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Research Article

Adaptive Fuzzy Tracking Control for Uncertain Nonlinear Time-Delay Systems with Unknown Dead-Zone Input

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The tracking control problem of uncertain nonlinear time-delay systems with unknown dead-zone input is tackled by a robust adaptive fuzzy control scheme. Because the nonlinear gain function and the uncertainties of the controlled system including matched and unmatched uncertainties are supposed to be unknown, fuzzy logic systems are employed to approximate the nonlinear gain function and the upper bounded functions of these uncertainties. Moreover, the upper bound of the uncertainty caused by the fuzzy modeling error is also estimated. According to these learning fuzzy models and some feasible adaptive laws, a robust adaptive fuzzy tracking controller is developed in this paper without constructing the dead-zone inverse. Based on the Lyapunov stability theorem, the proposed controller not only guarantees that the robust stability of the whole closed-loop system in the presence of uncertainties and unknown dead-zone input can be achieved, but it also obtains that the output tracking error can converge to a neighborhood of zero exponentially. Some simulation results are provided to demonstrate the effectiveness and performance of the proposed approach.

1. Introduction

In general systems, there exist some nonsmooth nonlinearities in the actuators, such as dead-zone, saturation, and backlash [1–7]. The information of the dead-zone is usually poorly known and time variant. Recently, high accuracy position control is required, such as DC servosystems, pressure control systems, power systems, chemical reactor systems, and machine tools [1–3, 8]. However, the dead-zone characteristics in actuators may severely limit the performance of the systems and let the output of the systems not reach our requirements. The robust adaptive control was proposed to deal with nonlinear systems with unknown dead-zone [2]. In Corradini and Orlando [3], the sliding mode controller was presented to robustly stabilize a nonlinear uncertain input. Robust adaptive dead-zone compensation method was used in a DC servo-motor control system [4]. Variable structure control laws were proposed for uncertain large-scale system with dead-zone input [5]. In [8, 9], adaptive control approach was used to cope with nonlinear systems with nonsymmetric dead-zone input. The proposed controllers in [10, 11] tackled the plants with unknown dead-zone via dead-zone inverse. However, the common feature of most previous results [1, 2, 4–6, 8, 9, 12] is the nonlinear gain function which is assumed to be a constant. Although the Previous restrictive assumption can be relaxed in [3, 7, 10, 11], the unmatched uncertainty is not taken into account. Therefore, the motivation of this paper is to synthesize a controller to handle the tracking control problem for a class of uncertain nonlinear state time-delay systems in the presence of an unknown dead-zone input and unmatched uncertainties without constructing the dead-zone inverse.

It is well known that a real system is difficult to be described by the exact mathematical model, owing to the existence of uncertain elements, such as parameter variation, modeling errors, unmodeled dynamics, and external disturbances. These uncertainties may affect the stability of the systems. Robust stabilization of the nonlinear uncertain system has widely been investigated [13–16]. In [13], the purpose of this direct robust adaptive fuzzy controller was to deal with a class of nonlinear systems containing both unconstructed state-dependent unknown nonlinear uncertain and gain functions. Bartolini et al. [14] suggested the second-order sliding mode controller to cope with the uncertain system nonaffine in the control law and the presence of the unmodeled dynamic actuator. The methods of robust adaptive control [15, 16] were utilized to solve the nonlinear uncertain problem. In [15], the robust adaptive controller for SISO nonlinear uncertain system was presented by the input/output linearization approach. In the case where the nonlinear uncertain systems include constant linearly parameterized uncertainty and nonlinear state-dependent parametric uncertainty, the direct robust adaptive control framework was developed in [16].

In recent years, the design problem of nonlinear time-delay systems has received considerable attention in [17–23] because time-delay characteristic usually confronted in engineering systems may degrade the control performance and make the systems unstable. By employing the input-output approach and the scaled small gain theorem, the filtering problem for discrete-time T-S fuzzy systems with time-varying delay has been studied [17]. In [18], the stabilization of LTI systems with time delay was considered by using a low-order controller. The stability analysis and robust control for time-delay systems attracted a large number of researchers over the past years [19–21]. Recently, the problem of stability analysis for stochastic neural networks with discrete interval and distributed time-varying was investigated by applying the idea of delay partitioning method [23].

On the other hand, the fuzzy control techniques have been widely used in many control problems in recent years [24–26]. The fuzzy logic system is constructed from a collection of fuzzy IF-THEN rules. It becomes a useful way to approximate the unknown nonlinear functions and uncertainties in the nonlinear systems. An adaptive interval type-2 fuzzy sliding mode controller for a class of unknown nonlinear discrete-time systems corrupted by external disturbances was presented [24]. In [25], an adaptive neural-fuzzy control design was examined for tracking of nonlinear affine in the control dynamic systems with unknown nonlinearities. Based on a novel fuzzy Lyapunov-Krasovskii functional, a delay partitioning method has been developed for the delay-dependent stability analysis of fuzzy time-varying state delay systems [26].

In this paper, the problem of output tracking control is investigated for a class of uncertain nonlinear state time-delay systems containing unknown dead-zone input and unmatched uncertainties. The main features of the proposed robust adaptive fuzzy controller are summarized as follows. (i) By utilizing a description of a dead-zone feature, an adaptive law is used to estimate the properties of the dead-zone model intuitively and mathematically, without constructing a dead-zone inverse. (ii) Fuzzy logic systems with some appropriate learning laws are applied to approximate the nonlinear gain function and the upper bounded functions of matched and unmatched uncertainties. (iii) The unknown upper bound of the uncertainties caused by approximation (or fuzzy modeling) error is estimated by a simple adaptive law. (iv) By means of Lyapunov stability theorem, the proposed controller cannot only guarantee the robust stability of the whole closed-loop system but also obtain the good tracking performance.

This paper is organized as follows. In Section 2, the form of the uncertain nonlinear state time-delay system with unknown dead-zone input is described. The fuzzy logic systems and fuzzy basis functions are also reviewed. Section 3 presents the robust adaptive fuzzy tracking controller to deal with a class of nonlinear uncertain state time-delay systems containing unknown dead-zone input. By Lyapunov stability theorem, the presented controller can ensure the stability of the controlled systems. Simulation results are demonstrated along with the effectiveness and performance of the proposed controller in Section 4. Finally, a conclusion is given in Section 5.

2. Problem Statement and Preliminaries

2.1. Problem Statement. Consider a class of uncertain nonlinear state time-delay systems containing an unknown dead-zone in the following form:

$$\begin{aligned}
\dot{x}_1 &= x_2 + \Delta\phi_1(\mathbf{x}), \\
\dot{x}_2 &= x_3 + \Delta\phi_2(\mathbf{x}), \\
\dot{x}_3 &= x_4 + \Delta\phi_3(\mathbf{x}), \\
&\vdots \\
\dot{x}_{n-1} &= x_n + \Delta\phi_{n-1}(\mathbf{x}), \\
\dot{x}_n &= \sum_{i=1}^{M}\theta_{1i}f_{1i}(\mathbf{x}(t)) + \Delta f_1(\mathbf{x}(t)) \\
&\quad + \sum_{j=1}^{N}\theta_{2j}f_{2j}(\mathbf{x}(t-\tau)) + \Delta f_2(\mathbf{x}(t-\tau)) \\
&\quad + g(\mathbf{x})Z(v(t)) + \Delta\phi_n(\mathbf{x}), \\
y &= x_1,
\end{aligned} \tag{1}$$

or equivalently,

$$\begin{aligned}
\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\Bigg[&\sum_{i=1}^{M}\theta_{1i}f_{1i}(\mathbf{x}(t)) + \Delta f_1(\mathbf{x}(t)) \\
&+ \sum_{j=1}^{N}\theta_{2j}f_{2j}(\mathbf{x}(t-\tau)) \\
&+ \Delta f_2(\mathbf{x}(t-\tau)) + g(\mathbf{x})Z(v(t))\Bigg] + \Theta(\mathbf{x}), \\
y = \mathbf{C}\mathbf{x},&
\end{aligned} \tag{2}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in R^{n\times n}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in R^{n\times 1},$$
$$\mathbf{C}^T = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \in R^{n\times 1}, \tag{3}$$

where $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in R^n$ is the system state vector which is assumed to be available for measurement, and $v(t) \in R$ and $y(t) \in R$ are the input and output of the system, respectively. τ is the value of time delay. The unknown nonlinear system functions are assumed to be in the linearly parameterized form and consist of two parts: (i) the sum of $\theta_{1i} f_{1i}(\mathbf{x}(t))$ for $i = 1, 2, \ldots, M$; (ii) the sum of $\theta_{2j} f_{2j}(\mathbf{x}(t-\tau))$ for $j = 1, 2, \ldots, N$. The parameters $\theta_{1\mathrm{i}}$ and $\theta_{2\mathrm{j}}$ are unknown but constant. $f_{1i}(\mathbf{x}(t))$ and $f_{2j}(\mathbf{x}(t-\tau))$ are known continuous, linear or nonlinear functions. $\Delta f_1(\mathbf{x}(t))$ and $\Delta f_2(\mathbf{x}(t-\tau))$ are the unknown matched uncertainties. $g(\mathbf{x}(t))$ is the unknown nonlinear gain function, and $\Theta(\mathbf{x}) = [\Delta\phi_1(\mathbf{x}), \Delta\phi_2(\mathbf{x}), \ldots, \Delta\phi_n(\mathbf{x})]^T \in R^{n\times 1}$ is the vector of unknown unmatched uncertainties. Without loss of generality, it is assumed that the sign of $g(\mathbf{x}(t))$ is positive. $Z(v(t)) : R \to R$ is the nonlinear input function containing a dead-zone.

Now, let the output of the system and its derivatives be expressed as follows:

$$\begin{aligned}
y &= x_1, \\
y^{(1)} &= \dot{x}_1 = x_2 + \Delta\phi_1, \\
y^{(2)} &= \dot{x}_2 + (\Delta\phi_1)^{(1)} = x_3 + \Delta\phi_2 + (\Delta\phi_1)^{(1)}, \\
y^{(3)} &= \dot{x}_3 + (\Delta\phi_2)^{(1)} + (\Delta\phi_1)^{(2)} \\
&= x_4 + \Delta\phi_3 + (\Delta\phi_2)^{(1)} + (\Delta\phi_1)^{(2)}, \\
&\vdots \\
y^{(n-1)} &= \dot{x}_{(n-1)} + \left(\Delta\phi_{(n-2)}\right)^{(1)} + \left(\Delta\phi_{(n-3)}\right)^{(2)} \\
&\quad + \cdots + \left(\Delta\phi_2\right)^{(n-3)} + \left(\Delta\phi_1\right)^{(n-2)} \\
&= x_n + \left(\Delta\phi_{(n-1)}\right) + \left(\Delta\phi_{(n-2)}\right)^{(1)} \\
&\quad + \left(\Delta\phi_{(n-3)}\right)^{(2)} + \cdots + \left(\Delta\phi_2\right)^{(n-3)} + \left(\Delta\phi_1\right)^{(n-2)}, \\
y^{(n)} &= \dot{x}_n + \left(\Delta\phi_{(n-1)}\right)^{(1)} + \left(\Delta\phi_{(n-2)}\right)^{(2)} \\
&\quad + \left(\Delta\phi_{(n-3)}\right)^{(3)} + \cdots + \left(\Delta\phi_2\right)^{(n-2)} + \left(\Delta\phi_1\right)^{(n-1)} \\
&= \sum_{i=1}^{M} \theta_{1i} f_{1i}(\mathbf{x}(t)) + \Delta f_1(\mathbf{x}(t)) \\
&\quad + \sum_{j=1}^{N} \theta_{2j} f_{2j}(\mathbf{x}(t-\tau)) + \Delta f_2(\mathbf{x}(t-\tau)) \\
&\quad + g(\mathbf{x}) Z(v(t)) + \Delta\phi_n + (\Delta\phi_{(n-1)})^{(1)} \\
&\quad + (\Delta\phi_{(n-2)})^{(2)} + (\Delta\phi_{(n-3)})^{(3)} \\
&\quad + \cdots + (\Delta\phi_2)^{(n-2)} + (\Delta\phi_1)^{(n-1)} \\
&= \sum_{i=1}^{M} \theta_{1i} f_{1i}(\mathbf{x}(t)) + \Delta f_1(\mathbf{x}(t)) \\
&\quad + \sum_{j=1}^{N} \theta_{2j} f_{2j}(\mathbf{x}(t-\tau)) + \Delta f_2(\mathbf{x}(t-\tau)) \\
&\quad + g(\mathbf{x}) Z(v(t)) + \Delta\Phi,
\end{aligned} \tag{4}$$

where

$$\Delta\Phi = \Delta\phi_1^{(n-1)} + \Delta\phi_2^{(n-2)} + \cdots + \Delta\phi_{n-1}^{(1)} + \Delta\phi_n. \tag{5}$$

The dead-zone with input $v(t)$ and output as shown in Figure 1 is described by

$$Z(v(t)) = \begin{cases} m_r(v(t) - c_a) & \text{for } v(t) \geq c_a, \\ 0 & \text{for } c_b < v(t) < c_a, \\ m_l(v(t) - c_b) & \text{for } v(t) \leq c_b, \end{cases} \tag{6}$$

where $c_a > 0, c_b < 0$ and $m_r > 0$, $m_l > 0$ are parameters and slopes of the dead-zone, respectively. In order to investigate the key features of the dead-zone in the control problems, the following assumptions should be made.

Assumption 1. The dead-zone output $Z(v(t))$ is not available to obtain.

Assumption 2. The dead-zone slopes are of the same value; that is, $m_r = m_l = m$.

Assumption 3. There exist known constants $c_{a\min}, c_{a\max}, c_{b\min}, c_{b\max}, m_{\min}$, and $m_{\max}$ such that the unknown dead-zone parameters c_a, c_b, and m are bounded; that is, $c_a \in [c_{a\min}, c_{a\max}]$, $c_b \in [c_{b\min}, c_{b\max}]$, and $m \in [m_{\min}, m_{\max}]$.

Based on the previous assumptions, the expression (6) can be represented as

$$Z(v(t)) = mv(t) + z(v(t)), \tag{7}$$

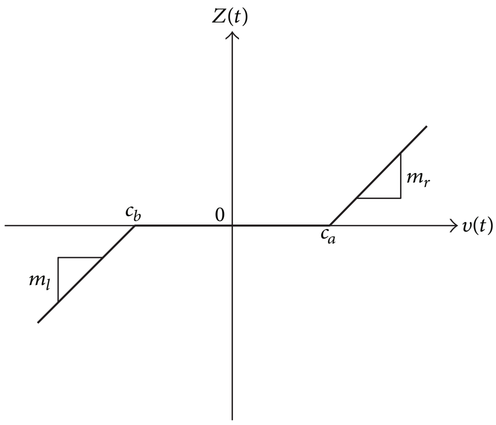

Figure 1: Dead-zone model.

where $z(v(t))$ can be calculated from (6) and (7) as

$$z(v(t)) = \begin{cases} -mc_a & \text{for } v(t) \geq c_a, \\ -mv(t) & \text{for } c_b < v(t) < c_a, \\ -mc_b & \text{for } v(t) \leq c_b. \end{cases} \quad (8)$$

From Assumptions 2 and 3, we can conclude that $z(v(t))$ is bounded and satisfies $|z(v(t))| \leq \rho$, where ρ is the upper bound which can be chosen as

$$\rho = \max\{m_{\max} c_{a\max}, -m_{\max} c_{b\max}\}, \quad (9)$$

where $c_{b\min}$ is a negative value.

Then, let y_m be a given bounded reference signal and contain finite derivatives up to the nth order, define the tracking error as

$$e_i = y_m^{(i-1)} - y^{(i-1)}, \quad \text{for } i = 1, 2, \ldots, n, \quad (10)$$

and denote $\mathbf{e} = [e_1, e_2, \ldots, e_n]^T$, $\mathbf{y} = [y, \dot{y}, \ldots, y^{(n-1)}]^T$, and $\mathbf{y}_m = [y_m, \dot{y}_m, \ldots, y_m^{(n-1)}]^T$.

The control objective of this paper is to design a control law $v(t)$ such that y can follow a given desired reference signal y_m and guarantee that all the signals involved in the whole closed-loop system are bounded.

2.2. Description of Fuzzy Logic Systems. The basic configuration of the fuzzy logic system consists of four main components: fuzzy rule base, fuzzy inference engine, fuzzifier, and defuzzifier [27]. The fuzzy logic system performs a mapping from $U \subset R^n$ to $V \subset R$. Let $U = U_1 \times \cdots \times U_n$, where $U_i \subset R, i = 1, 2, \ldots, n$. The fuzzy rule base consists of a collection of fuzzy IF-THEN rules as follows:

$$R^{(l)}: \text{ IF } x_1 \text{ is } F_1^l \text{ and} \ldots \text{and } x_n \text{ is } F_n^l, \text{ THEN } y \text{ is } G^l, \quad (11)$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T \in U$ and $y \in V \subset R$ are the input and output of the fuzzy logic system, and F_i^l and G^l are fuzzy sets in U_i and V, respectively. The fuzzifier maps a crisp point $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$ into a fuzzy set in U. The fuzzy inference engine performs a mapping from fuzzy sets in U to fuzzy sets in V, based upon the fuzzy IF-THEN rules in the fuzzy rule base and the compositional rule of inference. The defuzzifier maps a fuzzy set in V to a crisp point in V.

The fuzzy systems with center-average defuzzifier, product inference, and singleton fuzzifier are of the following form:

$$y(\mathbf{x}) = \frac{\sum_{l=1}^{M} \theta^l \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)}, \quad (12)$$

where M is the number of rules, θ^l is the point at which the fuzzy membership function $\mu_{G^l}(\theta^l)$ of fuzzy sets G^l achieves its maximum value, and it is assumed that $\mu_{G^l}(\theta^l) = 1$. Equation (12) can be rewritten as

$$y(\mathbf{x}) = \boldsymbol{\theta}^T \boldsymbol{\xi}(\mathbf{x}), \quad (13)$$

where $\boldsymbol{\theta} = [\theta^1, \theta^2, \ldots, \theta^M]^T$ is a parameter vector, and $\boldsymbol{\xi}(\mathbf{x}) = [\xi^1(\mathbf{x}), \ldots, \xi^M(\mathbf{x})]^T$ is a regressive vector with the regressor $\xi^l(\mathbf{x})$, which is defined as fuzzy basis function

$$\xi^l(\mathbf{x}) = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{n} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)}. \quad (14)$$

3. Adaptive Fuzzy Tracking Controller Design and Stability Analysis

According to (2.1), (7), and (10), the tracking error dynamic equation can be expressed as

$$\begin{aligned} \dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{B} \Bigg[& y_m^{(n)} - \sum_{i=1}^{M} \theta_{1i} f_{1i}(\mathbf{x}(t)) - \Delta f_1(\mathbf{x}(t)) \\ & - \sum_{j=1}^{N} \theta_{2j} f_{2j}(\mathbf{x}(t-\tau)) - \Delta f_2(\mathbf{x}(t-\tau)) \\ & - g(\mathbf{x}) mv(t) - g(\mathbf{x}) z(v(t)) - \Delta\Phi \Bigg]. \end{aligned} \quad (15)$$

Now, let us choose a vector $\mathbf{K} = [k_1, k_2, \ldots, k_n] \in R^{1\times n}$ such that $\mathbf{A}_m = \mathbf{A} - \mathbf{B}\mathbf{K}$ is Hurwitz; then, the tracking error dynamic equation (15) can be rewritten as

$$\begin{aligned} \dot{\mathbf{e}} = \mathbf{A}_m \mathbf{e} + \mathbf{B} \Bigg[& \mathbf{K}\mathbf{e} + y_m^{(n)} - \sum_{i=1}^{M} \theta_{1i} f_{1i}(\mathbf{x}(t)) - \Delta f_1(\mathbf{x}(t)) \\ & - \sum_{j=1}^{N} \theta_{2j} f_{2j}(\mathbf{x}(t-\tau)) - \Delta f_2(\mathbf{x}(t-\tau)) \\ & - g(\mathbf{x}) mv(t) - g(\mathbf{x}) z(v(t)) - \Delta\Phi \Bigg]. \end{aligned} \quad (16)$$

It is worth noting that $\Delta f_1(\mathbf{x}(t))$, $f_{2j}(\mathbf{x}(t-\tau))$, and $\Delta\Phi$ are unknown uncertainties and satisfy the following assumption.

Assumption 4. $|\Delta\Phi| \leq h_1(\mathbf{x}), |\Delta f_1(\mathbf{x}(t))| \leq h_2(\mathbf{x}(t))$, and $|\Delta f_2(\mathbf{x}(t-\tau))| \leq h_3(\mathbf{x}(t-\tau))$, where $h_1(\mathbf{x})$, $h_2(\mathbf{x}(t))$, and $h_3(\mathbf{x}(t-\tau))$ are unknown smooth positive functions and can be estimated by fuzzy logic systems with some adaptive laws which will be determined later.

First, the nonlinear gain function $g(\mathbf{x})$ and the upper bounded functions $h_1(\mathbf{x}), h_2(\mathbf{x}(t))$, and $h_3(\mathbf{x}(t-\tau))$ of unmatched and matched uncertainties can be approximated, over a compact set $\Omega_{\mathbf{x}}$, by the fuzzy logic systems as follows:

$$\begin{aligned}
\hat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right) &= \boldsymbol{\theta}_g^T \boldsymbol{\xi}(\mathbf{x}), \\
\hat{h}_1\left(\mathbf{x} \mid \boldsymbol{\theta}_{h1}\right) &= \boldsymbol{\theta}_{h1}^T \boldsymbol{\xi}(\mathbf{x}), \\
\hat{h}_2\left(\mathbf{x} \mid \boldsymbol{\theta}_{h2}\right) &= \boldsymbol{\theta}_{h2}^T \boldsymbol{\xi}(\mathbf{x}), \\
\hat{h}_3\left(\mathbf{x}(t-\tau) \mid \boldsymbol{\theta}_{h3}\right) &= \boldsymbol{\theta}_{h3}^T \boldsymbol{\xi}(\mathbf{x}(t-\tau)),
\end{aligned} \tag{17}$$

where $\boldsymbol{\xi}(\mathbf{x})$ and $\boldsymbol{\xi}(\mathbf{x}(t-\tau))$ are the fuzzy basis vectors, and $\boldsymbol{\theta}_g, \boldsymbol{\theta}_{h1}, \boldsymbol{\theta}_{h2}$, and $\boldsymbol{\theta}_{h3}$ are the corresponding adjustable parameter vectors of each fuzzy logic system. It is assumed that $\boldsymbol{\theta}_g, \boldsymbol{\theta}_{h1}, \boldsymbol{\theta}_{h2}$, and $\boldsymbol{\theta}_{h3}$ belong to compact sets $\Omega_{\boldsymbol{\theta}_g}, \Omega_{\boldsymbol{\theta}_{h1}}, \Omega_{\boldsymbol{\theta}_{h2}}$, and $\Omega_{\boldsymbol{\theta}_{h3}}$, respectively, which are defined as

$$\begin{aligned}
\Omega_{\boldsymbol{\theta} g} &= \left\{\boldsymbol{\theta}_g \in R^M : \left\|\boldsymbol{\theta}_g\right\| \leq N_1 < \infty\right\}, \\
\Omega_{\boldsymbol{\theta}_{h1}} &= \left\{\boldsymbol{\theta}_{h1} \in R^M : \left\|\boldsymbol{\theta}_{h1}\right\| \leq N_2 < \infty\right\}, \\
\Omega_{\boldsymbol{\theta}_{h2}} &= \left\{\boldsymbol{\theta}_{h2} \in R^M : \left\|\boldsymbol{\theta}_{h2}\right\| \leq N_3 < \infty\right\}, \\
\Omega_{\boldsymbol{\theta}_{h3}} &= \left\{\boldsymbol{\theta}_{h3} \in R^M : \left\|\boldsymbol{\theta}_{h3}\right\| \leq N_4 < \infty\right\},
\end{aligned} \tag{18}$$

where N_1, N_2, N_3, and N_4 are the designed parameters, and M is the number of fuzzy inference rules. Let us define the optimal parameter vectors $\boldsymbol{\theta}_g^*, \boldsymbol{\theta}_{h1}^*, \boldsymbol{\theta}_{h2}^*$, and $\boldsymbol{\theta}_{h3}^*$ as follows:

$$\begin{aligned}
\boldsymbol{\theta}_g^* &= \arg \min_{\boldsymbol{\theta}_g \in \Omega_{\boldsymbol{\theta}_g}} \left\{\sup_{\mathbf{x} \in \Omega_{\mathbf{x}}} \left|g(\mathbf{x}) - \hat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right)\right|\right\}, \\
\boldsymbol{\theta}_{h1}^* &= \arg \min_{\boldsymbol{\theta}_{h1} \in \Omega_{\boldsymbol{\theta}_{h1}}} \left\{\sup_{\mathbf{x} \in \Omega_{\mathbf{x}}} \left|h_1(\mathbf{x}) - \hat{h}_1\left(\mathbf{x} \mid \boldsymbol{\theta}_{h1}\right)\right|\right\}, \\
\boldsymbol{\theta}_{h2}^* &= \arg \min_{\boldsymbol{\theta}_{h2} \in \Omega_{\boldsymbol{\theta}_{h2}}} \left\{\sup_{\mathbf{x} \in \Omega_{\mathbf{x}}} \left|h_2(\mathbf{x}) - \hat{h}_2\left(\mathbf{x} \mid \boldsymbol{\theta}_{h2}\right)\right|\right\}, \\
\boldsymbol{\theta}_{h3}^* &= \arg \min_{\boldsymbol{\theta}_{h3} \in \Omega_{\boldsymbol{\theta}_{h3}}} \left\{\sup_{\mathbf{x} \in \Omega_{\mathbf{x}}} \left|h_3(\mathbf{x}(t-\tau)) - \hat{h}_1\left(\mathbf{x}(t-\tau) \mid \boldsymbol{\theta}_{h1}\right)\right|\right\},
\end{aligned} \tag{19}$$

where $\boldsymbol{\theta}_g^*, \boldsymbol{\theta}_{h1}^*, \boldsymbol{\theta}_{h2}^*$, and $\boldsymbol{\theta}_{h3}^*$ are bounded in the suitable closed sets $\Omega_{\boldsymbol{\theta}_g}, \Omega_{\boldsymbol{\theta}_{h1}}, \Omega_{\boldsymbol{\theta}_{h2}}$, and $\Omega_{\boldsymbol{\theta}_{h3}}$, respectively. The parameter estimation errors can be defined as

$$\begin{aligned}
\tilde{\boldsymbol{\theta}}_g &= \boldsymbol{\theta}_g - \boldsymbol{\theta}_g^*, \\
\tilde{\boldsymbol{\theta}}_{h1} &= \boldsymbol{\theta}_{h1} - \boldsymbol{\theta}_{h1}^*, \\
\tilde{\boldsymbol{\theta}}_{h2} &= \boldsymbol{\theta}_{h2} - \boldsymbol{\theta}_{h2}^*, \\
\tilde{\boldsymbol{\theta}}_{h3} &= \boldsymbol{\theta}_{h3} - \boldsymbol{\theta}_{h3}^*, \\
\left|\omega_1\right| &+ \left|\omega_2\right| \leq \omega,
\end{aligned} \tag{20}$$

where ω is an unknown positive constant, and

$$\begin{aligned}
\omega_1 &= \left(h_1(\mathbf{x}) - \hat{h}_1\left(\mathbf{x} \mid \boldsymbol{\theta}_{h1}^*\right)\right) \\
&\quad + \left(h_2(\mathbf{x}(t)) - \hat{h}_2\left(\mathbf{x} \mid \boldsymbol{\theta}_{h2}^*\right)\right) \\
&\quad + \left(h_3 \mathbf{x}(\mathbf{x}(t-\tau)) - \hat{h}_3\left(\mathbf{x} \mid \boldsymbol{\theta}_{h3}^*\right)\right), \\
\omega_2 &= \left(g(\mathbf{x}) - \hat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g^*\right)\right)(mv(t) + z(v(t)))
\end{aligned} \tag{21}$$

as the minimum approximation errors, which correspond to approximation errors obtained when optimal parameters are used.

Secondly, we define

$$\begin{aligned}
\tilde{\phi} &= \hat{\phi} - \phi, \\
\tilde{\boldsymbol{\theta}}_1 &= \hat{\boldsymbol{\theta}}_1 - \boldsymbol{\theta}_1, \\
\tilde{\boldsymbol{\theta}}_2 &= \hat{\boldsymbol{\theta}}_2 - \boldsymbol{\theta}_2, \\
\tilde{\omega} &= \hat{\omega} - \omega,
\end{aligned} \tag{22}$$

where $\hat{\phi}$ is an estimate of ϕ, which is defined as $\phi = (m)^{-1}$. $\hat{\boldsymbol{\theta}}_1$ and $\hat{\boldsymbol{\theta}}_2$ are the estimates of $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$, respectively, which are defined as

$$\begin{aligned}
\boldsymbol{\theta}_1 &= \left[(m)^{-1}\theta_{11}, (m)^{-1}\theta_{12}, \ldots, (m)^{-1}\theta_{1M}\right]^T \in R^M, \\
\boldsymbol{\theta}_2 &= \left[(m)^{-1}\theta_{21}, (m)^{-1}\theta_{22}, \ldots, (m)^{-1}\theta_{2N}\right]^T \in R^N,
\end{aligned} \tag{23}$$

and $\hat{\omega}$ is an estimate of ω.

Based on the previous discussion and under Assumptions 1–4, we are in a position to propose the robust adaptive fuzzy controller in the following form:

$$v = v_1 + v_2 + v_3 + v_4 + v_5, \tag{24}$$

where

$$\begin{aligned}
v_1 = \frac{1}{\hat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right)} \hat{\phi} \Bigg[& \mathbf{K}\mathbf{e} + y_m^{(n)} + \frac{\left(\mathbf{e}^T \mathbf{P}\mathbf{B}\right)^T}{\left\|\mathbf{e}^T \mathbf{P}\mathbf{B}\right\|} \\
& \times \left(\hat{h}_1\left(\mathbf{x} \mid \boldsymbol{\theta}_{h1}\right) + \hat{h}_2\left(\mathbf{x} \mid \boldsymbol{\theta}_{h2}\right)\right. \\
& \left. + \hat{h}_3\left(\mathbf{x}(t-\tau) \mid \boldsymbol{\theta}_{h3}\right)\right) \Bigg],
\end{aligned} \tag{25}$$

$$v_2 = -\frac{1}{\hat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right)}\mathbf{f}_1^T\left(\mathbf{x}(t)\right)\hat{\boldsymbol{\theta}}_1,$$

$$v_3 = -\frac{1}{\hat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right)}\mathbf{f}_2^T\left(\mathbf{x}(t-\tau)\right)\hat{\boldsymbol{\theta}}_2,$$

$$v_4 = \frac{1}{m_{\min}}\frac{1}{\hat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right)}\frac{\left(\mathbf{e}^T\mathbf{PB}\right)^T}{\left\|\mathbf{e}^T\mathbf{PB}\right\|}\hat{\omega},$$

$$v_5 = \frac{\rho}{m_{\min}}\tanh\left(\frac{\mathbf{e}^T\mathbf{PB}}{\varepsilon}\right), \tag{26}$$

where $\mathbf{f}_1(\mathbf{x}(t)) = [f_{11}, f_{12}, \ldots, f_{1M}]^T \in R^M$ and $\mathbf{f}_2(\mathbf{x}(t-\tau)) = [f_{21}, f_{22}, \ldots, f_{2N}]^T \in R^N$, ρ is defined in (9), and $\mathbf{P}$ is a symmetric positive definite matrix, which is a solution of the following Lyapunov equation:

$$\mathbf{A}_m^T\mathbf{P} + \mathbf{P}\mathbf{A}_m = -\mathbf{Q}, \tag{27}$$

where $\mathbf{Q}$ is a positive definite matrix, and the parameter update laws are as follows:

$$\dot{\boldsymbol{\theta}}_g = -\gamma_g\mathbf{e}^T\mathbf{PB}\boldsymbol{\xi}\left(\mathbf{x}\right)\left(v(t) + z_1\left(v(t)\right)\right), \tag{28}$$

$$\begin{aligned}\dot{\boldsymbol{\theta}}_{h1} &= \gamma_{h1}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}\left(\mathbf{x}(t)\right),\\ \dot{\boldsymbol{\theta}}_{h2} &= \gamma_{h2}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}\left(\mathbf{x}(t)\right),\end{aligned} \tag{29}$$

$$\dot{\boldsymbol{\theta}}_{h3} = \gamma_{h3}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}\left(\mathbf{x}(t-\tau)\right), \tag{30}$$

$$\dot{\hat{\boldsymbol{\theta}}}_1 = -\gamma_1\mathbf{e}^T\mathbf{PB}\mathbf{f}_1\left(\mathbf{x}(t)\right), \tag{31}$$

$$\dot{\hat{\boldsymbol{\theta}}}_2 = -\gamma_2\mathbf{e}^T\mathbf{PB}\mathbf{f}_2\left(\mathbf{x}(t-\tau)\right), \tag{32}$$

$$\dot{\hat{\omega}} = \gamma_\omega\left\|\mathbf{e}^T\mathbf{PB}\right\|, \tag{33}$$

$$\begin{aligned}\dot{\hat{\phi}} = \eta\left(\mathbf{e}^T\mathbf{PB}\right)\Bigg\{&\left[\mathbf{Ke} + y_m^{(n)}\right] + \frac{\left(\mathbf{e}^T\mathbf{PB}\right)}{\left\|\mathbf{e}^T\mathbf{PB}\right\|}\hat{h}_1\left(\mathbf{x} \mid \boldsymbol{\theta}_{h1}\right)\\ &+ \frac{\left(\mathbf{e}^T\mathbf{PB}\right)}{\left\|\mathbf{e}^T\mathbf{PB}\right\|}\hat{h}_2\left(\mathbf{x}(t) \mid \boldsymbol{\theta}_{h2}\right)\\ &+ \frac{\left(\mathbf{e}^T\mathbf{PB}\right)}{\left\|\mathbf{e}^T\mathbf{PB}\right\|}\hat{h}_3\left(\mathbf{x}(t-\tau) \mid \boldsymbol{\theta}_{h3}\right)\Bigg\},\end{aligned} \tag{34}$$

where the scalars $\gamma_{h1}, \gamma_{h2}, \gamma_{h3}, \gamma_g, \gamma_1, \gamma_2, \gamma_\omega$, and η are positive constants, determining the rates of adaptations, and

$$z_1\left(v(t)\right) = \begin{cases} -c_a & \text{for } v(t) \geq c_a, \\ -v(t) & \text{for } c_b < v(t) < c_a, \\ -c_b & \text{for } v(t) \leq c_b. \end{cases} \tag{35}$$

Remark 1. Without loss of generality, the adaptive laws used in this paper are assumed that the parameter vectors are within the constraint sets or on the boundaries of the constraint sets but moving toward the inside of the constraint sets. If the parameter vectors are on the boundaries of the constraint sets but moving toward the outside of the constraint sets, we have to use the projection algorithm [27] to modify the adaptive laws such that the parameter vectors will remain inside of the constraint sets. The proposed adaptive law (28)–(30) can be modified as the following form:

$$\dot{\boldsymbol{\theta}}_g = \begin{cases} -\gamma_g\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}\left(\mathbf{x}(t)\right)\left(v(t) + z_1\left(v(t)\right)\right), \\ \quad \text{if } \left(\left\|\boldsymbol{\theta}_g\right\| < N_1\right) \text{ or} \\ \quad \left(\begin{array}{c}\left\|\boldsymbol{\theta}_g\right\| = N_1 \text{ and} \\ \left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\theta}_g^T\boldsymbol{\xi}\left(\mathbf{x}(t)\right)\left(v(t) + z_1\left(v(t)\right)\right) \geq 0\end{array}\right), \\ P\left\{-\gamma_g\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}\left(\mathbf{x}(t)\right)\left(v(t) + z_1\left(v(t)\right)\right)\right\}, \\ \quad \text{if } \left(\begin{array}{c}\left\|\boldsymbol{\theta}_g\right\| = N_1 \text{ and} \\ \left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\theta}_g^T\boldsymbol{\xi}\left(\mathbf{x}(t)\right)\left(v(t) + z_1\left(v(t)\right)\right) < 0\end{array}\right), \end{cases} \tag{36}$$

where $P\{-\gamma_g\|\mathbf{e}^T\mathbf{PB}\|\boldsymbol{\xi}(\mathbf{x}(t))(v(t) + z_1(v(t)))\}$ is defined as

$$\begin{aligned}P&\left\{-\gamma_g\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}\left(\mathbf{x}(t)\right)\left(v(t) + z_1\left(v(t)\right)\right)\right\}\\ &= -\gamma_g\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}\left(\mathbf{x}(t)\right)\left(v(t) + z_1\left(v(t)\right)\right)\\ &\quad + \gamma_g\left\|\mathbf{e}^T\mathbf{PB}\right\|\frac{\boldsymbol{\theta}_g\boldsymbol{\theta}_g^T}{\left\|\boldsymbol{\theta}_g\right\|^2}\boldsymbol{\xi}\left(\mathbf{x}(t)\right)\\ &\quad \times\left(v(t) + z_1\left(v(t)\right)\right),\end{aligned}$$

$$\dot{\boldsymbol{\theta}}_{h1} = \begin{cases} \gamma_{h1}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}\left(\mathbf{x}(t)\right), \\ \quad \text{if } \left(\left\|\boldsymbol{\theta}_{h1}\right\| < N_2\right) \text{ or} \\ \quad \left(\left\|\boldsymbol{\theta}_{h1}\right\| = N_2 \text{ and } \left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\theta}_{h1}^T\boldsymbol{\xi}\left(\mathbf{x}(t)\right) \leq 0\right), \\ P\left\{\gamma_{h1}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}\left(\mathbf{x}(t)\right)\right\}, \\ \quad \text{if } \left(\left\|\boldsymbol{\theta}_{h1}\right\| = N_2 \text{ and } \left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\theta}_{h1}^T\boldsymbol{\xi}\left(\mathbf{x}(t)\right) > 0\right), \end{cases} \tag{37}$$

where $P\{\gamma_{h1}\|\mathbf{e}^T\mathbf{PB}\|\boldsymbol{\xi}(\mathbf{x}(t))\}$ is defined as

$$\begin{aligned}P&\left\{\gamma_{h1}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}\left(\mathbf{x}(t)\right)\right\}\\ &= \gamma_{h1}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}\left(\mathbf{x}(t)\right)\\ &\quad - \gamma_{h1}\left\|\mathbf{e}^T\mathbf{PB}\right\|\frac{\boldsymbol{\theta}_{h1}\boldsymbol{\theta}_{h1}^T}{\left\|\boldsymbol{\theta}_{h1}\right\|^2}\boldsymbol{\xi}\left(\mathbf{x}(t)\right),\end{aligned}$$

$$\dot{\boldsymbol{\theta}}_{h2} = \begin{cases} \gamma_{h2}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}\left(\mathbf{x}(t)\right), \\ \quad \text{if } \left(\left\|\boldsymbol{\theta}_{h2}\right\| < N_3\right) \text{ or} \\ \quad \left(\left\|\boldsymbol{\theta}_{h2}\right\| = N_3 \text{ and } \left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\theta}_{h2}^T\boldsymbol{\xi}\left(\mathbf{x}(t)\right) \leq 0\right), \\ P\left\{\gamma_{h2}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}\left(\mathbf{x}(t)\right)\right\}, \\ \quad \text{if } \left(\left\|\boldsymbol{\theta}_{h2}\right\| = N_3 \text{ and } \left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\theta}_{h2}^T\boldsymbol{\xi}\left(\mathbf{x}(t)\right) > 0\right), \end{cases} \tag{38}$$

where $P\{\gamma_{h2}\|\mathbf{e}^T\mathbf{PB}\|\boldsymbol{\xi}(\mathbf{x}(t))\}$ is defined as

$$P\left\{\gamma_{h2}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}(\mathbf{x}(t))\right\} = \gamma_{h2}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}(\mathbf{x}(t)) - \gamma_{h2}\left\|\mathbf{e}^T\mathbf{PB}\right\|\frac{\boldsymbol{\theta}_{h2}\boldsymbol{\theta}_{h2}^T}{\left\|\boldsymbol{\theta}_{h2}\right\|^2}\boldsymbol{\xi}(\mathbf{x}(t)),$$

$$\dot{\boldsymbol{\theta}}_{h3} = \begin{cases} \gamma_{h3}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}(\mathbf{x}(t-\tau)), \\ \quad \text{if } \left(\left\|\boldsymbol{\theta}_{h3}\right\| < N_4\right) \text{ or} \\ \quad \left(\left\|\boldsymbol{\theta}_{h3}\right\| = N_4 \text{ and } \left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\theta}_{h3}^T\boldsymbol{\xi}(\mathbf{x}(t-\tau)) \le 0\right), \\ P\left\{\gamma_{h3}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}(\mathbf{x}(t-\tau))\right\}, \\ \quad \text{if } \left(\left\|\boldsymbol{\theta}_{h3}\right\| = N_4 \text{ and} \right. \\ \quad \left. \left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\theta}_{h3}^T\boldsymbol{\xi}(\mathbf{x}(t-\tau)) > 0\right), \end{cases} \tag{39}$$

where $P\{\gamma_{h3}\|\mathbf{e}^T\mathbf{PB}\|\boldsymbol{\xi}(\mathbf{x}(t-\tau))\}$ is defined as

$$P\left\{\gamma_{h3}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}(\mathbf{x}(t-\tau))\right\} = \gamma_{h3}\left\|\mathbf{e}^T\mathbf{PB}\right\|\boldsymbol{\xi}(\mathbf{x}(t-\tau)) - \gamma_{h3}\left\|\mathbf{e}^T\mathbf{PB}\right\|\frac{\boldsymbol{\theta}_{h3}\boldsymbol{\theta}_{h3}^T}{\left\|\boldsymbol{\theta}_{h3}\right\|^2}\boldsymbol{\xi}(\mathbf{x}(t-\tau)). \tag{40}$$

The main result of the proposed robust adaptive fuzzy tracking control scheme is summarized in the following theorem.

Theorem 2. *Consider the uncertain nonlinear state time-delay system* (1) *with unknown dead-zone input* (7)*. If Assumptions 1–4 are satisfied, then the proposed robust adaptive fuzzy tracking controller defined by* (24)–(3) *with some adaptation laws* (28)–(34) *ensures that all the signals of the whole closed-loop system are bounded, and the output tracking errors converge to a neighborhood of zero exponentially.*

Proof. Consider the Lyapunov function candidate

$$\begin{aligned} V = \frac{1}{2}\Bigg(&\frac{1}{m}\mathbf{e}^T\mathbf{P}\mathbf{e} + \frac{1}{\gamma_1}\tilde{\boldsymbol{\theta}}_1^T\tilde{\boldsymbol{\theta}}_1 + \frac{1}{\gamma_2}\tilde{\boldsymbol{\theta}}_2^T\tilde{\boldsymbol{\theta}}_2 + \frac{1}{m\cdot\gamma_{h1}}\tilde{\boldsymbol{\theta}}_{h1}^T\tilde{\boldsymbol{\theta}}_{h1} \\ &+ \frac{1}{m\cdot\gamma_{h2}}\tilde{\boldsymbol{\theta}}_{h2}^T\tilde{\boldsymbol{\theta}}_{h2} + \frac{1}{m\cdot\gamma_{h3}}\tilde{\boldsymbol{\theta}}_{h3}^T\tilde{\boldsymbol{\theta}}_{h3} \\ &+ \frac{1}{\gamma_g}\tilde{\boldsymbol{\theta}}_g^T\tilde{\boldsymbol{\theta}}_g + \frac{1}{\eta}\tilde{\phi}^2 + \frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}^2\Bigg). \end{aligned} \tag{41}$$

Differentiating the Lyapunov function V with respect to time, we can obtain

$$\begin{aligned} \dot{V} = &\frac{1}{2m}\dot{\mathbf{e}}^T\mathbf{P}\mathbf{e} + \frac{1}{2m}\mathbf{e}^T\mathbf{P}\dot{\mathbf{e}} + \frac{1}{\gamma_1}\tilde{\boldsymbol{\theta}}_1^T\dot{\tilde{\boldsymbol{\theta}}}_1 \\ &+ \frac{1}{\gamma_2}\tilde{\boldsymbol{\theta}}_2^T\dot{\tilde{\boldsymbol{\theta}}}_2 + \frac{1}{m\gamma_{h1}}\tilde{\boldsymbol{\theta}}_{h1}^T\dot{\tilde{\boldsymbol{\theta}}}_{h1} \\ &+ \frac{1}{m\gamma_{h2}}\tilde{\boldsymbol{\theta}}_{h2}^T\dot{\tilde{\boldsymbol{\theta}}}_{h2} + \frac{1}{m\gamma_{h3}}\tilde{\boldsymbol{\theta}}_{h3}^T\dot{\tilde{\boldsymbol{\theta}}}_{h3} \\ &+ \frac{1}{\gamma_g}\tilde{\boldsymbol{\theta}}_g^T\dot{\tilde{\boldsymbol{\theta}}}_g + \frac{1}{\eta}\tilde{\phi}\dot{\tilde{\phi}} + \frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\tilde{\omega}}. \end{aligned} \tag{42}$$

From (16) and by the fact that $\dot{\tilde{\boldsymbol{\theta}}}_1 = \dot{\hat{\boldsymbol{\theta}}}_1$, $\dot{\tilde{\boldsymbol{\theta}}}_2 = \dot{\hat{\boldsymbol{\theta}}}_2$, $\dot{\tilde{\boldsymbol{\theta}}}_{h1} = \dot{\boldsymbol{\theta}}_{h1}$, $\dot{\tilde{\boldsymbol{\theta}}}_{h2} = \dot{\boldsymbol{\theta}}_{h2}$, $\dot{\tilde{\boldsymbol{\theta}}}_{h3} = \dot{\boldsymbol{\theta}}_{h3}$, $\dot{\tilde{\boldsymbol{\theta}}}_g = \dot{\boldsymbol{\theta}}_g$, $\dot{\tilde{\phi}} = \dot{\hat{\phi}}$, and $\dot{\tilde{\omega}} = \dot{\hat{\omega}}$, the previous equation becomes

$$\begin{aligned} \dot{V} = &\frac{1}{2m}\mathbf{e}^T\left[\mathbf{A}_m^T\mathbf{P} + \mathbf{P}\mathbf{A}_m\right]\mathbf{e} \\ &+ \frac{1}{m}\mathbf{e}^T\mathbf{PB}\Bigg[\mathbf{K}\mathbf{e} + y_m^{(n)} - \sum_{i=1}^{M}\theta_{1i}f_{1i}(\mathbf{x}(t)) \\ &\qquad - \Delta f_1(\mathbf{x}(t)) - \sum_{j=1}^{N}\theta_{2j}f_{2j}(\mathbf{x}(t-\tau)) \\ &\qquad - \Delta f_2(\mathbf{x}(t-\tau)) - g(\mathbf{x})mv(t) \\ &\qquad - g(\mathbf{x})z(v(t)) - \Delta\Phi\Bigg] \\ &+ \frac{1}{\gamma_1}\tilde{\boldsymbol{\theta}}_1^T\dot{\hat{\boldsymbol{\theta}}}_1 + \frac{1}{\gamma_2}\tilde{\boldsymbol{\theta}}_2^T\dot{\hat{\boldsymbol{\theta}}}_2 + \frac{1}{m\gamma_{h1}}\tilde{\boldsymbol{\theta}}_{h1}^T\dot{\boldsymbol{\theta}}_{h1} \\ &+ \frac{1}{m\gamma_{h2}}\tilde{\boldsymbol{\theta}}_{h2}^T\dot{\boldsymbol{\theta}}_{h2} + \frac{1}{m\gamma_{h3}}\tilde{\boldsymbol{\theta}}_{h3}^T\dot{\boldsymbol{\theta}}_{h3} \\ &+ \frac{1}{\gamma_g}\tilde{\boldsymbol{\theta}}_g^T\dot{\boldsymbol{\theta}}_g + \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} + \frac{1}{m_{\min}\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}}. \end{aligned} \tag{43}$$

Applying (27) and Assumption 4 to (43) yields

$$\begin{aligned} \dot{V} \le &-\frac{1}{2m}\mathbf{e}^T\mathbf{Q}\mathbf{e} + \frac{1}{m}\mathbf{e}^T\mathbf{PB} \\ &\times\Bigg[\mathbf{K}\mathbf{e} + y_m^{(n)} - \sum_{i=1}^{M}\theta_{1i}f_{1i}(\mathbf{x}(t)) \\ &\qquad - \sum_{j=1}^{N}\theta_{2j}f_{2j}(\mathbf{x}(t-\tau)) \\ &\qquad - g(\mathbf{x})mv(t) - g(\mathbf{x})z(v(t))\Bigg] \\ &+ \frac{1}{m}\left\|\mathbf{e}^T\mathbf{PB}\right\|h_1(\mathbf{x}) + \frac{1}{m}\left\|\mathbf{e}^T\mathbf{PB}\right\|h_2(\mathbf{x}(t)) \\ &+ \frac{1}{m}\left\|\mathbf{e}^T\mathbf{PB}\right\|h_3(\mathbf{x}(t-\tau)) + \frac{1}{\gamma_1}\tilde{\boldsymbol{\theta}}_1^T\dot{\hat{\boldsymbol{\theta}}}_1 \\ &+ \frac{1}{\gamma_2}\tilde{\boldsymbol{\theta}}_2^T\dot{\hat{\boldsymbol{\theta}}}_2 + \frac{1}{m\gamma_{h1}}\tilde{\boldsymbol{\theta}}_{h1}^T\dot{\boldsymbol{\theta}}_{h1} + \frac{1}{m\gamma_{h2}}\tilde{\boldsymbol{\theta}}_{h2}^T\dot{\boldsymbol{\theta}}_{h2} \end{aligned}$$

$$+\frac{1}{m\gamma_{h3}}\tilde{\boldsymbol{\theta}}_{h3}^T\dot{\boldsymbol{\theta}}_{h3}+\frac{1}{\gamma_g}\tilde{\boldsymbol{\theta}}_g^T\dot{\boldsymbol{\theta}}_g+\frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}}+\frac{1}{m_{\min}\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}}. \quad (44)$$

Substituting (17) and (23) into (44), we obtain

$$\begin{aligned}
\dot{V} \le & -\frac{1}{2m}\mathbf{e}^T\mathbf{Q}\mathbf{e}+\mathbf{e}^T\mathbf{P}\mathbf{B} \\
& \times\left\{\frac{1}{m}\left[\mathbf{K}\mathbf{e}+y_m^{(n)}\right]-\mathbf{f}_1^T(\mathbf{x}(t))\boldsymbol{\theta}_1-\mathbf{f}_2^T(\mathbf{x}(t-\tau))\boldsymbol{\theta}_2\right\} \\
& +\frac{1}{m}\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\left(|\omega_1|+|\omega_2|\right)-\frac{1}{m}\mathbf{e}^T\mathbf{P}\mathbf{B} \\
& \times\left[\hat{g}\left(\mathbf{x}\mid\boldsymbol{\theta}_g\right)(mv(t)+z(v(t)))\right] \\
& +\frac{1}{m}\mathbf{e}^T\mathbf{P}\mathbf{B}\left[\tilde{\boldsymbol{\theta}}_g^T\boldsymbol{\xi}(\mathbf{x})(mv(t)+z(v(t)))\right] \\
& +\frac{1}{m}\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\left[\hat{h}_1(\mathbf{x}\mid\boldsymbol{\theta}_{h1})-\tilde{\boldsymbol{\theta}}_{h1}^T\boldsymbol{\xi}(\mathbf{x})\right] \\
& +\frac{1}{m}\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\left[\hat{h}_2(\mathbf{x}(t)\mid\boldsymbol{\theta}_{h2})-\tilde{\boldsymbol{\theta}}_{h2}^T\boldsymbol{\xi}(\mathbf{x}(t))\right] \\
& +\frac{1}{m}\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\left[\hat{h}_3(\mathbf{x}(t-\tau)\mid\boldsymbol{\theta}_{h3})-\tilde{\boldsymbol{\theta}}_{h3}^T\boldsymbol{\xi}(\mathbf{x}(t-\tau))\right] \\
& +\frac{1}{\gamma_1}\tilde{\boldsymbol{\theta}}_1^T\dot{\hat{\boldsymbol{\theta}}}_1+\frac{1}{\gamma_2}\tilde{\boldsymbol{\theta}}_2^T\dot{\hat{\boldsymbol{\theta}}}_2+\frac{1}{m\cdot\gamma_{h1}}\tilde{\boldsymbol{\theta}}_{h1}^T\dot{\boldsymbol{\theta}}_{h1} \\
& +\frac{1}{m\cdot\gamma_{h2}}\tilde{\boldsymbol{\theta}}_{h2}^T\dot{\boldsymbol{\theta}}_{h2}+\frac{1}{m\cdot\gamma_{h3}}\tilde{\boldsymbol{\theta}}_{h3}^T\dot{\boldsymbol{\theta}}_{h3} \\
& +\frac{1}{\gamma_g}\tilde{\boldsymbol{\theta}}_g^T\dot{\boldsymbol{\theta}}_g+\frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}}+\frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}} \\
\le & -\frac{1}{2m}\mathbf{e}^T\mathbf{Q}\mathbf{e}+\mathbf{e}^T\mathbf{P}\mathbf{B} \\
& \times\left\{\frac{1}{m}\left[\mathbf{K}\mathbf{e}+y_m^{(n)}\right]-\mathbf{f}_1^T(\mathbf{x}(t))\boldsymbol{\theta}_1-\mathbf{f}_2^T(\mathbf{x}(t-\tau))\boldsymbol{\theta}_2\right\} \\
& +\frac{1}{m}\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\omega-\frac{1}{m}\mathbf{e}^T\mathbf{P}\mathbf{B} \\
& \times\left[\hat{g}\left(\mathbf{x}\mid\boldsymbol{\theta}_g\right)(mv(t)+z(v(t)))\right] \\
& +\frac{1}{m}\mathbf{e}^T\mathbf{P}\mathbf{B}\left[\tilde{\boldsymbol{\theta}}_g^T\boldsymbol{\xi}(\mathbf{x})\left(mv(t)+m\cdot z_1(v(t))\right)\right] \\
& +\frac{1}{m}\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\left[\hat{h}_1(\mathbf{x}\mid\boldsymbol{\theta}_{h1})-\tilde{\boldsymbol{\theta}}_{h1}^T\boldsymbol{\xi}(\mathbf{x})\right] \\
& +\frac{1}{m}\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\left[\hat{h}_2(\mathbf{x}(t)\mid\boldsymbol{\theta}_{h2})-\tilde{\boldsymbol{\theta}}_{h2}^T\boldsymbol{\xi}(\mathbf{x}(t))\right] \\
& +\frac{1}{m}\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\left[\hat{h}_3(\mathbf{x}(t-\tau)\mid\boldsymbol{\theta}_{h3})-\tilde{\boldsymbol{\theta}}_{h3}^T\boldsymbol{\xi}(\mathbf{x}(t-\tau))\right] \\
& +\frac{1}{\gamma_1}\tilde{\boldsymbol{\theta}}_1^T\dot{\hat{\boldsymbol{\theta}}}_1+\frac{1}{\gamma_2}\tilde{\boldsymbol{\theta}}_2^T\dot{\hat{\boldsymbol{\theta}}}_2+\frac{1}{m\cdot\gamma_{h1}}\tilde{\boldsymbol{\theta}}_{h1}^T\dot{\boldsymbol{\theta}}_{h1} \\
& +\frac{1}{m\cdot\gamma_{h2}}\tilde{\boldsymbol{\theta}}_{h2}^T\dot{\boldsymbol{\theta}}_{h2}+\frac{1}{m\cdot\gamma_{h3}}\tilde{\boldsymbol{\theta}}_{h3}^T\dot{\boldsymbol{\theta}}_{h3} \\
& +\frac{1}{\gamma_g}\tilde{\boldsymbol{\theta}}_g^T\dot{\boldsymbol{\theta}}_g+\frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}}+\frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}}.
\end{aligned} \quad (45)$$

According to adaptive laws (28)–(30), (45) can be rewritten as

$$\begin{aligned}
\dot{V} \le & -\frac{1}{2m}\mathbf{e}^T\mathbf{Q}\mathbf{e}+\frac{1}{m}\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\omega \\
& +\frac{1}{m}\mathbf{e}^T\mathbf{P}\mathbf{B}\left\{\left[\mathbf{K}\mathbf{e}+y_m^{(n)}\right]\right. \\
& +\frac{\left(\mathbf{e}^T\mathbf{P}\mathbf{B}\right)^T}{\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|}\hat{h}_1(\mathbf{x}\mid\boldsymbol{\theta}_{h1}) \\
& +\frac{\left(\mathbf{e}^T\mathbf{P}\mathbf{B}\right)^T}{\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|}\hat{h}_2(\mathbf{x}(t)\mid\boldsymbol{\theta}_{h2}) \\
& \left.+\frac{\left(\mathbf{e}^T\mathbf{P}\mathbf{B}\right)^T}{\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|}\hat{h}_3(\mathbf{x}(t-\tau)\mid\boldsymbol{\theta}_{h3})\right\} \\
& +\mathbf{e}^T\mathbf{P}\mathbf{B}\left\{-\hat{g}\left(\mathbf{x}\mid\boldsymbol{\theta}_g\right)v(t)-\hat{g}\left(\mathbf{x}\mid\boldsymbol{\theta}_g\right)\frac{z(v(t))}{m}\right\} \\
& -\mathbf{e}^T\mathbf{P}\mathbf{B}\mathbf{f}_1^T(\mathbf{x}(t))\boldsymbol{\theta}_1-\mathbf{e}^T\mathbf{P}\mathbf{B}\mathbf{f}_2^T(\mathbf{x}(t-\tau))\boldsymbol{\theta}_2 \\
& +\frac{1}{\gamma_1}\tilde{\boldsymbol{\theta}}_1^T\dot{\hat{\boldsymbol{\theta}}}_1+\frac{1}{\gamma_2}\tilde{\boldsymbol{\theta}}_2^T\dot{\hat{\boldsymbol{\theta}}}_2+\frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} \\
& +\frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}} \\
\le & -\frac{1}{2m}\mathbf{e}^T\mathbf{Q}\mathbf{e}+\frac{1}{m}\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\omega+\frac{1}{m}\mathbf{e}^T\mathbf{P}\mathbf{B} \\
& \times\left\{\left[\mathbf{K}\mathbf{e}+y_m^{(n)}\right]+\frac{\left(\mathbf{e}^T\mathbf{P}\mathbf{B}\right)^T}{\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|}\hat{h}_1(\mathbf{x}\mid\boldsymbol{\theta}_{h1})\right. \\
& +\frac{\left(\mathbf{e}^T\mathbf{P}\mathbf{B}\right)^T}{\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|}\hat{h}_2(\mathbf{x}(t)\mid\boldsymbol{\theta}_{h2}) \\
& \left.+\frac{\left(\mathbf{e}^T\mathbf{P}\mathbf{B}\right)^T}{\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|}\hat{h}_3(\mathbf{x}(t-\tau)\mid\boldsymbol{\theta}_{h3})\right\} \\
& -\mathbf{e}^T\mathbf{P}\mathbf{B}\hat{g}\left(\mathbf{x}\mid\boldsymbol{\theta}_g\right)v(t)+\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\hat{g}\left(\mathbf{x}\mid\boldsymbol{\theta}_g\right) \\
& \times\frac{|z(v(t))|}{|m|}-\mathbf{e}^T\mathbf{P}\mathbf{B}\mathbf{f}_1^T(\mathbf{x}(t))\boldsymbol{\theta}_1 \\
& -\mathbf{e}^T\mathbf{P}\mathbf{B}\mathbf{f}_2^T(\mathbf{x}(t-\tau))\boldsymbol{\theta}_2+\frac{1}{\gamma_1}\tilde{\boldsymbol{\theta}}_1^T\dot{\hat{\boldsymbol{\theta}}}_1 \\
& +\frac{1}{\gamma_2}\tilde{\boldsymbol{\theta}}_2^T\dot{\hat{\boldsymbol{\theta}}}_2+\frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}}+\frac{1}{m_{\min}\cdot\gamma_\omega}\tilde{\omega}\dot{\hat{\omega}}.
\end{aligned} \quad (46)$$

Using the control laws (24)–(3), the previous equation can be rewritten as

$$\begin{aligned}
\dot{V} \leq & -\frac{1}{2m}\mathbf{e}^T\mathbf{Q}\mathbf{e} + \frac{1}{m_{\min}}\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\| \cdot (\omega - \widehat{\omega}) \\
& + \mathbf{e}^T\mathbf{P}\mathbf{B}\left(\phi - \widehat{\phi}\right) \\
& \times \left\{ \left[\mathbf{K}\mathbf{e} + y_m^{(n)}\right] + \frac{\left(\mathbf{e}^T\mathbf{P}\mathbf{B}\right)^T}{\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|}\widehat{h}_1\left(\mathbf{x} \mid \boldsymbol{\theta}_{h1}\right) \right. \\
& \quad + \frac{\left(\mathbf{e}^T\mathbf{P}\mathbf{B}\right)^T}{\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|}\widehat{h}_2\left(\mathbf{x}(t) \mid \boldsymbol{\theta}_{h2}\right) \\
& \quad \left. + \frac{\left(\mathbf{e}^T\mathbf{P}\mathbf{B}\right)^T}{\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|}\widehat{h}_3\left(\mathbf{x}(t-\tau) \mid \boldsymbol{\theta}_{h3}\right) \right\} \\
& + \left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\| \widehat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right) \frac{|z(v(t))|}{|m|} \\
& - \mathbf{e}^T\mathbf{P}\mathbf{B}\left(\boldsymbol{\theta}_1^T - \widehat{\boldsymbol{\theta}}_1^T\right) \cdot \mathbf{f}_1(\mathbf{x}(t)) \\
& - \mathbf{e}^T\mathbf{P}\mathbf{B}\left(\boldsymbol{\theta}_2^T - \widehat{\boldsymbol{\theta}}_2^T\right) \cdot \mathbf{f}_2(\mathbf{x}(t-\tau)) \\
& - \mathbf{e}^T\mathbf{P}\mathbf{B} \cdot \widehat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right) \cdot \frac{\rho}{m_{\min}} \\
& \cdot \tanh\left(\frac{\mathbf{e}^T\mathbf{P}\mathbf{B}}{\varepsilon}\right) + \frac{1}{\gamma_1}\widetilde{\boldsymbol{\theta}}_1^T\dot{\widehat{\boldsymbol{\theta}}}_1 + \frac{1}{\gamma_2}\widetilde{\boldsymbol{\theta}}_2^T\dot{\widehat{\boldsymbol{\theta}}}_2 \\
& + \frac{1}{\eta}\widetilde{\phi}\dot{\widehat{\phi}} + \frac{1}{m_{\min} \cdot \gamma_\omega}\widetilde{\omega}\dot{\widehat{\omega}} \\
= & -\frac{1}{2m}\mathbf{e}^T\mathbf{Q}\mathbf{e} - \frac{1}{m_{\min}}\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\| \cdot \widetilde{\omega} \\
& - \mathbf{e}^T\mathbf{P}\mathbf{B}\widetilde{\phi}\left\{ \left[\mathbf{K}\mathbf{e} + y_m^{(n)}\right] + \frac{\left(\mathbf{e}^T\mathbf{P}\mathbf{B}\right)^T}{\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|}\widehat{h}_1\left(\mathbf{x} \mid \boldsymbol{\theta}_{h1}\right) \right. \\
& \quad + \frac{\left(\mathbf{e}^T\mathbf{P}\mathbf{B}\right)^T}{\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|}\widehat{h}_2\left(\mathbf{x}(t) \mid \boldsymbol{\theta}_{h2}\right) \\
& \quad \left. + \frac{\left(\mathbf{e}^T\mathbf{P}\mathbf{B}\right)^T}{\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|}\widehat{h}_3\left(\mathbf{x}(t-\tau) \mid \boldsymbol{\theta}_{h3}\right) \right\} \\
& + \left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\| \widehat{g}\left(\mathbf{x} \mid \theta_g\right) \frac{|z(v(t))|}{|m|} \\
& + \mathbf{e}^T\mathbf{P}\mathbf{B} \cdot \widetilde{\boldsymbol{\theta}}_1^T \cdot \mathbf{f}_1(\mathbf{x}(t)) + \mathbf{e}^T\mathbf{P}\mathbf{B} \cdot \widetilde{\boldsymbol{\theta}}_2^T \cdot \mathbf{f}_2(\mathbf{x}(t-\tau)) \\
& - \mathbf{e}^T\mathbf{P}\mathbf{B} \cdot \widehat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right) \cdot \frac{\rho}{m_{\min}} \cdot \tanh\left(\frac{\mathbf{e}^T\mathbf{P}\mathbf{B}}{\varepsilon}\right) \\
& + \frac{1}{\gamma_1}\widetilde{\boldsymbol{\theta}}_1^T\dot{\widehat{\boldsymbol{\theta}}}_1 + \frac{1}{\gamma_2}\widetilde{\boldsymbol{\theta}}_2^T\dot{\widehat{\boldsymbol{\theta}}}_2 \\
& + \frac{1}{\eta}\widetilde{\phi}\dot{\widehat{\phi}} + \frac{1}{m_{\min} \cdot \gamma_\omega}\widetilde{\omega}\dot{\widehat{\omega}}.
\end{aligned} \tag{47}$$

According to adaptive laws (31)–(33), we have

$$\begin{aligned}
\dot{V} \leq & -\frac{1}{2m}\mathbf{e}^T\mathbf{Q}\mathbf{e} + \left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\| \widehat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right) \frac{|z(v(t))|}{|m|} \\
& - \mathbf{e}^T\mathbf{P}\mathbf{B} \cdot \widehat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right) \cdot \frac{\rho}{m_{\min}} \cdot \tanh\left(\frac{\mathbf{e}^T\mathbf{P}\mathbf{B}}{\varepsilon}\right) \\
\leq & -\frac{1}{2m}\mathbf{e}^T\mathbf{Q}\mathbf{e} + \left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\| \widehat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right) \frac{\rho}{m_{\min}} \\
& - \mathbf{e}^T\mathbf{P}\mathbf{B} \cdot \widehat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right) \cdot \frac{\rho}{m_{\min}} \cdot \tanh\left(\frac{\mathbf{e}^T\mathbf{P}\mathbf{B}}{\varepsilon}\right).
\end{aligned} \tag{48}$$

By considering the inequality $|\phi| - \phi\tanh(\phi/\varepsilon) \leq 0.2785\varepsilon$. We obtain

$$\begin{aligned}
\dot{V} \leq & -\frac{1}{2m}\mathbf{e}^T\mathbf{Q}\mathbf{e} + 0.2785\widehat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right) \cdot \frac{\rho}{m_{\min}} \cdot \varepsilon \\
= & -\frac{1}{2m}\mathbf{e}^T\mathbf{Q}\mathbf{e} - \frac{1}{2\gamma_1}\widetilde{\boldsymbol{\theta}}_1^T\widetilde{\boldsymbol{\theta}}_1 - \frac{1}{2\gamma_2}\widetilde{\boldsymbol{\theta}}_2^T\widetilde{\boldsymbol{\theta}}_2 \\
& - \frac{1}{2m\gamma_{h1}}\widetilde{\boldsymbol{\theta}}_{h1}^T\widetilde{\boldsymbol{\theta}}_{h1} - \frac{1}{2m\gamma_{h2}}\widetilde{\boldsymbol{\theta}}_{h2}^T\widetilde{\boldsymbol{\theta}}_{h2} \\
& - \frac{1}{2m\gamma_{h3}}\widetilde{\boldsymbol{\theta}}_{h3}^T\widetilde{\boldsymbol{\theta}}_{h3} - \frac{1}{2\gamma_g}\widetilde{\boldsymbol{\theta}}_g^T\widetilde{\boldsymbol{\theta}}_g - \frac{1}{2\eta}\widetilde{\phi}^2 \\
& - \frac{1}{2m_{\min} \cdot \gamma_\omega}\widetilde{\omega}^2 + \frac{1}{2\gamma_1}\widetilde{\boldsymbol{\theta}}_1^T\widetilde{\boldsymbol{\theta}}_1 + \frac{1}{2\gamma_2}\widetilde{\boldsymbol{\theta}}_2^T\widetilde{\boldsymbol{\theta}}_2 \\
& + \frac{1}{2m\gamma_{h1}}\widetilde{\boldsymbol{\theta}}_{h1}^T\widetilde{\boldsymbol{\theta}}_{h1} + \frac{1}{2m\gamma_{h2}}\widetilde{\boldsymbol{\theta}}_{h2}^T\widetilde{\boldsymbol{\theta}}_{h2} \\
& + \frac{1}{2m\gamma_{h3}}\widetilde{\boldsymbol{\theta}}_{h3}^T\widetilde{\boldsymbol{\theta}}_{h3} + \frac{1}{2\gamma_g}\widetilde{\boldsymbol{\theta}}_g^T\widetilde{\boldsymbol{\theta}}_g + \frac{1}{2\eta}\widetilde{\phi}^2 \\
& + \frac{1}{2m_{\min} \cdot \gamma_\omega}\widetilde{\omega}^2 + 0.2785\widehat{g}\left(\mathbf{x} \mid \boldsymbol{\theta}_g\right) \cdot \frac{\rho}{m_{\min}} \cdot \varepsilon.
\end{aligned} \tag{49}$$

Let

$$\begin{aligned}
L = & \frac{1}{2\gamma_1}\widetilde{\boldsymbol{\theta}}_1^T\widetilde{\boldsymbol{\theta}}_1 + \frac{1}{2\gamma_2}\widetilde{\boldsymbol{\theta}}_2^T\widetilde{\boldsymbol{\theta}}_2 \\
& + \frac{1}{2m\gamma_{h1}}\widetilde{\boldsymbol{\theta}}_{h1}^T\widetilde{\boldsymbol{\theta}}_{h1} + \frac{1}{2m\gamma_{h2}}\widetilde{\boldsymbol{\theta}}_{h2}^T\widetilde{\boldsymbol{\theta}}_{h2} \\
& + \frac{1}{2m\gamma_{h3}}\widetilde{\boldsymbol{\theta}}_{h3}^T\widetilde{\boldsymbol{\theta}}_{h3} + \frac{1}{2\gamma_g}\widetilde{\boldsymbol{\theta}}_g^T\widetilde{\boldsymbol{\theta}}_g \\
& + \frac{1}{2\eta}\widetilde{\phi}^2 + \frac{1}{2m_{\min} \cdot \gamma_\omega}\widetilde{\omega}^2 \\
& + 0.2785\widehat{g}\left(\mathbf{x} \mid \theta_g\right) \cdot \frac{\rho}{m_{\min}} \cdot \varepsilon.
\end{aligned} \tag{50}$$

Then,

$$\begin{aligned}\dot{V} \le & -\frac{1}{2m}\mathbf{e}^T\mathbf{Q}\mathbf{e} - \frac{1}{2\gamma_1}\widetilde{\boldsymbol{\theta}}_1^T\widetilde{\boldsymbol{\theta}}_1 \\ & - \frac{1}{2\gamma_2}\widetilde{\boldsymbol{\theta}}_2^T\widetilde{\boldsymbol{\theta}}_2 - \frac{1}{2m\gamma_{h1}}\widetilde{\boldsymbol{\theta}}_{h1}^T\widetilde{\boldsymbol{\theta}}_{h1} \\ & - \frac{1}{2m\gamma_{h2}}\widetilde{\boldsymbol{\theta}}_{h2}^T\widetilde{\boldsymbol{\theta}}_{h2} - \frac{1}{2m\gamma_{h3}}\widetilde{\boldsymbol{\theta}}_{h3}^T\widetilde{\boldsymbol{\theta}}_{h3} \\ & - \frac{1}{2\gamma_g}\widetilde{\boldsymbol{\theta}}_g^T\widetilde{\boldsymbol{\theta}}_g - \frac{1}{2\eta}\widetilde{\phi}^2 - \frac{1}{2m_{\min}\cdot\gamma_\omega}\widetilde{\omega}^2 + L \\ \le & -\frac{1}{2m}\lambda_{\min}(\mathbf{Q})\,\mathbf{e}^T\mathbf{e} - \frac{1}{2\gamma_1}\widetilde{\boldsymbol{\theta}}_1^T\widetilde{\boldsymbol{\theta}}_1 \\ & - \frac{1}{2\gamma_2}\widetilde{\boldsymbol{\theta}}_2^T\widetilde{\boldsymbol{\theta}}_2 - \frac{1}{2m\gamma_{h1}}\widetilde{\boldsymbol{\theta}}_{h1}^T\widetilde{\boldsymbol{\theta}}_{h1} \\ & - \frac{1}{2m\gamma_{h2}}\widetilde{\boldsymbol{\theta}}_{h2}^T\widetilde{\boldsymbol{\theta}}_{h2} - \frac{1}{2m\gamma_{h3}}\widetilde{\boldsymbol{\theta}}_{h3}^T\widetilde{\boldsymbol{\theta}}_{h3} \\ & - \frac{1}{2\gamma_g}\widetilde{\boldsymbol{\theta}}_g^T\widetilde{\boldsymbol{\theta}}_g - \frac{1}{2\eta}\widetilde{\phi}^2 - \frac{1}{2m_{\min}\cdot\gamma_\omega}\widetilde{\omega}^2 + L \\ \le & -\frac{1}{2m}\frac{\lambda_{\min}(\mathbf{Q})}{\lambda_{\max}(\mathbf{P})}\mathbf{e}^T\mathbf{P}\mathbf{e} - \frac{1}{2\gamma_1}\widetilde{\boldsymbol{\theta}}_1^T\widetilde{\boldsymbol{\theta}}_1 \\ & - \frac{1}{2\gamma_2}\widetilde{\boldsymbol{\theta}}_2^T\widetilde{\boldsymbol{\theta}}_2 - \frac{1}{2m\gamma_{h1}}\widetilde{\boldsymbol{\theta}}_{h1}^T\widetilde{\boldsymbol{\theta}}_{h1} \\ & - \frac{1}{2m\gamma_{h2}}\widetilde{\boldsymbol{\theta}}_{h2}^T\widetilde{\boldsymbol{\theta}}_{h2} - \frac{1}{2m\gamma_{h3}}\widetilde{\boldsymbol{\theta}}_{h3}^T\widetilde{\boldsymbol{\theta}}_{h3} \\ & - \frac{1}{2\gamma_g}\widetilde{\boldsymbol{\theta}}_g^T\widetilde{\boldsymbol{\theta}}_g - \frac{1}{2\eta}\widetilde{\phi}^2 - \frac{1}{2m_{\min}\cdot\gamma_\omega}\widetilde{\omega}^2 + L.\end{aligned} \tag{51}$$

Let $\lambda_v = \lambda_{\min}(\mathbf{Q})/\lambda_{\max}(\mathbf{P})$. We obtain

$$\begin{aligned}\dot{V} \le & -\frac{1}{2m}\lambda_v\mathbf{e}^T\mathbf{P}\mathbf{e} - \frac{1}{2\gamma_1}\widetilde{\boldsymbol{\theta}}_1^T\widetilde{\boldsymbol{\theta}}_1 \\ & - \frac{1}{2\gamma_2}\widetilde{\boldsymbol{\theta}}_2^T\widetilde{\boldsymbol{\theta}}_2 - \frac{1}{2m\gamma_{h1}}\widetilde{\boldsymbol{\theta}}_{h1}^T\widetilde{\boldsymbol{\theta}}_{h1} \\ & - \frac{1}{2m\gamma_{h2}}\widetilde{\boldsymbol{\theta}}_{h2}^T\widetilde{\boldsymbol{\theta}}_{h2} - \frac{1}{2m\gamma_{h3}}\widetilde{\boldsymbol{\theta}}_{h3}^T\widetilde{\boldsymbol{\theta}}_{h3} \\ & - \frac{1}{2\gamma_g}\widetilde{\boldsymbol{\theta}}_g^T\widetilde{\boldsymbol{\theta}}_g - \frac{1}{2\eta}\widetilde{\phi}^2 - \frac{1}{2m_{\min}\cdot\gamma_\omega}\widetilde{\omega}^2 + L \\ \le & -\min\left\{\lambda_v, \frac{1}{\gamma_1}, \frac{1}{\gamma_2}, \frac{1}{m\cdot\gamma_{h1}}, \frac{1}{m\cdot\gamma_{h2}}, \right. \\ & \left. \frac{1}{m\cdot\gamma_{h3}}, \frac{1}{\gamma_g}, \frac{1}{\eta}, \frac{1}{m\cdot\gamma_\omega}\right\} \\ & \times\left[\frac{1}{2m}\mathbf{e}^T\mathbf{P}\mathbf{e} + \frac{1}{2\gamma_1}\widetilde{\boldsymbol{\theta}}_1^T\widetilde{\boldsymbol{\theta}}_1 + \frac{1}{2\gamma_2}\widetilde{\boldsymbol{\theta}}_2^T\widetilde{\boldsymbol{\theta}}_2 \right. \\ & + \frac{1}{2m\gamma_{h1}}\widetilde{\boldsymbol{\theta}}_{h1}^T\widetilde{\boldsymbol{\theta}}_{h1} + \frac{1}{2m\gamma_{h2}}\widetilde{\boldsymbol{\theta}}_{h2}^T\widetilde{\boldsymbol{\theta}}_{h2} \\ & + \frac{1}{2m\gamma_{h3}}\widetilde{\boldsymbol{\theta}}_{h3}^T\widetilde{\boldsymbol{\theta}}_{h3} + \frac{1}{2\gamma_g}\widetilde{\boldsymbol{\theta}}_g^T\widetilde{\boldsymbol{\theta}}_g \\ & \left. + \frac{1}{2\eta}\widetilde{\phi}^2 + \frac{1}{2m_{\min}\cdot\gamma_\omega}\widetilde{\omega}^2\right] + L.\end{aligned} \tag{52}$$

Setting $c = \min\{\lambda_v, 1/\gamma_1, 1/\gamma_2, 1/(m\cdot\gamma_{h1}), 1/(m\cdot\gamma_{h2}), 1/(m\cdot\gamma_{h3}), 1/\gamma_g, 1/\eta, 1/(m\cdot\gamma_\omega)\}$, it yields that

$$\dot{V} \le -cV + L. \tag{53}$$

Then, it is easy from (53) to show that

$$V(t) \le e^{-ct}V(0) + \frac{L}{c}. \tag{54}$$

Therefore, the output tracking error converges to a neighborhood of zero exponentially. □

Remark 3. In the future work, the control problem of uncertain T-S fuzzy time-varying delay systems with unknown dead-zone input is an important topic and is worth to be studied. Based on a novel fuzzy Lyapunov-Krasovskii functional, a delay partitioning method has been developed for the delay-dependent stability analysis of fuzzy time-varying state delay systems [26]. Obviously, it provides a useful idea to deal with the aforementioned future research.

4. An Example and Simulation Results

Consider the second-order uncertain nonlinear time-delay system containing an unknown dead-zone that is modified from the simulation example in [7] as follows:

$$\begin{aligned}\dot{x}_1 &= x_2 + \Delta\phi_1(\mathbf{x}), \\ \dot{x}_2 &= x_1 + f_{11}(\mathbf{x}(t)) + f_{22}(\mathbf{x}(t-\tau)) \\ &\quad + \Delta f_1(\mathbf{x}(t)) + \Delta f_2(\mathbf{x}(t-\tau)) \\ &\quad + g(\mathbf{x})Z(v(t)) + \Delta\phi_2(\mathbf{x}), \\ y &= x_1,\end{aligned} \tag{55}$$

where the nonlinear functions $f_{11}(\mathbf{x}(t)) = -0.3\sin x_1(t)$, $f_{12}(\mathbf{x}(t)) = 0, f_{21}(\mathbf{x}(t-\tau)) = 0$, and $f_{22}(\mathbf{x}(t-\tau)) = 0.1x_1^2(t-\tau)$ are assumed to be known, and $\Delta f_1(\mathbf{x}(t)) = -0.1x_1\sin(3x_2(t))$, $\Delta f_2(\mathbf{x}(t-\tau)) = -0.1x_1\sin(3x_2(t-\tau))$ are unknown system uncertainties with unknown upper bound functions, where τ is the time delay. $\Delta\phi_1(\mathbf{x}) = 0.1x_1\sin(t)$ and $\Delta\phi_2(\mathbf{x}) = 0.3x_2\sin(t)$ are unknown external disturbances, and $g(\mathbf{x}(t)) = 2 - \sin^2(x_1(t))$. $|\Delta f_1(\mathbf{x}(t))| \le h_2(\mathbf{x}(t)), |\Delta f_2(\mathbf{x}(t-\tau))| \le h_3(\mathbf{x}(t-\tau))$, and $Z(v(t))$ is an output of a dead-zone. The goal of control is to maintain the system output y to follow the reference signal $y_m = 0.5[\sin(t) + \sin(0.5t)]$.

In the simulation, parameters of the dead-zone are $m = 1$, $c_r = 0.5$, and $c_l = -0.5$. And their bounds are chosen as $m_{\max} = 1.5, m_{\min} = 0.6, c_{r\max} = 0.9, c_{r\min} = 0.1, c_{l\max} = -0.1$,

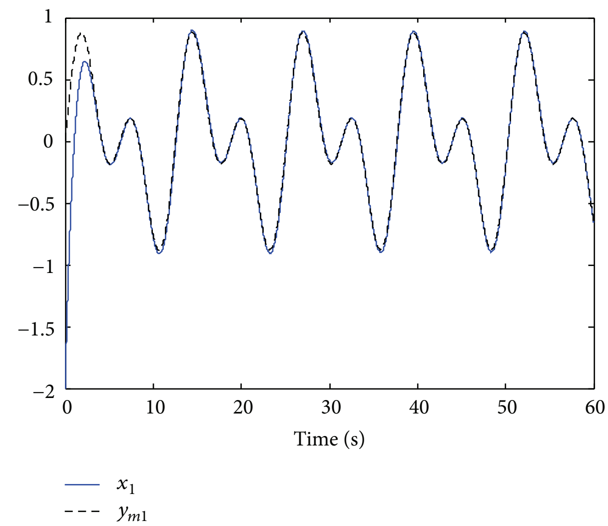

Figure 2: The trajectories of state x_1 and desired output y_{m1}.

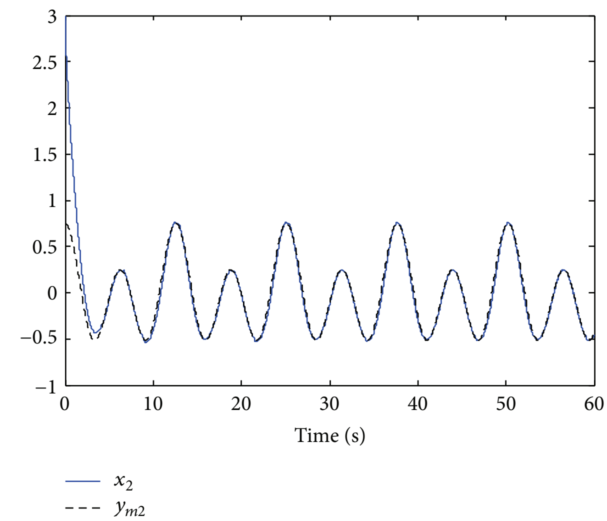

Figure 3: The trajectories of state x_2 and desired output y_{m2}.

and $c_{l\,\min} = -0.8$. In the implementation, six fuzzy sets are defined over interval $[-3, 3]$ for both x_1 and x_2, with labels $F1$, $F2$, $F3$, $F4$, $F5$, and $F6$, and their membership functions are

$$\begin{aligned}
\mu_{F1}(x_i) &= \frac{1}{1+\exp(5(x_i+2))},\\
\mu_{F2}(x_i) &= \exp\left(-(x_i+1.5)^2\right),\\
\mu_{F3}(x_i) &= \exp\left(-(x_i+0.5)^2\right),\\
\mu_{F4}(x_i) &= \exp\left(-(x_i-0.5)^2\right),\\
\mu_{F5}(x_i) &= \exp\left(-(x_i-1.5)^2\right),\\
\mu_{F6}(x_i) &= \frac{1}{1+\exp(-5(x_i-2))}, \quad i = 1, 2.
\end{aligned} \tag{56}$$

In this section, we apply the proposed robust adaptive fuzzy tracking control approach in Section 3 to deal with the output tracking control problem of the second-order uncertain nonlinear time-delay system as shown in (55). Choose $\mathbf{K} = [10, 10]$ and $\mathbf{Q} = \mathrm{diag}[5, 5]$; then, we solve the Lyapunov equation (27) to obtain

$$\mathbf{P} = \begin{bmatrix} 5.25 & 0.25 \\ 0.25 & 0.275 \end{bmatrix}. \tag{57}$$

In this example, the sampling time is 0.01 sec. Initial values are chosen as $\mathbf{x}(0) = [-2, 3]^{\mathrm{T}}$, $\boldsymbol{\theta}_g(0) = 1$, $\boldsymbol{\theta}_{h1}(0) = 0$, $\boldsymbol{\theta}_{\mathbf{h}2}(0) = 0$, and $\boldsymbol{\theta}_{\mathbf{h}3}(0) = 0$. The initial values of the parameters to be estimated are selected as $\widehat{\phi}(0) = 0.85$, $\widehat{\boldsymbol{\theta}}_1(0) = [0\ 0]^T$, $\widehat{\boldsymbol{\theta}}_2(0) = [0\ 0]^T$. $\gamma_g = 2$, $\gamma_{h1} = 1.5$, $\gamma_{h2} = 1.5$, $\gamma_{h3} = 1.5$, $\gamma_1 = 1.5$, $\gamma_2 = 1.5$, $\gamma_\omega = 1.5$, $\eta = 1.0$, $\tau = 0.5$ s, and $\varepsilon = 0.06$. The simulation results are shown in Figures 2–5. Figures 2 and 3 show the trajectories of states x_1 and x_2 and the desired outputs y_{m1} and y_{m2}, respectively. The phase plane of tracking errors of e_1 and e_2 is shown in Figure 4. Figure 5 shows the trajectory of the control signal. Obviously, the proposed robust adaptive fuzzy tracking control scheme can achieve the objective of good tracking performance and robust stability simultaneously in spite of the controlled system containing an unknown dead-zone and uncertainties.

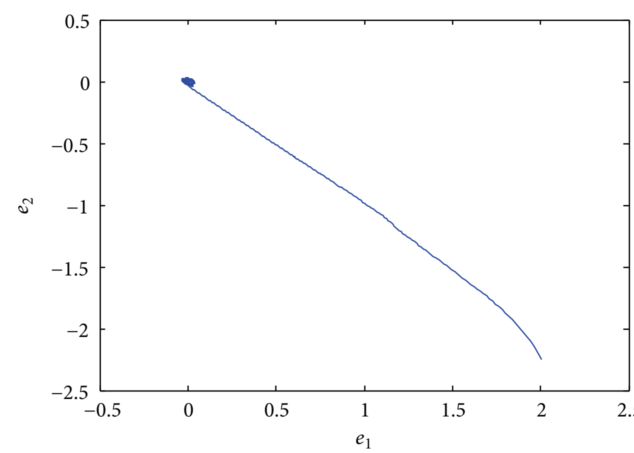

Figure 4: The phase plane of tracking errors e_1 and e_2.

5. Conclusion

The dead-zone input characteristics widely exist in the actuators of practical control systems, which are usually poorly known. The time-delay characteristics are usually confronted in engineering systems. The two characteristics may severely limit the performance of control. In this paper, the robust adaptive fuzzy tracking controller is designed to overcome the

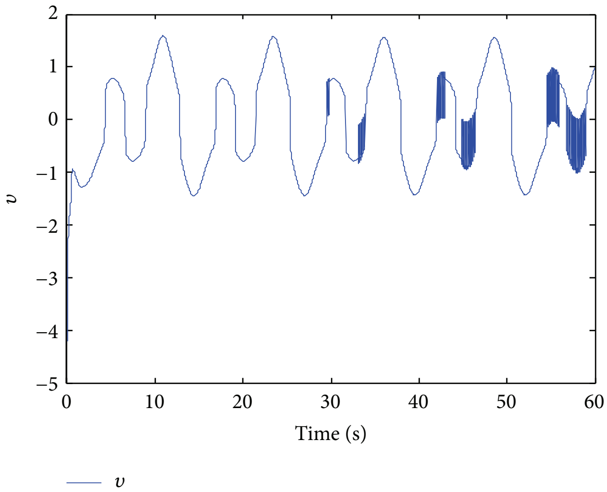

Figure 5: The trajectory of the control input $v(t)$.

stabilization problem of a class of uncertain nonlinear state time-delay systems containing unknown dead-zone input and unmatched uncertainties. By utilizing a description of a dead-zone feature to estimate the properties of the dead-zone model intuitively and mathematically, the adaptive fuzzy tracking controller is proposed without constructing the dead-zone inverse. The nonlinear uncertainties are approximated by the fuzzy logic system according to the adaptive laws. Based on the Lyapunov stability theorem, the proposed robust adaptive tracking fuzzy controller can ensure that the output tracking error of the resulting closed-loop system converges to a neighborhood of zero exponentially. Finally, some simulations results are illustrated to verify the effectiveness and performance of the proposed approach.

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