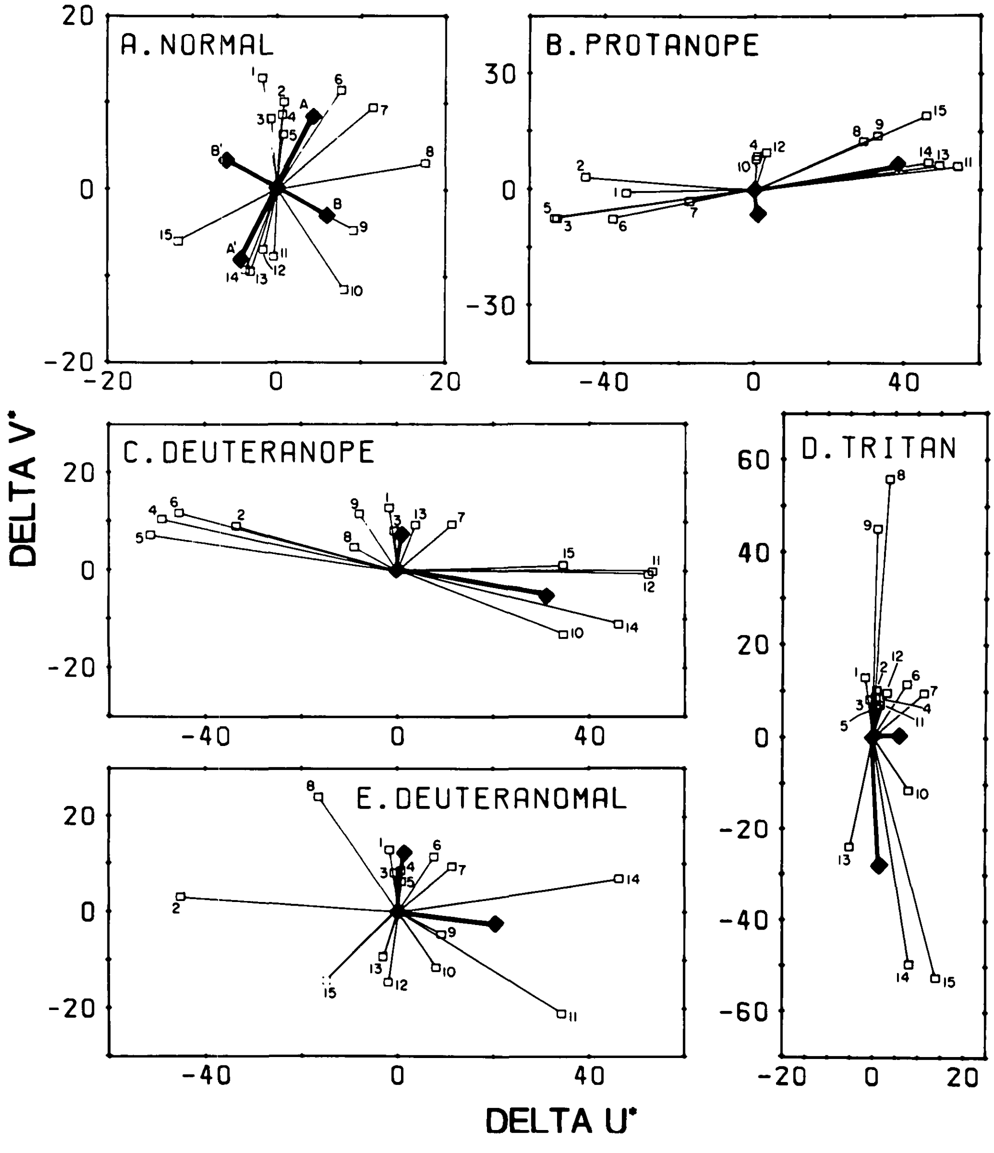

Fig. 4. Relative color difference vectors plotted for the following arrangements: (**A**) Normal (Fig. 1A), (**B**) Protanope (Fig. 1D), (**C**) Deuteranope (Fig. 1E), (**D**) Tritanope (Fig. 1F) and (**E**) Deuteranomal (Fig. 1G). Each vector terminates at a square (arrows omitted to prevent clutter) with numbers identifying the terminal cap. Resultant radii are indicated by solid lines and filled diamonds. All resultants have been shown in Figure 4A but only those lying to the right of vertical are given in other plots; see text for details.

ever, instead of each moment, we prefer to use the corresponding "radius of gyration" which is defined as that distance from the origin producing the same moment of inertia for the total mass (15 units) system. The advantage of using radii of gyration (compared to moments of inertia) is that they are expressed in the same units as the color difference vectors plotted in Figure 4 and so they are more readily understood in terms of these diagrams; the thick bars joining the origin to the diamonds in these figures correspond to the major and minor radii of gyration in each case. These radii of gyration can be represented on either side of the corresponding principal axis; for example, the major radius of gyration for the normal may be represented by the thick bars from the origin either to the diamond A or to the diamond A′ (Fig. 4A). Because only one of these two radii is needed to represent the major (A or A′) and minor (B or B′) resultants, we have chosen to standardize by using axes and radii whose angles are in the range −90° to 90° in Figure 4B–E and in Tables 1 and 2. Note that the major radius is plotted along the confusion angle (as defined above), thereby providing an index for the severity of color defect. The mathematical derivation of confusion angle and its principal radii is described in Appendix 1.

Practical Application and Normative Data

In order to demonstrate the result of applying our analysis and to present some normative values for the statistics that may be obtained by our method, we conducted a retrospective analysis of D-15 results obtained from 53 normal and 66 congenital color defective observers (12 protanopes, 10 protanomals, 23 deuteranopes, 17 deuteranomals and 4 tritans) who have been tested in our laboratory over the past 4 years. All observers gave informed consent, serving as controls in another unrelated study, and were free of any ocular or systemic condition that may affect their color perception.

Monocular testing was conducted under a MacBeth Daylight lamp (Newburgh, NY) (200 lux) with subjects given the same protocol of tests including: AO-HRR plates, Farnsworth's F2-Tritan plate, the Standard (SPP Types 1 and 2) plates, Nagel's anomaloscope and the D-15 panel (as well as other tests, some not relevant to this analysis), although only the D-15 results will be reported in this paper. Only observers who gave an unambiguous diagnosis at the anomaloscope have been included in the evaluation.

Subjects were told how to perform the test and had their result recorded without retest or practice. This can be considered as a demanding test situation because most people will improve their panel scores when tested binocularly or on retest.[6]

The age range of the 53 normals was 7 to 82 (median 33, Inter Quartile Range (IQR) 43.5) which is reasonably representative of the general population whereas the color defective group were younger with most (63/66) being under the age of 40; their range was 7 to 71 (median 22.5, IQR 8). Of the 27 anomalous trichromats, ten (37%) made no errors at the D-15 panel and their data are uninformative; therefore they were not included in the analysis, leaving 17 (five protanomals and 12 deuteranomals) of the original 27 anomalous trichromats. Likewise 45/53 (85%) normals have also been excluded because they made no errors in arranging the D-15 caps; only data from subjects who made *any* errors at the test were used. The D-15 results from four people with acquired color vision defects (two optic atrophies and two maculopathies) have also been included.

Results

This paper describes a method that can be used to quantitatively score any panel test of color vision. The cap tristimulus values needed for these calculations are available in the literature as Munsell values[8,17] and can be converted into 1931 CIE space (Table 1(6.6.1) of Wyszecki and Stiles[16]). Because the method requires a good deal of calculation we recommend using a microcomputer for this purpose and have developed a BASIC program to perform the necessary calculations which, in our case, were conducted on a North-Star Horizon computer (San Leandro, CA). The body of this program has been given in Appendix 2 for the D-15, D-15DS and FM100-Hue tests.

Results and Discussion of Moment Analysis

Scrutiny of Figures 4A to 4E indicates that our analysis provides an objective assessment of performance which is consistent with subjective evaluation of the results. For example, for the dichromats (Fig. 4B–D), the major radius aligns along an average of the directions of the long color difference vectors; the minor radius is much smaller, corresponding to the fact that there are few vectors whose angles differ much from the confusion angle and these vectors tend to be relatively short. The more uniform distribution of color difference vectors for the perfect arrangement (Fig. 4A) gives rise to major and minor radii which are more nearly equal (implying no obvious confusion axis); the major radius is much smaller than those for the dichromats (implying much better color discrimination). The major and minor radii of the deuteranomal (Fig. 4E) are intermediate between the dichromat and normal, implying a loss of red-green color discrimination which is not so severe as in, say, the deuteranope (Fig. 4C). One of the benefits of applying moment analysis to this problem is that both sets of data (ie confusions and correct cap placements) are used to determine the resultant axes with correct placements contributing to angle estimates.

The *angle* of the maximum radius provides an estimate for the average confusion axis of an observer whereas its length gives an estimate of the error score expressed as a *Confusion index* (C-index). The ratio of the major and minor radii, called the S-index for *Scatter index*, (S-index = major radius/minor radius) may also be used to describe the degree of scatter, polarity, randomness or selectivity evident in an observer's cap placements. If an anarchic or random pattern occurs (Fig. 5) then this index may be expected to be relatively small because no single axis of orientation predominates cap placement. High indices indicate strongly polar orientations typical of dichromatic observers (Fig. 4B–D) and serve to confirm the visual plots seen with standard record sheets.

A total error score (TES) can be calculated from the minor and major radii by obtaining the square root of their sum of squares. Whenever the S-index is large, ie with highly polar arrangements, this error score will be approximated by the length of the major radius because the minor radius will have little affect on the overall TES. Later we will argue for adopting the length of the major radius as an index of error rather than a root mean square value (TES).

Bowman et al propose the use of a Color Confusion Index (CCI, see p. 230 of Bowman et al[13]) to express error scores and we feel this index has several advantages over a raw score. Its prime advantage is to reduce the effects of local non-uniformities in a color space by normalizing results to a perfect cap arrangement. Therefore we propose adopting a similar index, but in order to avoid any possible confusion with the CCI, and to demonstrate the different origins of the

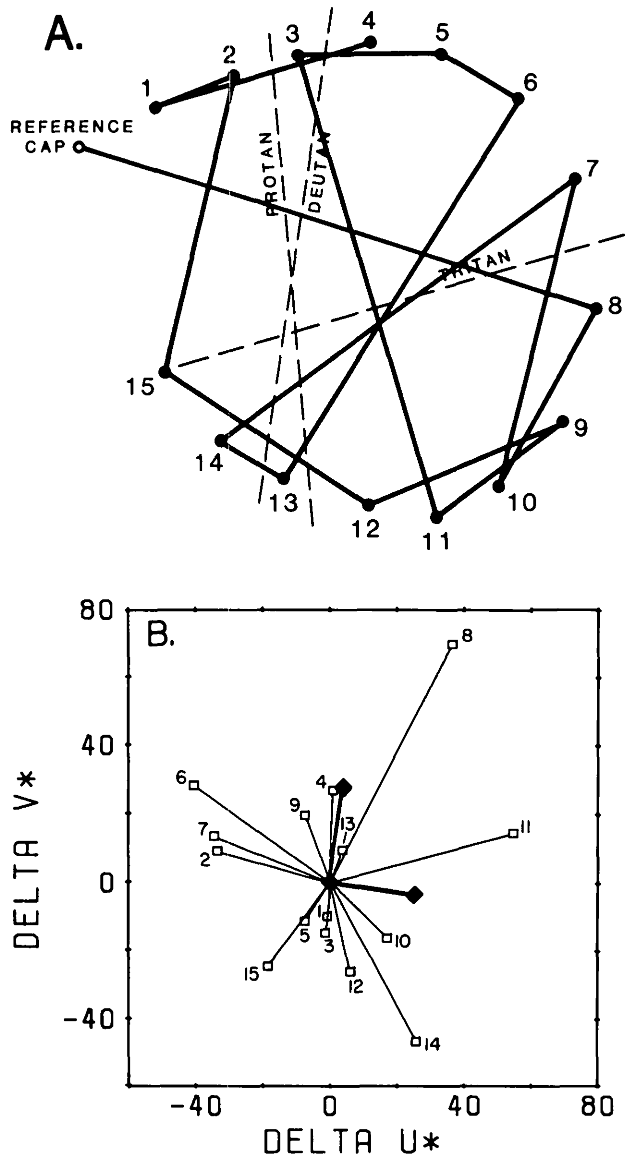

Fig. 5. An anarchic D-15 cap arrangement made by an observer with Inherited Optic Atrophy (DIDMOAD Syndrome). (A) Standard D-15 plot and (B) relative color difference vectors with resultant moments.

two ratios, we have called the ratio calculated from our technique the Confusion-index (C-index). The C-index is derived by dividing the length of a subject's maximum radius by the maximum radius obtained for a perfect arrangement of caps (ie no errors; C-index = Subj. Max. radius/Max. radius for no errors) and, by definition, a perfect arrangement of caps will give a C-index of 1.0. Expressing a score as a C-index allows comparisons of performance across different tests[13] and eliminates the existence of a high error score for a perfect arrangement, which is psychologically undesirable for any clinical test. The C-index could have been defined as a TES ratio, in which case it would better reflect anarchic arrangements where the TES can be expected to have a substantially higher value than the major radius. From some of our data we found that a TES ratio decreased the C-index obtained from polar arrangements and felt that emphasizing polarity was of greater clinical importance; therefore we consider the initial proposal satisfactory since a TES ratio provides few additional benefits (eg the arrangement in Figure 5 has a C-index of 3.01 using a ratio of radii and 3.28 using a TES ratio).

Figure 4 demonstrates the results of our analysis by indicating the principal radii for some of the cap arrangements shown in Figure 1. Normal color difference vectors (CDVs from Fig. 3A) are plotted in Figure 4A and the resultant axes do not reflect confusions but normal cap positioning. In this case the values of the axes and radii are rather meaningless other than to indicate normality of cap arrangement. Figure 4B–E demonstrate the ability of the technique to determine the color confusion axes of congenital color defective observers. The various parameters found by the analysis have been listed in Table 1.

From the data of Table 1 it is apparent that not only can the technique discriminate between the different types of congenital color defects by differences in their angles but it can also quantify various levels of severity of color defect by the confusion index (C-index). We propose that three values are needed to describe fully an observer's score on any panel test and these are shown in bold typeface in Table 1. The first is the *angle* which identifies the primary axis of color confusion. Red-green color defects tend to give horizontal axes with protans falling above the right horizontal (positive angles) and deutans below (negative angles) whereas blue-yellow defects tend to give vertical angles (see Table 1). The second value is the *S-index* which gives an idea of the selectivity or scatter in the cap arrangement. Random or non-polar arrangements listed in Table 1, such as a normal arrangement or that given in Figure 5, have a low S-index (1.09–1.38) whereas the polar arrangements of dichromatic observers have higher values (4.74–6.12). Even the polarity of a mild deuteranomalous arrangement is correctly reflected by the intermediate value of this index (1.68). The final value is the *C-index* which can be used to estimate the severity of a color confusion and to compare results obtained on different tests. From Table 1, a C-index greater than 1.77 may be expected to indicate an abnormal D-15 cap arrangement with congenital color defectives having values as high as 4.21.

Results of Retrospective Analysis

Figures 6 and 7 are polar plots showing the C-index and S-index as a function of confusion angle for all observers and Table 2 lists individual results for the

Table 1. Results of vector analysis* for the cap arrangements given in Figures 1 and 5

			Radius					
Type of cap arrangement	*Figure*	***Angle***	*Major*	*Minor*	*TES*	*TCDS*	***S-index***	***C-index***
Normals:								
No error	1A	**+62.0**	9.2	6.7	11.4	165.0	**1.38**	**1.00**
Minor error	1B	**−12.1**	9.8	9.2	13.4	182.8	**1.07**	**1.06**
Tritan error	1C	**−80.8**	16.3	6.4	17.5	201.7	**2.57**	**1.77**
Congenital color defectives:								
Protanope	1D	**+9.7**	38.9	6.4	39.4	537.2	**6.12**	**4.21**
Deuteranope	1E	**−8.8**	35.6	7.4	36.4	478.5	**4.82**	**3.86**
Tritanope	1F	**−86.8**	28.2	6.0	28.8	336.6	**4.74**	**3.06**
Deuteranomal	1G	**−8.7**	20.5	12.2	23.9	297.2	**1.68**	**2.22**
Acquired color vision loss:								
DIDMOAD	5	**+81.7**	27.7	25.4	37.6	487.2	**1.09**	**3.00**

* Values appearing in bold typeface in the Table are recommended by the authors for comparative purposes; see text for details.
TES = total error score; TCDS = Bowman's Total Color Difference Score[11] calculated in LUV space; Angle = resultant confusion angle; S-index = selectivity index (polarity); C-index = confusion index (severity).

normals and acquired defectives as well as group averages for congenital defectives by type of color defect. The results shown in Figures 6, 7 and Table 2 confirm that angle serves to dichotomize protans from deutans in all but three cases; these observers were mild anomalous trichromats and will be discussed later.

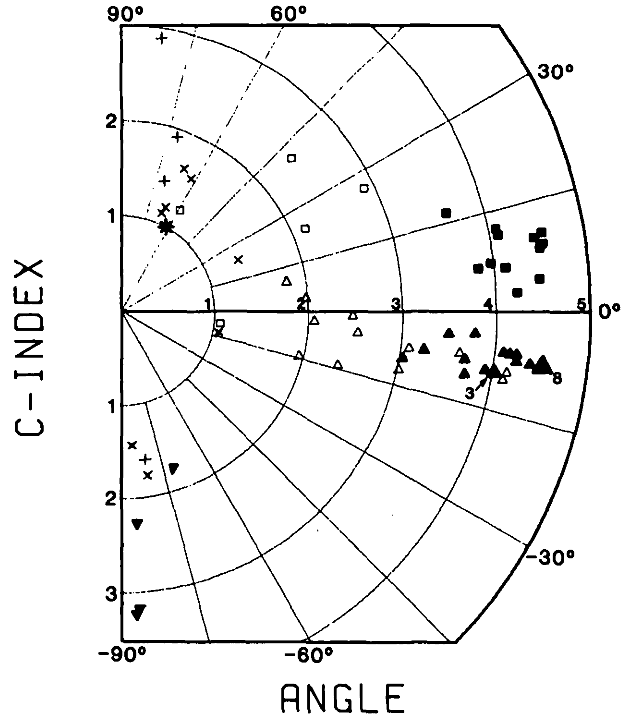

Fig. 6. Relationship between the C-index (severity) and angle (orientation of resultant major radius of gyration) found on the D-15 panel in our retrospective study. Symbols represent: normals-no errors (*); normals N1 to N8 of Table 2 (X); 12 protanopes (■); five protanomals (□); 23 deuteranopes (▲); 12 deuteranomals (△); four tritans (▼); and four observers with acquired losses of color vision (+). Values given next to the large deuteranopic triangles indicate the number of subjects with this common datum point.

The average protanopic angle is +8.8° (Table 2) with individual values ranging between +3° to +17° (Fig. 6) whereas the average deuteranopic angle is −7.4° (Table 2) and its range is −4° to −11° (Fig. 6). It would appear reasonable to suggest that the horizontal be used to distinguish protans from deutans since the divisor of the color defective group means (Protan-Deutan) is +0.7°. On the other hand, tritans give more vertical and negative angles (> −70°, see Fig. 6). Figure 6 also indicates that angle estimates are less likely to be accurate when fewer errors are made (ie low C-index) because cap placements are more

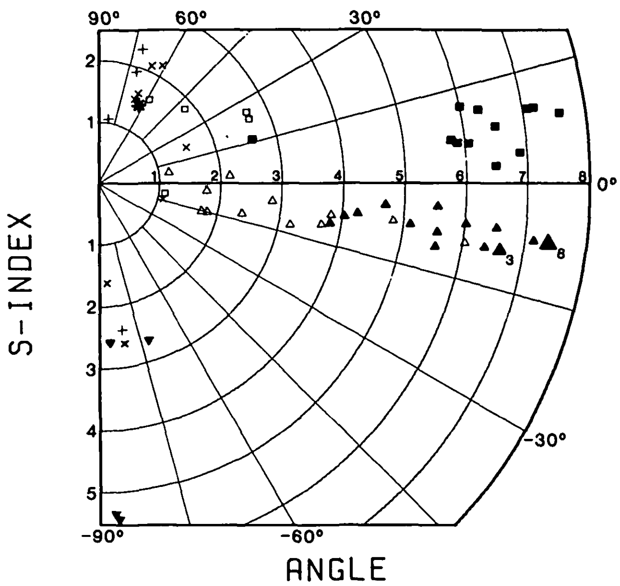

Fig. 7. Relationship between the S-index (polarity) and angle (orientation of resultant major radius of gyration) for the same observer groups given in Figure 6.

Table 2. Results of analysis performed retrospectively on the CDVs of various groups of observers who made errors at the D-15 panel (data for perfect arrangement given in bold typeface)

		*Results**						
					Radius			
Type of color vision†	*Number in sample*	*Angle*	*S-index*	*C-index*	*Major*	*Minor*	*TES*	*TCDS*
Normals:								
No error	**45**	**62.0**	**1.38**	**1.00**	**9.2**	**6.7**	**11.4**	**165**
N1(69)	1	−12.1	1.07	1.06	9.8	9.2	13.4	183
N2(52)	1	−80.8	2.57	1.77	16.3	6.4	17.5	202
N3(9)	1	65.8	2.12	1.64	15.2	7.2	16.8	213
N4(41)	1	23.2	1.55	1.36	12.5	8.1	14.9	198
N5(51)	1	67.4	1.53	1.12	10.4	6.8	12.4	177
N6(73)	1	61.7	2.20	1.58	14.6	6.7	16.1	202
N7(79)	1	−85.3	1.58	1.43	13.2	8.4	15.6	217
N8(82)	1	66.5	1.63	1.19	11.0	6.8	12.9	185
Acquired color defects:								
Optic atrophy A.	1	81.7	1.09	3.00	27.7	25.4	37.6	487
Optic atrophy B.	1	−80.8	2.35	1.60	16.3	6.4	17.5	202
Maculopathy A.	1	71.8	2.31	1.92	17.7	7.8	19.3	256
Maculopathy B.	1	71.3	1.95	1.44	13.3	6.8	14.9	211
Averages for congenital color defectives:								
Protanopes	12	+8.8	6.16	4.20	38.8	6.6	39.4	525
Protanomals	5	+28.3	1.97	1.95	18.0	8.2	20.4	253
Deuteranopes	23	−7.4	6.19	4.10	37.9	6.3	38.4	525
Deuteranomals	12	−5.8	2.99	2.75	25.4	9.6	27.5	350
Tritans	4	−82.8	3.94	2.60	24.0	6.4	24.9	300

* Individual data except for congenital color defectives where group means are listed. TES = total error score; TCDS = Bowman's Total Color Difference Score[11] calculated in LUV space; Angle = resultant confusion angle; S-index = selectivity index (polarity); C-index = confusion index (severity).
† The value in parentheses is the subject's age.

likely to be of a random nature, producing less polarity in the plot (low S-index for these individuals in Figure 7); this result confirms a similar impression gained by clinical experience, namely that it may be difficult to diagnose the type of defect in the presence of few or minor crossings.

Comparison of Results to a Clinical Assessment

It is important to establish that our analysis does reflect accurately the type, degree and polarity of the underlying color defect, ie that it does not introduce artifacts peculiar to the scoring method. It is also necessary to set normal limits for the C-index and S-index to separate normals from the severe *congenital color defective group* as was intended by Farnsworth.[14] These values may not be as useful with acquired color defects because acquired defects do not always show trends typical of congenital color deficiencies.

In the preceding section it was noted that three anomalous observers were misclassified as to type of color vision defect. The D-15 is known to misclassify anomalous trichromats as to type of defect (11% of anomalous trichromats may be misclassified[18]) and perhaps the inaccuracies noted here are inherent in the test and do not arise as a consequence of the method of analysis. To further explore this possibility we conducted a visual inspection of the plots for those three anomalous trichromats who had been misclassified. These results indicate the following: one observer, a protan, made one minor transpositional error and therefore the angle does not indicate a color confusion; a second observer, a deutan, (angle 3.8°, see Figs. 6, 7) made crossings between caps 1–15 and 13–2 which is consistent with a protan axis and constitutes a misdiagnosis on behalf of the test (1 of 56, 2%); the final observer, another deutan, made three crossings between caps 2–15, 12–7 and 3–8. This latter pattern is rather anarchic as is evidenced by its low S-index of 1.15 (see Fig. 7) and would have to be classified as ambiguous.

Therefore the results suggest that most of the calculated angles (53 of 56, 94%) correctly identified type of color defect and agreed with visual assessments of the standard plots; there was one case (2%) of misdiagnosis and one case (2%) of ambiguity due to the random nature of cap placements. Our findings compare well with Helve's study[18] where 6% (9 of 148) of all congenital color defective results were reported as ambiguous and 1% (2 of 148) were misdiagnosed as to their axis of orientation. It is possible that a greater discrepancy may be seen with a larger observer sample; however, we believe that angle estimates using our technique provide a reliable and accurate measure of the type of color defect especially when numerous errors (severe defect) are made.

If we assume Farnsworth's criterion of two crossings needed for a failure (where a crossing has been taken as anything greater than a minor transposition) and calculate the C-index for different crossing combinations, then the lowest theoretical values are found to be 1.60 along a red-green axis and 1.34 along a blue-yellow axis. Likewise, a similar analysis for the S-index indicates that the lowest failing values are 1.68 along a red-green axis and 1.82 along a blue-yellow axis. Since most congenital color defects lie along a red-green axis it would seem reasonable to suggest that the calculated value of 1.60 be used for the C-index to separate normals from congenitally color defective observers.

Applying this criterion (C-index = 1.60) to our sample of observers would fail 2/53 (4%) normals and pass 2/52 (4%) of the red-green color defectives (Table 2, Fig. 6). In fact the lowest C-index obtained by any of the five red-green color defective observers who *just failed* at the test (two crossings) was 1.79 (range 1.79–2.14), suggesting that a value of 1.6 sets a stringent cut-off criterion. If it is considered important to correctly identify all normals, as suggested by Farnsworth,[7] then a C-index of 1.78 could be used; this would pass all normals without affecting the pass rate of protans or deutans. More importantly, from a clinical standpoint, it would correctly dichotomize all those red-green color defectives who made substantial errors at the panel (Fig. 6), as was intended by Farnsworth.[14] Further refinement of the fail criterion was not possible from our data because of the small numbers of normals who failed the test. The S-index could be adopted to differentiate between normals and congenital color defectives but it produces a poorer dichotomy between these groups (Fig. 8) and we see no benefit in using it for this purpose.

Figure 8 plots the relationships between the major and minor radii and the indices outlined in this paper for our observer groups. In this figure the lines sloping up and to the right (numbered 1 to 8) give values for the S-index whereas the scale at the top of the figure relates to the C-index; the dashed line indicates one possible criterion for failure (C-index = 1.78). From Figure 8 it is evident that dichromats (with the exception of one 10-year-old protanope who gave a random arrangement) have high values for both indices, normals have low values for both and anomalous trichromats (who make errors at the D-15) score somewhere in between the normal and dichromatic group. It is also apparent that observers with acquired color vision defects can give both a high C-index but a low S-index (Fig. 8) due to their overall losses of color

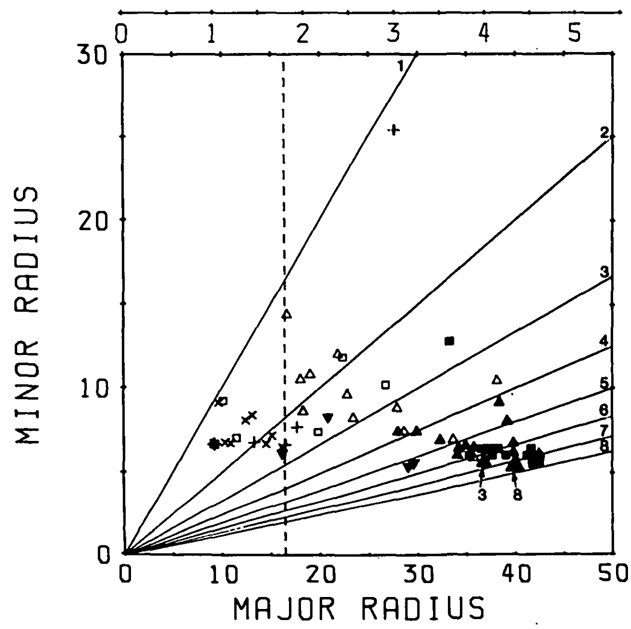

Fig. 8. Minor radius versus major radius for the same observer groups shown in Figures 6 and 7. The scale at the top of the diagram gives the C-index (severity) and the lines sloping upwards and to the right give equal S-index (polarity) values. The dashed line represents a C-index value of 1.78; see text for details.

perception (achromatopsia). This was found for one observer in our group who had advanced optic atrophy (uppermost point, Fig. 8) and may be useful in differentiating acquired from congenital color vision defects, especially when large losses of color perception are involved. The other cases of acquired defects gave tritan errors which place them in the congenital or normal regions of the plot.

If normals make errors at this panel, then from Figure 8 most (5/8) are seen to be of a random, nonpolar nature (S-index < 2.00) although some may show polarity along an axis (S-index > 2.00); a blue-yellow axis was found with those three observers who had large S-index values in our study (angles in Table 2 for subjects N2, N3 and N6).

Discussion

The purpose of this paper was not specifically to set normative data for the D-15 panel but only to propose a vector fitting program for the analysis of panel tests of color vision. However, we feel that our suggested values for the indices are realistic and prove a good starting point in the absence of a larger and more formal study. Further refinement of these values would need more extensive experimental data on variations in normal and anomalous trichromatic performance, especially the effect of age, since most of our errors were made by older (33% of normals

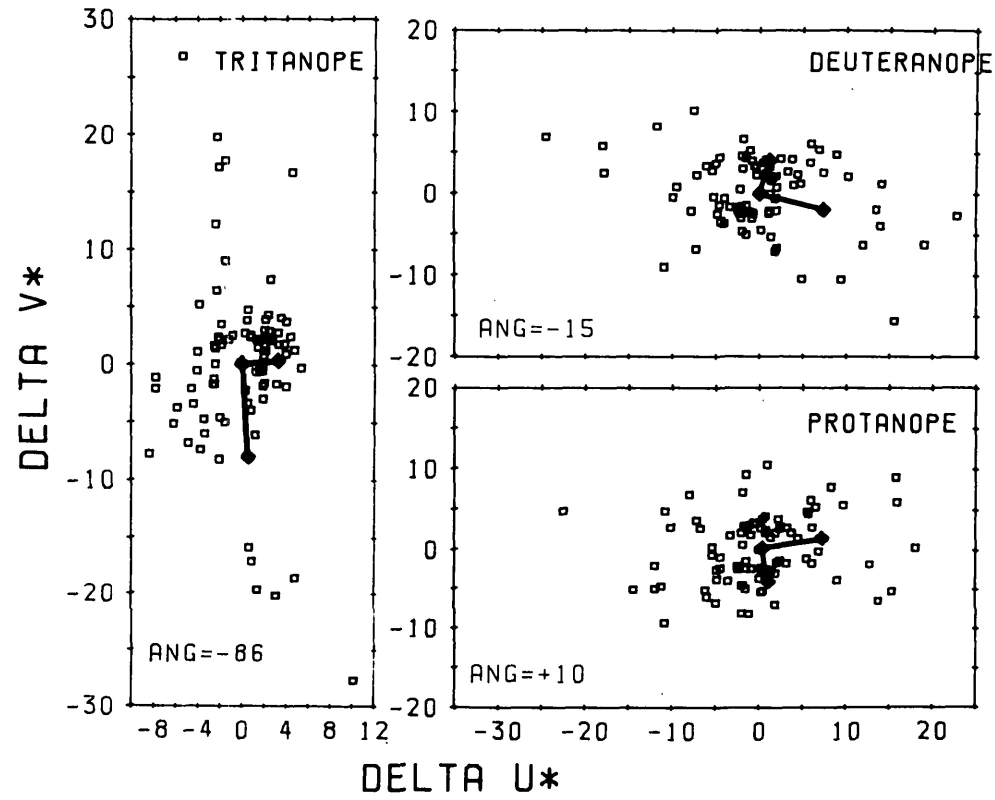

Fig. 9. Relative color difference vectors for protanopic, deuteranopic and tritanopic arrangements given on the FM100-Hue by three observers used in our study. The major and minor radii are shown in this diagram as thick lines with filled diamonds whereas other data points are plotted without vectors; see text for details.

over age 45 made errors—Table 2) or very young observers. There also may be value in setting different criteria depending upon whether a congenital (red-green) defect or an acquired color vision loss is being evaluated.

Although we do not report any findings for other tests, initial results obtained on the D-15DS and FM100-Hue panels are equally encouraging, giving a similar dichotomy in angle estimates as reported in this paper for the D-15 panel. We expect similar values of angle, S- and C-indices for most panel tests but note that there may be some between-test variability in these values because of different cap locations in the color space, local non-uniformities of color space or differences in test procedure. Figure 9 shows the relative color differences and resultant radii obtained with our procedure on the FM100-Hue panel for three dichromatic observers (vectors not plotted to prevent visual clutter). FM100-Hue data analysis yields smaller values for both the S-index and C-index primarily because the major radius is smaller and the minor radius is larger than that seen with the D-15 panel. The size difference arises because smaller vectors are obtained with the FM100-Hue test (compare Fig. 9 with Fig. 4B, C) due to the test procedure adopted with this panel. The FM100-Hue test is presented to the subject in sections or boxes, containing start and terminal caps, thereby denying any opportunity for making diametric errors as with the D-15 test. This means that our method yields smaller values for the FM100-Hue test and that it may not be as suited to the analysis of FM100-Hue data, although initial evaluation of the results of 16 color defective observers (eight dichromats and eight trichromats) indicates that the technique provides realistic and reliable estimates of angle as well as other test parameters. The suitability of this technique for the FM100-Hue test will only be apparent after a larger proving trial, but in the meantime we suggest that the stated values be used and modified, as needed, with the accumulation of more experimental results. In the interests of obtaining population norms for the indices mentioned in this paper the authors encourage correspondence from clinicians or researchers who decide to adopt this method of analysis.

Key words: color difference vectors, color vision, color vision testing, Farnsworth D-15 panel, Farnsworth-Munsell 100-Hue test

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Appendix 1

Calculation of the Confusion Axis and Other Parameters Using the Moment of Inertia Method

In Appendix Figure 1, OQ represents one of the 15 color difference vectors replotted from a diagram such as Figure

4; OU and OV are the horizontal and vertical axes. We require to find the moment of inertia, I, about an axis through the origin such as OX (inclined at an angle A to the horizontal) and then find the angles of OX which yield the maximum and minimum moments of inertia. Let the coordinates of the Q be u_n and v_n where n is the cap number (1 to 15) and let the distance of Q from the axis OX be y_n. Then the total moment of inertia about this axis is

$$I = \sum y_n^2$$

where the summation is over all 15 caps and it is assumed that the head of each vector has unit mass. It may be shown that

$$y_n = v_n \cos A - u_n \sin A$$

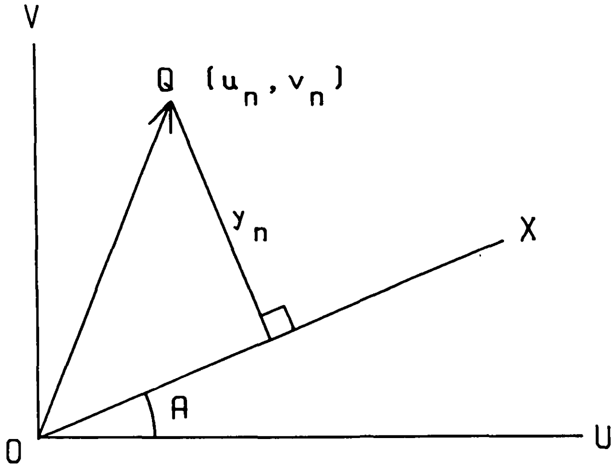
and this relation may be substituted into the preceding equation to yield

$$I = \sum (v_n \cos A - u_n \sin A)^2 = \cos^2 A \sum v_n^2 + \sin^2 A \sum u_n^2 - 2 \cos A \sin A \sum u_n v_n \quad (1)$$

The axis angles which give maximum and minimum inertia (the "principle axes") are obtained by differentiating the moment of inertia, I, with respect to the axis angle, A, and setting this derivative equal to zero. This yields

$$\tan 2A = \sum 2u_n v_n / \sum (u_n^2 - v_n^2)$$

We choose the two solutions of A which lie in the range −90° to 90° and the corresponding "principal moments of inertia" are obtained by substituting these angles into Equation (1).



Appendix Fig. 1. Illustration of the method used for calculating the moment of inertia of the vector OQ about the axis OX; see Appendix 1 for details.

Appendix 2

BASIC Program for Calculating Major and Minor Axes For the D-15, D-15DS and FM100-Hue Tests

The following BASIC program can be used for either the standard or desaturated D-15 and FM100-Hue tests and is designed for use on an IBM-PC computer. The initial printout entitled "SUMS OF U AND V" can be used to check the correct entry of the u and v data in the DATA statements; these sums should be 41.26 and −4.92 for the standard D-15, 26.86 and −38.69 on the desaturated test and 423.79 and 203.73 for the FM100-Hue. The program includes some checking of the data as they are entered to ensure that it will not accept cap numbers which are not whole numbers in the range 1 to 15 (D-15 panels) or 1 to 85 (FM100-Hue); if a cap number is entered which repeats a previous entry, the operator is given the choice of correcting the present number or starting the data entry again. The program can be checked by entering the perfect order (1, 2, 3 . . . 15 or 85) for each test and comparing the results obtained with those in Table 1 for the D-15 tests. A perfect order check on the FM100-Hue gives: Angle = 54.15°, major axis radius = 2.53, minor axis radius = 1.97, total error score = 3.20, S-index = 1.28 and C-index = 1.0. Readers may obtain a direct copy of this program by sending a formatted disc for an IBM-PC or compatible computer (use a padded envelope) to P.E. King-Smith.

```
10  DIM U(85),V(85),C(85)
20  INPUT "TYPE 1 FOR D-15, 2 FOR D-15DS, 3 FOR FM100-HUE",TE
30  IF TE<>1 AND TE<>2 AND TE<>3 THEN 20: REM ILLEGAL TEST NUMBER
40  IF TE=TO GOTO 100: REM SAME TYPE AS LAST TEST
45  REM RECALL U AND V
50  IF TE=1 THEN RESTORE 500
52  IF TE=2 THEN RESTORE 600
54  IF TE=3 THEN RESTORE 700
60  READ H: REM NUMBER OF CAPS
70  SU=0: SV=0
80  FOR N=0 TO H: READ U(N),V(N): SU=SU+U(N): SV=SV+V(N): NEXT N
90  PRINT "SUMS OF U AND V",SU,SV
100 PRINT "ENTER CAP NUMBERS FROM PILOT CAP END"
105 PRINT "(FM 100 STARTS AT POSITION 85)"
108 REM DATA ENTRY
110 FOR N=1 TO H
120 PRINT USING "##";N;: INPUT C(N)
```

```
130 IF C(N)>0 AND C(N)<=H AND C(N) = INT(C(N)) GOTO 150
140 PRINT "INPUT ERROR": GOTO 130
150 REM CHECK FOR REPEATED ENTRY
160 FOR M=1 TO N-1: IF C(M)<>C(N) GOTO 200
170 PRINT "REPEATED ENTRY, TYPE 1 TO CORRECT CAP VALUE, 2 TO START AGAIN";
180 INPUT P: IF P<>1 AND P<>2 THEN 170: REM INCORRECT RESPONSE
190 ON P GOTO 120,100
200 NEXT M
210 NEXT N
212 IF TE=3 THEN C(0)=C(85) ELSE C(0)=0: REM CHOOSE FIRST CAP NUMBER
215 REM CALCULATE SUMS OF SQUARES AND CROSS PRODUCTS
220 U2=0: V2=0: UV=0
230 FOR N=1 TO H
240 DU=U(C(N))-U(C(N-1)): DV=V(C(N))-V(C(N-1)): REM COLOR DIFFERENCE VECTORS
250 U2=U2+DU^2: V2=V2+DV^2: UV=UV+DU*DV
260 NEXT N
270 REM CALCULATE MAJOR AND MINOR RADII AND ANGLE
280 D=U2-V2: IF D=0 THEN A0=.7854 ELSE A0=ATN(2*UV/D)/2: REM ANGLE
290 I0=U2*SIN(A0)^2+V2*COS(A0)^2-2*UV*SIN(A0)*COS(A0): REM MAJOR MOMENT
300 IF A0<0 THEN A1=A0+1.5708 ELSE A1=A0-1.5708: REM PERPENDICULAR ANGLE
310 I1=U2*SIN(A1)^2+V2*COS(A1)^2-2*UV*SIN(A1)*COS(A1): REM MINOR MOMENT
320 IF I0>I1 THEN 340: REM CHECK THAT MAJOR MOMENT GREATER THAN MINOR
330 P=A0: A0=A1: A1=P: P=I0: I0=I1: I1=P: REM SWAP ANGLES & MOMENTS
340 R0=SQR(I0/H): R1=SQR(I1/H): R=SQR(R0^2+R1^2): REM RADII & TOTAL ERROR
350 IF TE=1 THEN R2=9.234669: PRINT "STANDARD D-15"
360 IF TE=2 THEN R2=5.121259: PRINT "DESATURATED D-15"
370 IF TE=3 THEN R2=2.525249: PRINT "FM-100 HUE"
380 PRINT "  ANGLE  MAJ RAD  MIN RAD  TOT ERR  S-INDEX  C-INDEX"
390 PRINT USING "######.##"; 57.3*A1, R0, R1, R, R0/R1, R0/R2
400 T0=TE: GOTO 20
500 DATA 15, -21.54, -38.39: REM STANDARD D-15
510 DATA        -23.26,-25.56,        -22.41,-15.53        -23.11,-7.45
520 DATA        -22.45,1.10,          -21.67,7.35          -14.08,18.74
530 DATA         -2.72,28.13           14.84,31.13          23.87,26.35
540 DATA         31.82,14.76           31.42,6.99,          29.79,0.10
550 DATA         26.64,-9.38,          22.92,-18.65,        11.20,-24.61
600 DATA 15, -4.77,-16.63: REM DESATURATED D-15
610 DATA         -8.63,-14.65,        -12.08,-11.94,       -12.86,-6.74
620 DATA        -12.26,-2.67,         -11.18,2.01,          -7.02,9.12
630 DATA          1.30,15.78,           9.90,16.46,         15.03,12.05
640 DATA         15.48,2.56,           14.76,-2.24,         13.56,-5.04
650 DATA         11.06,-9.17,           8.95,-12.39,         5.62,-15.20
700 DATA 85,43.57,4.76: REM FM100-HUE
710 DATA   43.18,8.03,     44.37,11.34,    44.07,13.62,    44.95,16.04,    44.11,18.52
720 DATA   42.92,20.64,    42.02,22.49,    42.28,25.15,    40.96,27.78,    37.68,29.55
730 DATA   37.11,32.95,    35.41,35.94,    33.38,38.03,    30.88,39.59,    28.99,43.07
740 DATA   25.00,44.12,    22.87,46.44,    18.86,45.87,    15.47,44.97,    13.01,42.12
750 DATA   10.91,42.85,     8.49,41.35,     3.11,41.70,      .68,39.23,    -1.70,39.23
760 DATA   -4.14,36.66,    -6.57,32.41,    -8.53,33.19,   -10.98,31.47,   -15.07,27.89
770 DATA  -17.13,26.31,   -19.39,23.82,   -21.93,22.52,   -23.40,20.14,   -25.32,17.76
780 DATA  -25.10,13.29,   -26.58,11.87,   -27.35,9.52,    -28.41,7.26,    -29.54,5.10
790 DATA  -30.37,2.63,    -31.07,0.10,    -31.72,-2.42,   -31.44,-5.13,   -32.26,-8.16
800 DATA  -29.86,-9.51,   -31.13,-10.59,  -31.04,-14.30,  -29.10,-17.32,  -29.67,-19.59
810 DATA  -28.61,-22.65,  -27.76,-26.66,  -26.31,-29.24,  -23.16,-31.24,  -21.31,-32.92
820 DATA  -19.15,-33.17,  -16.00,-34.90   -14.10,-35.21,  -12.47,-35.84,  -10.55,-37.74
830 DATA   -8.49,-34.78,   -7.21,-35.44,   -5.16,-37.08,   -3.00,-35.95,    -.31,-33.94
840 DATA    1.55,-34.50,    3.68,-30.63,    5.88,-31.18,    8.46,-29.46,    9.75,-29.46
850 DATA   12.24,-27.35,   15.61,-25.68,   19.63,-24.79,   21.20,-22.83,   25.60,-20.51
860 DATA   26.94,-18.40,   29.39,-16.29,   32.93,-12.30,   34.96,-11.57,   38.24,-8.88
870 DATA   39.06,-6.81,    39.51,-3.03,    40.90,-1.50,    42.80,0.60,     43.57,4.76
```

References

1. Birch JM, Chisholm IA, Kinnear P, Pinckers AJLG, Pokorny J, Smith VC, and Verriest G: Clinical testing methods. *In* Congenital and Acquired Color Vision Defects, Pokorny J, Smith VC, Verriest G, and Pinckers AJLG, editors. New York, Grune and Stratton, 1979, pp. 83–135.
2. Farnsworth D: The Farnsworth-Munsell 100-Hue Test for the Examination of Color Discrimination Manual. Baltimore, Munsell Color Co., 1957, pp. 1–7.
3. Allan D: Fourier analysis and the Farnsworth-Munsell 100-Hue test. Ophthalmic Physiol Opt 5:337, 1985.
4. Benzschawel T: Computerized analysis of the Farnsworth-Munsell 100-Hue Test. Am J Optom Physiol Opt 62:254, 1985.
5. Smith VC, Pokorny J, and Pass AS: Color-axis determination on the Farnsworth-Munsell 100-Hue test. Am J Ophthalmol 100:176, 1985.
6. Verriest G, van Laethem J, and Uvijls A: A new assessment of the normal ranges of the Farnsworth-Munsell 100-Hue test scores. Am J Ophthalmol 93:635, 1982.
7. Farnsworth D: The Farnsworth Dichotomous Test for Color Blindness Panel D-15 Manual. New York, The Psychological Corp., 1947, pp. 1–8.
8. DuBois-Poulsen A and Lanthony P: Le Farnsworth-15 desature. Bull Soc Ophtalmol Fr 73:861, 1973.
9. Frisen L and Kalm H: Sahlgren's saturation test for detecting and grading acquired dyschromatopsia. Am J Ophthalmol 92:252, 1981.
10. Adams AJ and Rodic R: Use of desaturated and saturated versions of the D-15 test in glaucoma and glaucoma-suspect patients. *In* Doc Ophthalmol Proc Series (Vol 33), Verriest G, editor. The Hague, Netherlands, Dr. W. Junk Publishers, 1982, pp. 419–424.
11. Bowman KJ: A method for quantitative scoring of the Farnsworth panel D-15. Acta Ophthalmol 60:907, 1982.
12. Bowman KJ and Cameron KD: A Quantitative assessment of colour discrimination in normal vision and senile macular degeneration using some colour confusion tests. *In* Colour Vision Deficiencies VII, Verriest G, editor. The Hague, Netherlands, Dr. W. Junk Publishers, 1984, pp. 363–370.
13. Bowman KJ, Collins J, and Henry CJ: The effect of age on the performance on the panel D-15 and desaturated D-15: A quantitative evaluation. *In* Colour Vision Deficiencies VII, Verriest G, editor. The Hague, Netherlands, Dr. W. Junk Publishers, 1984, pp. 227–231.
14. Farnsworth D: The Farnsworth-Munsell 100-Hue and dichotomous tests for color vision. J Opt Soc Am 33:568, 1943.
15. Commision Internationale De L'Eclairage (CIE): Recommendations on Uniform Color Spaces, Color-Difference Equations, Psychometric Color Terms. Supplement 2 of CIE Publication 15 (E-1.3.1), 1971. Paris, Bureau Central de la CIE, 1978.
16. Wyszecki G and Stiles WS: Color Science: Concepts and Methods, Quantitative Data and Formulae. New York, Wiley and Sons, 1982.
17. Paulson HM: Comparison of color vision tests used by the armed forces. *In* Color Vision, Judd DB, editor. Washington, D.C., National Academy of Sciences, 1973.
18. Helve J: A comparative study of several diagnostic tests of colour vision used for measuring types and degrees of congenital red-green defects. Acta Ophthalmol (Suppl) 115:1, 1972.