Formulation of a Basic Building Block Model for Interaction of High Speed Vehicles on Flexible Structures

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In traditional analyses of vehicle/structure interaction, especially when there are constraints between vehicle masses and the structure, vehicle nominal motion is prescribed a priori, *and therefore unaffected by the structure flexibility. In this paper, a concept of* nominal *motion is defined, and a methodology is proposed in which the above restriction is removed, allowing the vehicle nominal motion to become unknown, and encompassing the traditional approach as a particular case. General nonlinear equations of motion of a building block model, applicable to both wheel-on-rail and magnetically levitated vehicles, are derived. These equations are simplified to a set of mildly nonlinear equations upon introducing additional assumptions – essentially on small structural deformation. An example is given to illustrate the present formulation.*

1 Introduction

In recent years considerable interest has been developed in implementing energy-efficient, high-speed, low-noise systems for airport-city or intercity transportation – in particular, the magnetically levitated (Maglev) vehicle systems (cf. Eastham and Hayes (1987)). Currently, to ensure success of Maglev systems, guideway structures must be designed to be stiff so that deflections remain within narrow margins of tolerance. The cost of a stiff guideway structure can easily exceed 70 percent of the total cost of a system (Zicha (1986)). More flexible guideways are less expensive, but present complex vehicle/structure interaction.[1] The interaction between high speed moving vehicles and flexible supporting structures is the focus of the present paper. Even though the impetus behind this work is geared toward high speed vehicles, the problem of moving loads does find applications in various fields of engineering (cf. Frýba (1972), Blejwas et al. (1979)). Extensive lists of references on the subject of moving loads over elastic structures are contained in the classical monograph by Frýba (1972), and in several review papers, e.g., Kortüm and Wormley (1981), Ting and Yener (1983), report of Subcommittee on Vibration Problems (1985) and Kortüm (1986).

Formulation of vehicle/structure interaction for wheel-on-rail vehicles, or for electromagnetic Maglev vehicles with tight gap control, leads to a complex system of equations of motion. This complexity stems mainly from the constraints between moving masses and the flexible structure, and from the existence of convective terms, which are important for high speed regimes. Such problem does not arise for vehicle models connected to the structure via suspension systems where there are no constraints between moving masses and the structure. In addition, efficient numerical algorithms must be developed to deal with the resulting complex system of equations of motion; analytical solution (for simple cases) is possible only when convective terms are neglected (e.g., Stanisic (1985)). So far, research effort has been based on the assumption that vehicle nominal motion is known *a priori* (e.g., Ting, Genin and Ginsberg (1974), Venancio-Filho (1978), Olsson (1985–1986), and Wallrapp (1986)). Since mathematical models in these work require prescribed vehicle nominal motion and do not admit driving forces, there is no possibility to study effects of structure flexibility on vehicle nominal motion, or effects of applied accelerating or braking forces on the vehicle/structure system. We have not come across any reference where the assumption of known vehicle nominal motion is not used.

We propose herein a methodology to analyze the complete vehicle/structure interaction, valid for high speed regime, without resorting to the usual assumption of known nominal motion. This general setting clearly includes the case where nominal motion is prescribed *a priori*. The scope of this paper is restricted to a basic model of planar motion of a rigid wheel, or a Maglev magnet unit with tight gap control, moving over a flexible beam. Energy dissipative mechanisms are not considered here. The present prototype model serves as a basic building block for more complex vehicle/structure models.

[1] Progress in suspension control technology will make possible the use of flexible guideways, and the efficiency of Maglev systems will increase with advance in superconductor research.

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We note that the wheel model also finds application in electrodynamic (repulsive) Maglev vehicles since these vehicles move on wheels up to a maximum lift-off speed of about 80 km/h (Alscher et al. (1983)). Further, both high speed Maglev vehicles and wheel-on-rail vehicles may possibly run on the same bivalent guideway structure.

Nonlinear equations of motion of the basic model, valid for large deformation of the beam, are derived for a class of general (nonlinear) contact constraints via Hamilton's principle of stationary action.[2] In the present work, structural response in the small deformation range is, however, our main interest. With assumptions of small deformation, the nonlinear equations of motion are then reduced, in a consistent manner, to a system of mildly nonlinear equations. This consistency is an important feature that distinguishes the present approach from traditional practice of complete linearization: Nonlinear terms of physical relevance, essential for high speed regime, are retained in the equation for nominal motion of the basic model. Finally, an example of vehicle/structure interaction at different initial velocities is given to illustrate the present formulation.

Note that the study of dynamic motion of the complete system, driven by external forces, as done here, is the only way to explain the Timoshenko paradox: Consider a constant vertical force traversing, with some prescribed motion, a simply-supported beam. Since the net work done by the force is zero, where does the energy which leaves the beam in a vibratory state after the traversing come from? The same question can be asked for a moving mass with prescribed motion. In fact, the "excess" of energy comes precisely from the work done by (unmodeled) external forces needed for the vehicle to follow the motion prescribed (cf. Maunder (1960)).

2 Description of Basic Problem

In this section, we describe the basic problem of planar motion of a high speed moving load – a single rigid wheel or a suspended magnet with tight gap control – over a flexible beam. Attention is focused, however, to the dynamics of the more complex case of a rolling wheel. Several possible models of a Magleve magnet ("magnetic wheel") can be obtained from this basic model. Recall that the present basic model serves as a building-block for more complex vehicle/structure models.

2.1. Basic Assumptions. Let $\{\mathbf{E}_1, \mathbf{E}_2\}$ be orthonormal basis vectors, and (X^1, X^2) the coordinates along these axes. These objects define a coordinate system for the material (undeformed) configuration of a beam. The line of centroids of the beam, of length L and initially straight, is assumed to lie along the axis $\mathbf{E}_1$; the coordinate of a material point on the line of centroids is denoted by $S \equiv X^1 \in [0, L]$. Let $\{\mathbf{e}_1, \mathbf{e}_2\}$ be the set of orthonormal vectors spanning the spatial (deformed) configuration, and conveniently chosen such that $\mathbf{E}_i \equiv \mathbf{e}_i$, for $i = 1, 2$. The displacement of a material point S is denoted by $\mathbf{u}(S,t) = u^\alpha(S,t)\mathbf{e}_\alpha$,[3] where $t \in [0, +\infty)$ is the time parameter.

Consider a rigid wheel with mass M, radius R, and rotatory inertia about its center of mass I_w. Let $\mathbf{Y}(t) = Y^\alpha(t)\mathbf{E}_\alpha$ be the position of the wheel center of mass in the material configuration of the beam; its position in the spatial configuration is denoted by $\mathbf{y}(t) = y^\alpha(t)\mathbf{e}_\alpha$. We consider the following general form of constraint

$$y^\alpha(t) = Y^\alpha(t) + g^\alpha(\mathbf{u}(Y^1(t),t), \mathbf{u},_S\,(Y^1(t),t)), \tag{1}$$

for $\alpha = 1,2$, where $g^\alpha(\bullet,\bullet)$ are some functions of the structural displacement $\mathbf{u}$ and its spatial derivative $\mathbf{u},_S \equiv \partial\mathbf{u}/\partial S = (\partial u^\beta/\partial S)\mathbf{e}_\beta$, such that $g^\alpha(0, 0) \equiv 0$. We call $\mathbf{Y}(t)$, the motion of the wheel in the material configuration of the beam, the *nominal motion* of the wheel. Thus, for $\mathbf{u}(S,t) \equiv 0$, we have $y^\alpha(t) \equiv Y^\alpha(t)$. Given the functions $y^1(t)$, $\mathbf{u}(S,t)$, and $g^1(\mathbf{u}, \mathbf{u}_S)$, relation (1) with $\alpha = 1$ could be taken as a definition of the (unknown) nominal motion $Y^1(t)$, i.e., $Y^1(t)$ is defined to be a solution of (1). In this formulation, we consider only the case where $Y^2 \equiv \bar{R}$, for some constant $\bar{R}$. Let θ denote the angle of revolution of the wheel, which is considered to be a function of the nominal position Y^1 and the structural deformation $(\mathbf{u}, \mathbf{u},_S)$. We will often employ the shorthand notation $g^\alpha(Y^1,t) \equiv \hat{g}^\alpha(\mathbf{u}(Y^1,t), \mathbf{u},_S(Y^1,t))$, and similarly with $\theta(Y^1,t) \equiv \hat{\theta}(Y^1, \mathbf{u}(Y^1,t), \mathbf{u},_S(Y^1,t))$. Thus,

$$\frac{\partial g^\alpha}{\partial S} \equiv \frac{\partial \hat{g}^\alpha}{\partial u^\beta}\frac{\partial u^\beta}{\partial S} + \frac{\partial \hat{g}^\alpha}{\partial u^\beta,_S}\frac{\partial^2 u^\beta}{\partial S^2}, \tag{2a}$$

$$\frac{\partial \theta}{\partial S} \equiv \frac{\partial \hat{\theta}}{\partial S} + \frac{\partial \hat{\theta}}{\partial u^\beta}\frac{\partial u^\beta}{\partial S} + \frac{\partial \hat{\theta}}{\partial u^\beta,_S}\frac{\partial^2 u^\beta}{\partial S^2}. \tag{2b}$$

2.2 Kinetic Energy and Potential Energy. The kinetic energy $I\!K$ of the basic system (wheel and flexible beam) is given by

$$I\!K := \frac{1}{2}M\left\{[\dot{Y}^1 + \dot{g}^1(Y^1,t)]^2 + [\dot{g}^2(Y^1,t)]^2\right\} + \frac{1}{2}I_w[\dot{\theta}(Y^1,t)]^2$$
$$+ \frac{1}{2}\int_{[0,L]} A_\rho \left\{[u^1,_t(S,t)]^2 + [u^2,_t(S,t)]^2\right\} dS, \tag{3}$$

where the superposed "$\bullet$" denotes the total time derivative (i.e., $(\dot{\ }) \equiv d/dt\,(\bullet)$); $u^\alpha,_t \equiv \partial u^\alpha/\partial t$ denotes the partial derivative of u^α in time, and A_ρ the mass per unit length of the beam.[4] Now, consider a function f: $[0, L] \times [0, \infty) \to I\!R$, smooth enough in both arguments. The first and second total time derivatives of $f(S,t)$, evaluated at $S = Y^1(t)$, are obtained as follows

$$\dot{f}(Y^1, \dot{Y}^1, t) = \frac{\partial f(Y^1,t)}{\partial S}\dot{Y}^1 + \frac{\partial f(Y^1,t)}{\partial t}, \tag{4a}$$

$$\ddot{f}(Y^1, \dot{Y}^1, \ddot{Y}^1, t) = \frac{\partial f(Y^1,t)}{\partial S}\ddot{Y}^1 + \frac{\partial^2 f(Y^1,t)}{\partial S^2}(\dot{Y}^1)^2$$
$$+ 2\frac{\partial^2 f(Y^1,t)}{\partial S\partial t}\dot{Y}^1 + \frac{\partial^2 f(Y^1,t)}{\partial t^2}. \tag{4b}$$

We will often omit to specify $(\dot{Y}^1, \ddot{Y}^1)$ in the argument lists of quantities such as $\dot{f}$ and $\ddot{f}$, and simply write $\dot{f}(Y^1,t)$ and $\ddot{f}(Y^1,t)$.[5] Thus, employing (2) and (4) with $f \equiv g^\alpha$ to evaluate $\dot{g}^\alpha(Y^1,t)$ and $\dot{\theta}(Y^1,t)$, one obtains an expanded form of the kinetic energy (3). The convective terms in (4) – i.e., the first term in (4*a*), and the first three terms in (4*b*) – play an important role in the interaction between high speed moving vehicles and the supporting flexible structures, as shown in Blejwas, Feng, and Ayre (1979), where numerical results corroborated experimental findings (see also Ting, Genin, and Ginsberg (1974)). Further, by the assumed smoothness of the function f in (4), total time derivatives and spatial derivatives are interchangeable,

$$\frac{d^i}{dt^i}\left[\frac{\partial^j f(Y^1,t)}{\partial S^j}\right] = \frac{\partial^j}{\partial S^j}\left[\frac{d^i f(Y^1,t)}{dt^i}\right], \tag{5}$$

[2] The term "contact" is also used here for Maglev magnets with tight gap control.

[3] Throughout the paper, summation convention is implied on repeated indices, which take values in $\{1, 2\}$.

[4] It should be noted that in (3) we do not consider the rotatory inertia of the beam cross-section; however, an analysis including this term could be carried out following the same methodology presented in this paper.

[5] This shorthand notation had been used in (3).

and thus notation such as $\dot{f}_{,S}(Y^1,t)$ can be used without confusion.

The wheel is subjected to an applied force $\mathbf{F} = F^\alpha \mathbf{e}_\alpha$, and a torque T about its center of mass. Without loss of generality, for the moment, the applied force and torque can be considered constant in time for the purpose of deriving the equations of motion. The work done by the external forces is then given by $W := \mathbf{F}\cdot\mathbf{y} + T\theta$. Further, let $\psi(\mathbf{u})$ denote the elastic strain energy stored in the beam. The formulation is so far valid for large deformation in the beam, and we have not yet introduced assumptions of small deformation at this stage. Explicit expression of $\psi(\mathbf{u})$ for finite deformation of a beam in plane motion can be found in Simo and Vu-Quoc (1986).

3 Derivation of Equations of Motion

In this section, we derive the equations of motion for the basic problem, valid for large structural deformation, by employing Hamilton's principle of stationary action. Additional assumptions of small deformation in the structure are subsequently introduced to further simplify the equations of motion. This simplification process is carefully carried out in a manner that is consistent with the assumptions. It should be indicated that even though particularized to small structural deformation the resulting equations of motion do retain some crucial nonlinear terms, for an adequate description of the dynamics at high speed regime.

3.1 The General Nonlinear Equations of Motion. The Lagrangian of the system can be written as

$$\mathbb{L}(Y^1,\mathbf{u}) := \mathbb{K}(Y^1,\mathbf{u}) - \psi(\mathbf{u}) + W(Y^1,\mathbf{u}),^{6} \tag{6}$$

Consider the time interval $[t_1,t_2]$. Let $(\psi(t),\eta^1(S,t),\eta^2(S,t))$ be the admissible variations corresponding to the functions (Y^1,u^1,u^2), and vanishing at time $t=t_1$ and $t=t_2$. The equations of motion are obtained from the stationary condition of the action integral, i.e., the Euler-Lagrange equations corresponding to (6):

$$\frac{d}{d\epsilon}\int_{[t_1,t_2]} \mathbb{L}(Y^1+\epsilon\psi,\mathbf{u}+\epsilon\eta)dt\,\bigg|_{\epsilon=0} = 0, \tag{7}$$

for all admissible variations (ψ,η), where $\eta = \eta^\beta \mathbf{e}_\beta$. It follows that the equations for nominal motion Y^1 and for structural displacement $\mathbf{u}$ are, respectively, given by

$$\frac{d}{d\epsilon}\int_{[t_1,t_2]} \mathbb{L}(Y^1+\epsilon\psi,\mathbf{u})dt\,\bigg|_{\epsilon=0} = 0,$$

$$\frac{d}{d\epsilon}\int_{[t_1,t_2]} \mathbb{L}(Y^1,\mathbf{u}+\epsilon\eta)dt\,\bigg|_{\epsilon=0} = 0,\ \forall \text{ admissible } (\psi,\eta) \tag{8}$$

Nominal Motion $\mathbf{Y^1}$. We first note that from (4*a*) one has

$$\frac{\partial \dot{f}(Y^1,\dot{Y}^1,t)}{\partial} \equiv \frac{\partial f(Y^1,t)}{\partial S}. \tag{9a}$$

Then, it follows from (9*a*) and (5) that

$$\frac{d}{dt}\left(\frac{\partial \dot{f}(Y^1,t)}{\partial}\right) = \frac{d}{dt}\left(\frac{\partial f(Y^1,t)}{\partial S}\right) \equiv \frac{\partial \dot{f}(Y^1,t)}{\partial S}. \tag{9b}$$

Further, the variation of f with respect to Y^1 is given by

$$\frac{d}{d\epsilon}\dot{f}(Y^1+\epsilon\psi,t)\bigg|_{\epsilon=0} = \frac{\partial \dot{f}(Y^1,t)}{\partial S}\psi + \frac{\partial f(Y^1,t)}{\partial S}\dot{\psi}, \tag{10}$$

where we have made use of (4*a*).[7] Next, after evaluation of the directional derivative $(8)_1$, and applying integration by parts with the boundary conditions $\psi(t_1)=\psi(t_2)\equiv 0$, we obtain

$$-\frac{d}{d\epsilon}\int_{[t_1,t_2]} \mathbb{K}(Y^1+\epsilon\psi,\mathbf{u})dt\,\bigg|_{\epsilon=0}$$
$$= \int_{[t_1,t_2]}\bigg\{M\left(1+\frac{\partial g^1(Y^1,t)}{\partial S}\right)[\ddot{Y}^1+\ddot{g}^1(Y^1,t)]$$
$$+M\frac{\partial g^2(Y^1,t)}{\partial S}\ddot{g}^2(Y^1,t) + I_w\frac{\partial\theta(Y^1,t)}{\partial S}\ddot{\theta}(Y^1,t)\bigg\}\psi\,dt, \tag{11a}$$

$$\frac{d}{d\epsilon}\int_{[t_1,t_2]} W(Y^1+\epsilon\psi,\mathbf{u})dt\,\bigg|_{\epsilon=0} = \int_{[t_1,t_2]}\bigg\{F^1\left(1+\frac{\partial g^1(Y^1,t)}{\partial S}\right)$$
$$+F^2\frac{\partial g^2(Y^1,t)}{\partial S} + T\frac{\partial\theta(Y^1,t)}{\partial S}\bigg\}\psi\,dt, \tag{11b}$$

where use has been made of (9) and (10) with $f\equiv g^\alpha$ to allow cancellation of certain terms. The stationary condition $(8)_1$ and relations (11) yield the equation for the nominal motion Y^1:

$$M\left(1+\frac{\partial g^1(Y^1,t)}{\partial S}\right)[\ddot{Y}^1+\ddot{g}^1(Y^1,t)]$$
$$+M\frac{\partial g^2(Y^1,t)}{\partial S}\ddot{g}^2(Y^1,t)+I_w\frac{\partial\theta(Y^1,t)}{\partial S}\ddot{\theta}(Y^1,t)$$
$$=F^1\left(1+\frac{\partial g^1(Y^1,t)}{\partial S}\right)+F^2\frac{\partial g^2(Y^1,t)}{\partial S}+T\frac{\partial\theta(Y^1,t)}{\partial S}. \tag{12}$$

Structural Motion (u^1,u^2). Similar to relations (9), one can prove that the following identifies hold

$$\frac{\partial\dot{g}^\alpha(\mathbf{u},\mathbf{u}_{,S})}{\partial\dot{u}^\beta} \equiv \frac{\partial g^\alpha(\mathbf{u},\mathbf{u}_{,S})}{\partial u^\beta}, \tag{13a}$$

$$\frac{d}{dt}\left(\frac{\partial\dot{g}^\alpha(\mathbf{u},\mathbf{u}_{,S})}{\partial\dot{u}^\beta}\right) \equiv \frac{\partial\dot{g}^\alpha(\mathbf{u},\mathbf{u}_{,S})}{\partial u^\beta}, \tag{13b}$$

$$\frac{\partial\dot{g}^\alpha(\mathbf{u},\mathbf{u}_{,S})}{\partial\dot{u}^\beta{}_{,S}} \equiv \frac{\partial g^\alpha(\mathbf{u},\mathbf{u}_{,S})}{\partial u^\beta{}_{,S}}, \tag{13c}$$

$$\frac{d}{dt}\left(\frac{\partial\dot{g}^\alpha(\mathbf{u},\mathbf{u}_{,S})}{\partial\dot{u}^\beta{}_{,S}}\right) \equiv \frac{\partial\dot{g}^\alpha(\mathbf{u},\mathbf{u}_{,S})}{\partial u^\beta{}_{,S}}. \tag{13d}$$

Now, computation of the directional derivative in $(8)_2$, and integration by parts with respect to time yield the following results

$$-\frac{d}{d\epsilon}\int_{[t_1,t_2]} \mathbb{K}(Y^1,\mathbf{u}+\epsilon\eta)dt\,\bigg|_{\epsilon=0}$$
$$=\int_{[t_1,t_2]}\bigg\{M\ddot{y}^\alpha\left(\eta^\beta(Y^1,t)\frac{\partial g^\alpha(Y^1,t)}{\partial u^\beta}\right.$$
$$\left.+\eta^\beta{}_{,S}(Y^1,t)\frac{\partial g^\alpha(Y^1,t)}{\partial u^\beta{}_{,S}}\right) + I_w\ddot{\theta}(Y^1,t)\left(\eta^\beta(Y^1,t)\frac{\partial\theta(Y^1,t)}{\partial u^\beta}\right.$$
$$\left.+\eta^\beta{}_{,S}(Y^1,t)\frac{\partial\theta(Y^1,t)}{\partial u^\beta{}_{,S}}\right)+\int_{[0,L]}A_\rho\eta^\beta u^\beta{}_{,tt}\,dS\bigg\}dt, \tag{14a}$$

$$\frac{d}{d\epsilon}\int_{[t_1,t_2]} W(Y^1,\mathbf{u}+\epsilon\eta)dt\,\bigg|_{\epsilon=0}$$
$$=\int_{[t_1,t_2]}\bigg\{F^\alpha\left(\eta^\beta(Y^1,t)\frac{\partial g^\alpha(Y^1,t)}{\partial u^\beta}+\eta^\beta{}_{,S}(Y^1,t)\frac{\partial g^\alpha(Y^1,t)}{\partial u^\beta{}_{,S}}\right)$$

[6] We omit the time derivatives of $(Y^1,\mathbf{u})$ in the argument lists of $\mathbb{L}$ and $\mathbb{K}$ to alleviate the notation.

[7] Another way to obtain (10) is by interchanging $d/d\epsilon$ and d/dt, and then using (5).

$$+T\left(\eta^{\beta}(Y^1,t)\,\frac{\partial\theta(Y^1,t)}{\partial u^{\beta}}+\eta^{\beta},_S(Y^1,t)\,\frac{\partial\theta(Y^1,t)}{\partial u^{\beta},_S}\right)\Big\}dt, \qquad (14b)$$

where we have made use of the (homogeneous) boundary conditions of (η^1, η^2) at $t=t_1$ and $t=t_2$, relation (4*a*), and the identities (13)[8]. Next, let the weak form of the stiffness operator be denoted by $G(\bullet,\bullet)$, and

$$G(\mathbf{u},\eta)=\frac{d}{d\epsilon}\,\psi(\mathbf{u}+\epsilon\eta)\Big|_{\epsilon=0}, \qquad (15a)$$

where we recall that $\psi(\mathbf{u})$ designates the strain energy of the beam – see Vu-Quoc (1986) and Simo and Vu-Quoc (1986) for an expression of $G(\mathbf{u}, \eta)$. Therefore, using $(8)_2$, (14) and (15*a*), the weak form of the structural equations of motion is then given by

$$[-F^1+M[\ddot{Y}^1+\ddot{g}^1(Y^1,t)]]\left[\eta^{\beta}(Y^1,t)\,\frac{\partial g^1(Y^1,t)}{\partial u^{\beta}}+\eta^{\beta},_S(Y^1,t)\,\frac{\partial g^1(Y^1,t)}{\partial u^{\beta},_S}\right]+[-F^2+M\ddot{g}^2(Y^1,t)]$$

$$\times\left[\eta^{\beta}(Y^1,t)\frac{\partial g^2(Y^1,t)}{\partial u^{\beta}}+\eta^{\beta},_S(Y^1,t)\,\frac{\partial g^2(Y^1,t)}{\partial u^{\beta},_S}\right]$$

$$+[-T+I_w\ddot{\theta}(Y^1,t)]\left[\eta^{\beta}(Y^1,t)\,\frac{\partial\theta(Y^1,t)}{\partial u^{\beta}}+\eta^{\beta},_S(Y^1,t)\frac{\partial\theta(Y^1,t)}{\partial u^{\beta},_S}\right]$$

$$+\int_{[0,L]}A_{\rho}\eta^{\beta}(S,t)u^{\beta},_{tt}(S,t)dS+G(\mathbf{u},\eta)=0,\ \forall\ \text{admissible}\ \eta. \qquad (15b)$$

The corresponding partial differential equations of motion can be easily obtained from (15) by integrating by parts in S, and by invoking the fundamental lemma of calculus of variations.[9] We prefer, however, to retain the structural equations of motion in its weak form for numerical work.

Remark 3.1. Energy Balance. The balance of system energy at time t can be written as follows

$$\mathbb{K}_t+\psi_t-\int_0^t[F^{\alpha}(\tau)\dot{y}^{\alpha}(\tau)+T(\tau)\dot{\theta}(\tau)]d\tau=\mathbb{K}_0+\psi_0, \qquad (16)$$

where $\mathbb{K}_t$ is as given in (3), ψ_t as given in Simo and Vu-Quoc (1986); the integral term is the work done by (time-varying) external forces. On the right-hand side of (16) are, respectively, the initial kinetic energy $\mathbb{K}_0$ and the initial potential energy ψ_0. The discrete form of the system energy balance (16) has proved to be a very useful criterion in the design of reliable numerical integration algorithms for the equations of motion; see Vu-Quoc and Olsson (1987, 1988a) for the details. ■

3.2 Contact Constraints and Contact Forces. The wheel is assumed to be in permanent contact with, and rolling without slipping on, the beam.[10] Clearly, without structural deformation ($\mathbf{u}(S,t)\equiv 0$), the revolution of the wheel is related to its nominal motion by $\theta = Y^1/R$. Let $\bar{R}\ (=Y^2)$ denote the distance from the beam centroidal line to the center of mass of the wheel (Fig. 1). For $\bar{R}=R$, the wheel is moving with its circumference tangent to the beam centroidal line. An explicit form of the function g^{α} in the general constraint equations (1) for wheel/beam contact, or magnet/beam with constant gap,

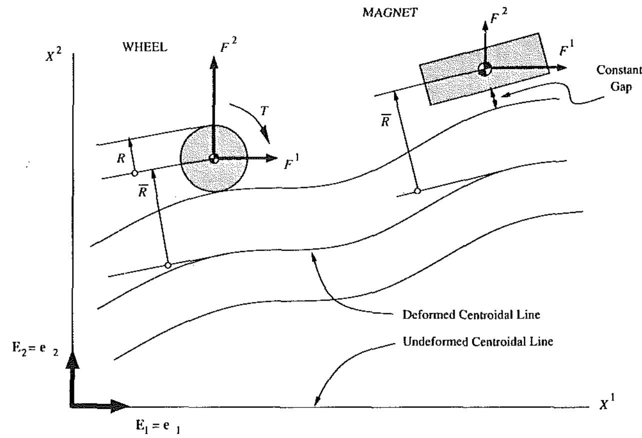

Fig. 1 ***Basic problem*****: Building block models for wheel-on-rail and Maglev vehicles.**

can be written exactly as follows

$$g^1(\mathbf{u},\ \mathbf{u},_S)=u^1-\bar{R}\ \sin\chi(\mathbf{u},_S), \qquad (17a)$$

$$g^2(\mathbf{u},\ \mathbf{u},_S)=u^2-\bar{R}[1-\cos\chi(\mathbf{u},_S)], \qquad (17b)$$

$$\text{where}\ \chi(\mathbf{u},_S):=\tan^{-1}\left(\frac{u^2,_S}{1+u^1,_S}\right), \qquad (17c)$$

represents the slope angle of the deformed beam (cf. Fig. 1). It should be noted that the expressions in (17) are written for beam theory without shear deformation, and are valid for a finitely deformed beam.

Remark 3.2 "Magnetic Wheel." The above formulation encompasses several possible models for a Maglev vehicle using electromagnetic suspension (attractive system) with tight gap control.[11] By letting $I_w\equiv 0$ (or $\theta\equiv 0$) in the kinetic energy (3), we have a model (A) of a moving magnet, where $\bar{R}$ represents the distance from the beam centroidal line to the magnet center of mass (Fig. 1). Next, by letting $I_w=\bar{R}\equiv 0$, in which case the constraints (1) becomes $y^1(t) = Y^1(t) + u^1(Y^1,t)$ and $y^2(t) = u^2(Y^1,t)$, we obtain yet another model (B) of a moving magnet. In practice, often even simpler constraints are chosen (model C) so that $y^1(t)\equiv Y^1(t)$ and $y^2(t) = u^2(Y^1,t)$ (cf., e.g., Wallrapp (1986)). Thus, there is no direct coupling between vehicle nominal motion and structural axial deformation. In this case, the equations of motion (12) and (15) (in weak form) simplify to

$$M\left[\ddot{Y}^1+\frac{\partial u^2(Y^1,t)}{\partial S}\,\ddot{u}^2(Y^1,t)\right]=F^1+F^2\,\frac{\partial u^2(Y^1,t)}{\partial S}, \qquad (18a)$$

$$\eta^2(Y^1,t)[-F^2+M\ddot{u}^2(Y^1,t)]+\int_{[0,L]}A_{\rho}\eta^{\beta}(S,t)\,u^{\beta},_{tt}(S,t)dS+G(\mathbf{u},\eta)=0, \qquad (18b)$$

which are also valid for a finitely deformed beam. In (18), the equation for axial displacement and the equation for transverse displacement are coupled through the nonlinear nature of $G(\mathbf{u},\eta)$ for the finite deformation case. ■

In the design of flexible structures under moving vehicles, it is important to quantify the (dynamic) contact forces. In particular, these forces are crucial in studying structural response to emergency braking of a vehicle. For the basic problem considered herein, let $\mathbf{F}_c = F_c^{\alpha}\mathbf{e}_{\alpha}$ be the contact force acting on the beam. Once Y^1 and u^{α} have been solved for, the contact force can be evaluated by $\mathbf{F}_c = \mathbf{F} - M\ddot{\mathbf{y}}$, obtained from considering the equilibrium of forces acting on the wheel. Recall that $\ddot{\mathbf{y}}$ is

[8]We could also obtain these results by making use of the interchangeability of $d/d\epsilon$ and d/dt.

[9]The containing space of the variations (η^1,η^2) should be suitably chosen and should include the essential boundary conditions at $S=0$ and $S=L$ (see, e.g., Rektorys (1980)).

[10]The velocity of the contact point on the wheel is only about one thousandth of the velocity of the wheel center of mass (rigid slip); see Kalker (1979).

[11]The gap between a magnet and the guideway is in the range of 10–15 mm, independently of vehicle speed (Eastham and Hayes (1987)). See also the review paper by Kortüm and Wormley (1981).

evaluated using (1), (17), and with the aid of (4b). In the case of a moving magnet, the contact force $\mathbf{F}_c$ is the required active control force that should be exerted on the magnet to maintain a constant gap.

3.3 Assumptions on Small Structural Deformation. Equations (12) and (15) form the complete set of coupled, fully nonlinear equations describing the motion of a rigid wheel moving over a flexible beam. In the present work, we consider the following additional assumptions to reduce the equations (12) and (15) to a mildly nonlinear form: **(A1)** $|u^\alpha,_S| << 1$, for $\alpha = 1,2$; **(A2)** The Bernoulli-Euler hypothesis is adopted for beam response,

$$\psi(\mathbf{u}) = \frac{1}{2}\int_{[0,L]} \left\{EA\,[u^1,_S]^2 + EI\,[u^2,_{SS}]^2\right\} dS, \qquad (19a)$$

where EA is the axial stiffness, and EI the bending stiffness; **(A3)** All nonlinear terms in the displacement u^α are neglected in the equations for structural motion; **(A4)** The wheel rolls without slipping and with little influence from structural deformation,

$$\theta(Y^1,t) \approx \frac{Y^1}{R},\ \frac{\partial\theta(Y^1,t)}{\partial S} \approx \frac{1}{R},\ \dot\theta(Y^1,t) \approx \frac{\dot Y^1}{R},\ \ddot\theta(Y^1,t) \approx \frac{\ddot Y^1}{R},$$

$$\frac{\partial\theta}{\partial u^\beta} \approx 0, \quad \text{and} \quad \frac{\partial\theta}{\partial u^\beta,_S} \approx 0. \qquad (19b)$$

Note that the aforementioned assumptions are not only physically relevant in real operational conditions of the system, but carry important implications on the numerical treatment as well (see Vu-Quoc and Olsson (1988a)).

3.4 The Mildly Nonlinear Equations of Motion. Considering the structural equations of motion (15b), assumption **(A3)** implies that we neglect nonlinear terms in u^1 and u^2 in the fully-expanded expressions of $\ddot g^1$ and of $\ddot g^2$ obtained from using (2a) and (4b) in (17). Thus, together with assumption **(A1)**, we arrive at the approximations

$$\ddot g^1 \approx \ddot u^1 - \bar R \ddot u^2,_S, \quad \ddot g^2 \approx \ddot u^2. \qquad (20)$$

Note that approximations (20) together with relations (4) when applied to g^1 and g^2 imply

$$\frac{\partial^{i+j}g^1}{\partial S^i \partial t^j} \approx \frac{\partial^{i+j}u^1}{\partial S^i \partial t^j} - \bar R\,\frac{\partial^{i+j+1}u^2}{\partial S^{i+1}\partial t^j},\ \frac{\partial^{i+j}g^2}{\partial S^i \partial t^j} \approx \frac{\partial^{i+j}u^2}{\partial S^i \partial t^j}, \qquad (21)$$

for (i,j) = (1,0), (2,0), (1,1), (0,2). Further, assumptions **(A1)** and **(A3)** lead to the following approximations

$$\frac{\partial g^1}{\partial u^1,_S} \approx \bar R u^2,_S, \quad \frac{\partial g^1}{\partial u^2,_S} \approx -\bar R, \qquad (22a)$$

$$\frac{\partial g^2}{\partial u^1,_S} \approx 0, \quad \frac{\partial g^2}{\partial u^2,_S} \approx -\bar R u^2,_S, \qquad (22b)$$

$$1 + \frac{\partial g^1}{\partial S} \approx 1 - \bar R u^2,_{SS}, \quad \frac{\partial g^2}{\partial S} \approx u^2,_S, \qquad (22c)$$

where (22c) are obtained with the additional aid of (21) (or (2a)). As a result of (4b), (20), (22), together with assumption **(A4)** (i.e., (19b)), the equation for nominal motion (12) can be approximated by

$$c_3(Y^1,t)\,\ddot Y^1 + c_2(Y^1,t)(\dot Y^1)^2 + c_1(Y^1,t)\dot Y^1 + c_0(Y^1,t) = 0, \qquad (23a)$$

where

$$c_0(Y^1,t) \approx -F^1[1-\bar R u^2,_{SS}(Y^1,t)] - F^2 u^2,_S(Y^1,t) - \frac{T}{R}$$
$$+ M\Big[[1-\bar R u^2,_{SS}(Y^1,t)]\,[u^1,_{tt}(Y^1,t) - \bar R u^2,_{Stt}(Y^1,t)]$$
$$+ u^2,_S(Y^1,t)u^2,_{tt}(Y^1,t)\Big], \qquad (23b)$$

$$c_1(Y^1,t) \approx 2M\Big[[1-\bar R u^2,_{SS}(Y^1,t)]\,[u^1,_{St}(Y^1,t)$$
$$- \bar R u^2,_{SSt}(Y^1,t)] + u^2,_S(Y^1,t)u^2,_{St}(Y^1,t)\Big], \qquad (23c)$$

$$c_2(Y^1,t) \approx M\Big[[1-\bar R u^2,_{SS}(Y^1,t)]\,[u^1,_{SS}(Y^1,t)$$
$$- \bar R u^2,_{SSS}(Y^1,t)] + u^2,_S(Y^1,t)u^2,_{SS}(Y^1,t)\Big], \qquad (23d)$$

$$c_3(Y^1,t) \approx M[1-\bar R u^2,_{SS}(Y^1,t)]^2 + \frac{I_w}{R^2}. \qquad (23e)$$

Remark 3.3. Consistency in the Formulation. The nonlinear term in g^2 in the equation for the nominal motion (12) is, according to (20) and (22), approximated by

$$\frac{\partial g^2(Y^1,t)}{\partial S}\ddot g^2(Y^1,t) \approx u^2,_S(Y^1,t)\ddot u^2(Y^1,t), \qquad (24a)$$

which is also nonlinear in u^2. Using (4b), we obtain the term (24a) in expanded form as given in (23). This term plays an important role in representing the influence of transverse structural displacement on vehicle nominal motion at high speed. To see this, we rewrite the equation for nominal motion (18a) of Maglev model C, for $F^1 = 0$, as follows

$$M\ddot Y^1 = u^2,_S(Y^1,t)\,[F^2 - M\ddot u^2(Y^1,t)] \equiv u^2,_S(Y^1,t)\,F_c^2(t). \qquad (24b)$$

At high speed, the amplitude of the vertical contact force F_c^2 may significantly exceed that of the vertical force F^2. We will present next an example with high speed vehicle motion where one has $|F_c^2(t)| > 1.5\,|F^2|$, for some time t. In other words, the inertia force $M\ddot u^2$ could be of the same order of magnitude as that of F^2, and should be retained in equation (23). Hence, it is shown that the formulation would not be appropriate for high speed regime, had we systematically removed all nonlinear terms in u^α from the equations of motion. This is a variance with the usual practice of complete linearization (see discussion in Kortüm (1986)), which is therefore inconsistent in the present situation. ■

Now, applying assumptions **(A1-A4)**, the weak form of the equations for structural motion, which is linear in the displacement u^α, is given by

$$\eta^1(Y^1,t)\,\Big(-F^1 + M[\ddot Y^1 + \ddot u^1(Y^1,t) - \bar R\ddot u^2,_S(Y^1,t)]\Big)$$
$$-\bar R[F^1 - M\ddot Y^1]\eta^1,_S(Y^1,t)u^2,_S(Y^1,t)$$
$$+\int_{[0,L]} A_\rho \eta^1(S,t)u^1,_{tt}(S,t)dS$$
$$+\int_{[0,L]} EA\,\eta^1,_S(S,t)u^1,_S(S,t)dS = 0, \qquad (25a)$$

and

$$-\bar R\eta^2,_S(Y^1,t)\Big(-F^1 + M[\ddot Y^1 + \ddot u^1(Y^1,t) - \bar R\ddot u^2,_S(Y^1,t)]\Big)$$
$$+\eta^2(Y^1,t)\Big(-F^2 + M\ddot u^2(Y^1,t)\Big)$$
$$+\bar R F^2\eta^2,_S(Y^1,t)u^2,_S(Y^1,t) + \int_{[0,L]} A_\rho\eta^2(S,t)u^2,_{tt}(S,t)dS$$
$$+\int_{[0,L]} EI\,\eta^2,_{SS}(S,t)u^2,_{SS}(S,t)dS = 0, \qquad (25b)$$

for all admissible variations (η^1, η^2). Next, using the relations (4), we can recast equations (25*a*), (25*b*) to the following expanded form

$$\begin{aligned}&\Big[M\eta^1(Y^1,t)\Big(u^1{,_{tt}}(Y^1,t)-\bar{R}u^2{,_{St}}(Y^1,t)\Big)\\&+\int_{[0,L]} A_\rho \eta^1(S,t)u^1{,_{tt}}(S,t)dS\Big]+2M\dot{Y}^1\eta^1(Y^1,t)\Big[u^1{,_{St}}(Y^1,t)\\&-\bar{R}u^2{,_{SSt}}(Y^1,t)\Big]+\Big[M\eta^1(Y^1,t)\Big(\ddot{Y}^1[u^1{,_S}(Y^1,t)-\bar{R}u^2{,_{SS}}(Y^1,t)]\\&\qquad+(\dot{Y}^1)^2\,[u^1{,_{SS}}(Y^1,t)-\bar{R}u^2{,_{SSS}}(Y^1,t)\Big)\\&-\bar{R}[F^1-M\ddot{Y}^1]\eta^1{,_S}(Y^1,t)u^2{,_S}(Y^1,t)\\&+\int_{[0,L]} EA\eta^1{,_S}(S,t)u^1{,_S}(S,t)dS\Big]=\eta^1(Y^1,t)[F^1-M\ddot{Y}^1],\end{aligned}\tag{26a}$$

and

$$\begin{aligned}&\Big[-\bar{R}M\eta^2{,_S}(Y^1,t)\Big(u^1{,_{tt}}(Y^1,t)-\bar{R}u^2{,_{Stt}}(Y^1,t)\Big)\\&+M\eta^2(Y^1,t)u^2{_{tt}}(Y^1,t)+\int_{[0,L]} A_\rho\eta^2(S,t)u^2{,_{tt}}(S,t)dS\Big]\\&+2M\dot{Y}^1\Big[-\bar{R}\eta^2{,_S}(Y^1,t)\Big(u^1{,_{St}}(Y^1,t)-\bar{R}u^2{,_{SSt}}(Y^1,t)\Big)\\&+\eta^2(Y^1,t)u^2{,_{St}}(Y^1,t)\Big]+\Big[M\ddot{Y}^1\Big\{-\bar{R}\eta^2{,_S}(Y^1,t)\Big(u^1{,_S}(Y^1,t)\\&-\bar{R}u^2{,_{SS}}(Y^1,t)\Big)+\eta^2(Y^1,t)u^2{,_S}(Y^1,t)\Big\}\\&+M(\dot{Y}^1)^2\Big\{-\bar{R}\eta^2{,_S}(Y^1,t)\Big(u^1{,_{SS}}(Y^1,t)\\&-\bar{R}u^2{,_{SSS}}(Y^1,t)\Big)+\eta^2(Y^1,t)u^2{,_{SS}}(Y^1,t)\Big\}\\&+\bar{R}F^2\eta^2{,_S}(Y^1,t)u^2{,_S}(Y^1,t)+\int_{[0,L]} EI\eta^2{,_{SS}}(S,t)u^2{,_{SS}}(S,t)dS\Big]\\&=-\bar{R}\eta^2{,_S}(Y^1,t)[F^1-M\ddot{Y}^1]+\eta^2(Y^1,t)F^2,\end{aligned}\tag{26b}$$

for all admissible variations (η^1, η^2), where terms are grouped in square brackets according to their nature (mass, velocity-convection, and stiffness terms on the left-hand side, and applied forces on the right-hand side). Note the geometric stiffness character of the term with factor $\bar{R}[F^1 - M\ddot{Y}^1]$, and of the term with factor $\bar{R}F^2$ in the stiffness operators of (26*a*) and (26*b*), respectively. Even though equations (23) and (26) are the simplified versions of the fully nonlinear equations (12) and (15), according to assumptions (**A1**) to (**A4**), they remain nonlinear and coupled. Moreover, these equations in spatially discrete form are not explicit ordinary differential equations, and special algorithms must be designed for numerical computation. The system is driven by the initial conditions $\{Y^1(0);\ \dot{Y}^1(0),\ \mathbf{u}(S,0),\ \mathbf{u}{,_t}(S,0)\}$ and the forces $\{F^1, F^2, T\}$ applied on the wheel.

Remark 3.4. In connection with Remark 3.3, we note that the linearized structural equations of motion (26*b*) contains the (low order) effect of the contact force $F_c^2 = F^2 - M\ddot{u}^2$ (the term $M\ddot{u}^2$ appears in (26*b*) in expanded form using (4*b*). Thus, the contact force F_c^2 is consistently accounted for in both equations (23) and (26). ■

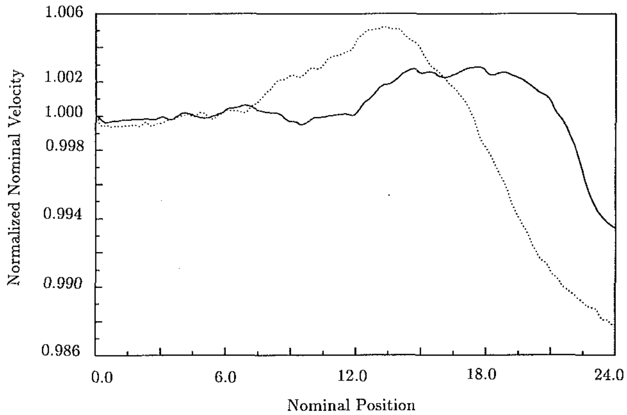

Fig. 2 ***Vehicle/structure interaction at different initial velocities:*** **Nominal velocity (normalized wrt initial values) versus Nominal position. Solid line: $\dot{Y}^1(0)$ = 100 m/s. Dotted line: $\dot{Y}^1(0)$ = 50 m/s. Beam length *L* = 24 m**

Remark 3.5. With assumptions (**A1-A3**), equation (18*b*) is decoupled into an equation of motion for axial vibration and an equation of motion for the transverse vibration. But this means that the Maglev model C, unlike models A and B (see Remark 3.2), cannot be used to study effects of vehicle accelerating or braking on the axial structural response. ■

4 An Illustrative Example

In this section, an example is given to illustrate the above basic model for interaction between a vehicle, starting with different initial velocities, and a flexible supporting structure. Emphasis is focused on results which are not achievable using formulations based on the traditional assumption of known vehicle nominal motion. The results, obtained by numerical methods, correspond to the set of mildly nonlinear, coupled equations (23) and (26). We refer to Vu-Quoc and Olsson (1987, 1988a) for details and discussions on the numerical algorithms employed in solving these equations.

Consider a basic model with parameters M = 3000 kg, I_w = 135 kgm^2, R = 0.3 m, $\bar{R}$ = 0.9 m, L = 24 m, A_ρ = 1250 kg/m, $EA = 5\times 10^9 N$, and $EI = 10^9 N\text{m}^2$. The beam has simple supports at its ends. The wheel is subjected to a constant vertical force $F^2 = -600{,}000\ N$ (with $F^1 = T \equiv 0$), whose magnitude is about 20 times that of the weight of the wheel (acceleration of gravity 9.81 m/s^2), creating a maximum midspan static deflection of 0.1728 m or about $L/140$. The lowest flexural frequency of the beam is 2.44 Hz; its lowest axial frequency is 20.8 Hz. Initial conditions are set to: $Y^1(0) = 0$, $\mathbf{u}(S,0) = \mathbf{u}{,_t}(S,0) \equiv 0$ with the origin of S being coincident with the left support. The vehicle moves mainly due to its own initial velocity $\dot{Y}^1(0)$.

Nominal Velocity. Figure 2 shows the variation of the nominal velocities, normalized with respect to their respective initial values (at the entry of the beam) of $\dot{Y}^1(0)$ = 50 m/s and 100 m/s, as functions of the nominal position Y^1. From this figure, one can clearly observe a loss in nominal velocity at the end of the traversing: An entry velocity of 50 m/s drops by 1.2 percent at the exit, while an entry velocity of 100 m/s drops by 0.7 percent at the exit. The peak-to-peak variations in nominal velocity for these two cases are, respectively, 1.7 percent and 1.0 percent of their initial velocities. These variations stand in contrast to traditional analyses where the velocity $\dot{Y}^1$ is prescribed to its initial value throughout the traversing.

The drop in velocity is related to a drop in vehicle kinetic

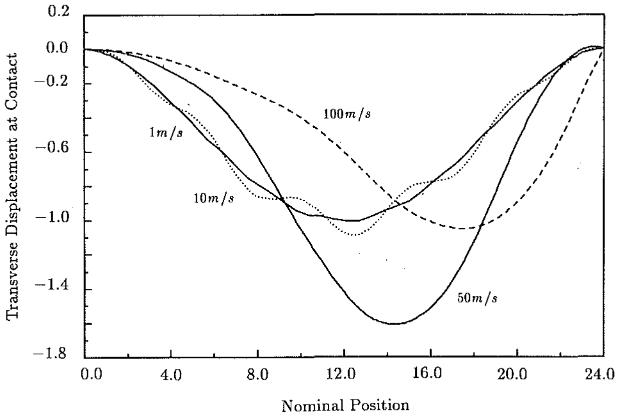

Fig. 3 ***Vehicle/structure interaction at different initial velocities:*** **Vertical displacement at contact point (normalized wrt 0.1728 m) versus Nominal position. $\dot{Y}^1(0)$ = 1 m/s, 10 m/s, 50 m/s, 100 m/s. L = 24 m.**

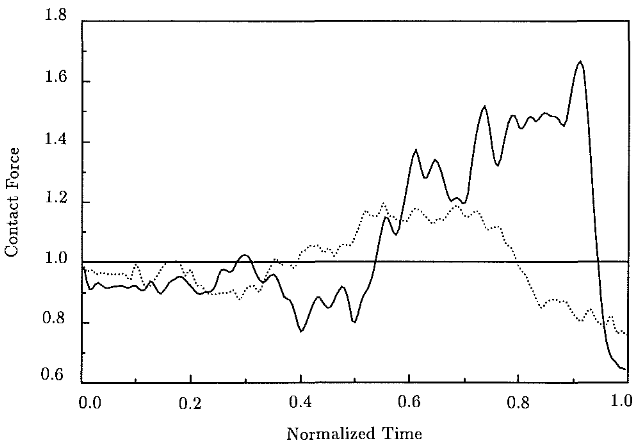

Fig. 4 ***Vehicle/structure interaction at different initial velocities:*** **Vertical contact force F_c^2 (normalized wrt vertical force F^2) versus Time (normalized wrt traversing time on rigid structure). Solid line: $\dot{Y}^1(0)$ = 100 m/s. Dotted line: $\dot{Y}^1(0)$ = 50 m/s.**

energy, as part of this initial kinetic energy is transferred to the beam; we refer to Vu-Quoc and Olsson (1987) for the details. This energy transfer, which keeps the beam in free vibration after the passage of the vehicle, effectively explains the Timoshenko paradox. We note that for a sufficiently long multiple-span structure, a vehicle moving under its initial velocity, without the aid of any other external force than a vertical one, and even in the absence of all energy-dissipative force, will experience a continuous drop in velocity as a result of this type of energy transfer (examples are given in Vu-Quoc and Olsson (1988a,b).

It is also interesting to note that at very low speed, one has a large relative increase in velocity during the traversing. For instance, for $\dot{Y}^1(0)$ = 1 m/s, the increase in nominal velocity is about 400 percent, i.e., the maximum velocity is about 5 m/s. As a result, the traversing time ($\approx$ 9s) is only about one-third of the traversing time on a rigid structure (24s). This increase in velocity is, however, drastically reduced to about 10 percent for $\dot{Y}^1(0)$ = 10 m/s (see Vu-Quoc and Olsson (1988a)).

Structural Deflection. The greater relative loss of velocity for $\dot{Y}^1(0)$ = 50 m/s is due to larger vertical displacement at contact point, compared to the same displacement for $\dot{Y}^1(0)$ = 100 m/s, as recorded in Fig. 3. Also plotted on this figure are displacement at contact point for $\dot{Y}^1(0)$ = 1 m/s (close to a static curve) and for $\dot{Y}^1(0)$ = 10 m/s. We note the shift of the location of maximum displacement closer to the exit as entry velocity increases.

Contact Force. Recorded in Fig. 4 are time histories of the vertical contact force F_c^2, for initial velocities of 50 m/s and 100 m/s. As noted in Remark 3.2, the inertia force $M\ddot{u}^2$ is non-negligible at high speed: For $\dot{Y}^1(0)$ = 100 m/s, this inertia force could reach 60 percent of the vertical force F^2 (Fig. 4). Again, this points to the consistency of the present formulation, which is crucial for a high speed regime.

5 Closure

We have presented a basic building block model for analyzing the interaction between high speed vehicles and supporting flexible structures. The present formulation departs completely from traditional practice of assuming known vehicle nominal motion. Nonlinear equations of motion for the basic model, with a general form of constraints and valid for large structural deformation, are derived using Hamilton's principle. Additional assumptions, essentially on small structural deformation, are introduced to simplify these equations to a mildly nonlinear form. The applicability of the present model is demonstrated through an example. In subsequent publications, we will present efficient algorithms to integrate the nonlinear equations of motion of the complete vehicle/structure interaction problem, and further results.

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