

# Inductor Current Sampled Feedback Control of Chaos in Current-Mode Boost Converter

Bo-Cheng Bao, Jian-Ping Xu, and Yan Liang

**Abstract**—A chaos control strategy for chaotic current-mode boost converter is presented by using inductor current sampled feedback control technique. The quantitative analysis of control mechanism is performed by establishing a discrete alternative map of the controlled system. The stability criterion, feedback gain, and corresponding critical duty ratio are obtained from the eigenvalue of the map. The simulation results verify the theoretical analysis results of the control strategy.

**Index Terms**—Control of chaos, current-mode boost converter, inductor current sampled feedback, stability criterion.

## 1. Introduction

Switching power converter circuits are strong nonlinear dynamics circuit with various chaotic behavior<sup>[1]-[8]</sup>. With the variation of circuit parameters, the current-mode boost converters will enter into chaos via period-doubling bifurcation<sup>[1]-[3]</sup>, which will enlarge the voltage and current ripple, and thus deteriorate the converter performance. Therefore, an effective control method to avoid the chaotic behavior is critical to switching power converter design.

Recently, various control methods have been developed for the chaos control in boost converter, such as slope compensation method<sup>[9]</sup>, modulated off-time current mode control technique<sup>[10]</sup>, linear state-feedback control method<sup>[11]</sup>, self-inductor current feedback method<sup>[12]</sup>, dynamic feedback control method<sup>[13]</sup>, time-delayed feedback control method<sup>[14],[15]</sup>, etc.. Until now, for the chaos control of boost converter, much attention is paid on the effectiveness of control technique<sup>[9]-[12]</sup>. As the quantitative analysis is complex, few research works are done on the quantitative analysis of the control parameters on the behavior of system<sup>[13],[14]</sup>.

In this paper, we will apply inductor current sampled feedback control to realize the control of chaos in the current-mode boost converter and focus on the quantitative analysis of control mechanism. A simple discrete map of the

controlled system is established. The stability criterion, feedback gain, and corresponding critical duty ratio are derived from eigenvalue analysis. The simulation is also performed to verify the analysis results. The theoretical analysis and numerical simulation results indicate that the inductor current sampled feedback control method can effectively control the current-mode boost converter to avoid chaos.

## 2. Chaotic Boost Converter

The current-mode control boost converter circuit is shown in Fig. 1, which includes an inductor, a diode, a DC source, a switch, a resistor, a capacitor, and a feedback path consisting of a flip-flop and a comparator. The converter is assumed to operate in continuous conduction mode (CCM), i.e. the inductor current  $i$  never falls to zero.

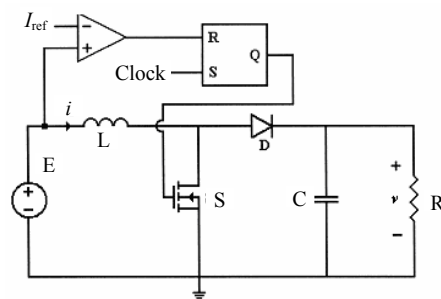


Fig. 1. Current-mode boost converter.

The inductor current  $i$  is chosen as the programming variable which, by comparing with reference current  $I_{ref}$  from current controller, generates the control signal of switch. The switch is turned on at the beginning of each switching cycle and turned off when  $i$  increases to  $I_{ref}$ , it will remain in off state until the beginning of next switching cycle.

Thus, there are two switching operation states in every switching cycle: (a) switch on and diode off; (b) switch off and diode on. The state equations of the current-mode boost converter in these two switching operation states can be given as<sup>[1]</sup>

$$\frac{d}{dt} \mathbf{x} = \mathbf{A}_1 \mathbf{x} + \mathbf{B}_1 \mathbf{E}, \quad nT \leq t \leq nT + dT \quad (1)$$

$$\frac{d}{dt} \mathbf{x} = \mathbf{A}_2 \mathbf{x} + \mathbf{B}_2 \mathbf{E}, \quad nT + dT \leq t \leq nT + T \quad (2)$$

where  $\mathbf{x} = [i \ v]^T$  are state variables,  $v$  is the voltage across the

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B.-C. Bao and Y. Liang are with Department of Electronic Engineering, Nanjing University of Science and Technology, Nanjing, 210094, China (e-mail: mervinbao@126.com and yliang@alcatel-lucent.com).

J.-P. Xu is with College of Electrical Engineering, Southwest Jiaotong University, Chengdu, 610031, China (e-mail: jpxu-swjtu@163.com).

capacitor  $C$ ,  $n$  is an integer, and  $d$  is the duty ratio. The state matrixes  $\mathbf{A}_1$  and  $\mathbf{A}_2$  can be obtained as

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}.$$

and the input matrixes  $\mathbf{B}_1$  and  $\mathbf{B}_2$  are

$$\mathbf{B}_1 = \mathbf{B}_2 = \begin{bmatrix} \frac{1}{L} & 0 \end{bmatrix}^T$$

To obtain a sampled data model, let the initial condition at the beginning of  $n$ th and  $(n+1)$ th switching cycle be  $i=i_n$ ,  $v=v_n$  and  $i_{n+1}$ ,  $v_{n+1}$ , respectively.

As the switch will be turn off when the inductor current increasing to the reference current  $I_{\text{ref}}$ , the turn-on time  $t_n$  can be obtained from (1) by integration, so the closed time  $t_n$  can thus be obtained as

$$t_n = \frac{(I_{\text{ref}} - i_n)L}{E}. \quad (3)$$

From [6], a stroboscopic map of current-mode boost converter can be derived into two cases:  $t_n \geq T$  and  $t_n < T$ .

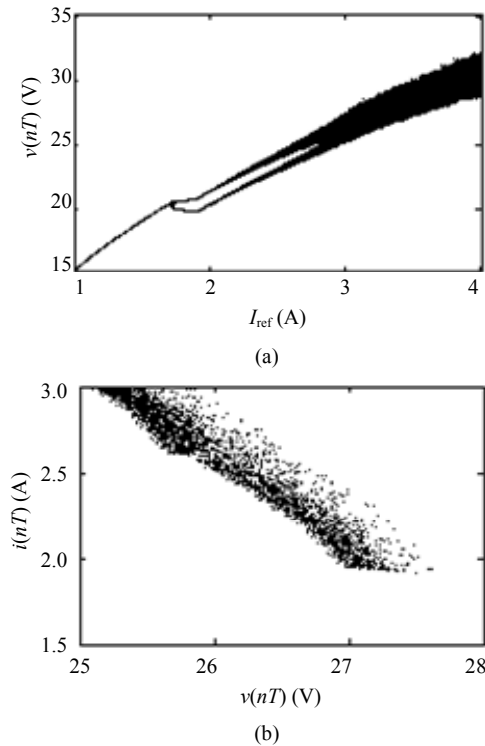


Fig. 2. Chaotic behavior in boost converter: (a) bifurcation diagram with  $I_{\text{ref}}$  as parameter and (b) chaos attractor at  $I_{\text{ref}}=3\text{A}$ .

This piecewise-smooth system can exhibit bifurcation to chaotic behavior with parameters variation. By taking reference current  $I_{\text{ref}}$  as the bifurcation parameter and circuit parameters as:  $E=10\text{ V}$ ,  $L=1.5\text{ mH}$ ,  $C=100\text{ }\mu\text{F}$ ,  $R=20\text{ }\Omega$ ,  $T=100\text{ }\mu\text{s}$ ,  $1\text{ A} \leq I_{\text{ref}} \leq 4\text{ A}$ , a bifurcation diagram can be obtained as shown in Fig. 2(a), from which we can know that the boost converter falls into chaos at  $I_{\text{ref}}=3\text{A}$ . Fig. 2(b)

shows the chaos attractor at this reference current value.

### 3. Chaos Control Strategy

#### 3.1 Schematic Model

Fig. 3 shows the schematic model of the chaos control strategy by using inductor current sampled feedback, where  $i(nT)$  is the sampled inductor current at the beginning of  $n$ th switching cycle,  $T$  is sampling period.  $i(nT)$  produces a feedback chaos control signal  $I_{\text{con}}$  with gain  $K$ . When the inductor current  $i$  increases to  $I_{\text{ref}} + I_{\text{con}}$ , a control signal of switch will be generated.

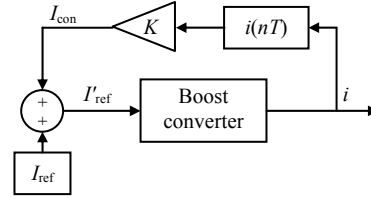


Fig. 3. Schematic model of the chaos control strategy.

By applying feedback chaos control signal  $I_{\text{con}}$  to the current-mode boost converter, the new reference current  $I'_{\text{ref}}$  can be obtained as

$$I'_{\text{ref}} = I_{\text{ref}} + I_{\text{con}} = I_{\text{ref}} + Ki_n. \quad (4)$$

#### 3.2 Mechanism of Chaos Control

If switching frequency is much higher than the highest natural frequency of the converter in each switching states, the output voltage can be regarded as constant in each switching cycle. Under this condition, the system becomes one-dimensional and the inductor current waveform becomes piecewise linear, therefore simply the modeling of the converters<sup>[5]</sup>.

Let the slopes of the inductor current during switch on and off states be  $m_1 = E/L$  and  $m_2 = (v-E)/L$ , respectively. Consider the boost converter with the feedback chaos control, the turn-on time  $t_n$  of switch is then

$$t_n = \frac{(I'_{\text{ref}} - i_n)L}{E} = \frac{[I_{\text{ref}} - (1-K)i_n]L}{m_1}. \quad (5)$$

In inductor current continuous conduction mode (CCM), there are two kinds of orbits between clock instants. If the turn-on time of switch satisfies  $t_n \geq T$ , the switch will remain in turn-on throughout the switching period, and the map in this case is given by

$$i_{n+1} = i_n + m_1 T. \quad (6)$$

If  $t_n < T$ , the inductor current will increase to  $I_{\text{ref}} + I_{\text{con}}$  and then decrease until the end of the switching cycle, in this case, we have

$$\begin{aligned} i_{n+1} &= I'_{\text{ref}} - m_2(T - t_n) \\ &= -\frac{m_2 - K(m_1 + m_2)}{m_1} i_n + (1 + \frac{m_2}{m_1}) I_{\text{ref}} - m_2 T \end{aligned} \quad (7)$$

The eigenvalue  $\lambda$  of the map can thus be given as

$$\lambda = \frac{di_{n+1}}{di_n} = 1, \text{ for } t_n \geq T \quad (8)$$

$$\lambda = \frac{di_{n+1}}{di_n} = -\frac{m_2 - K(m_1 + m_2)}{m_1}, \text{ for } t_n < T. \quad (9)$$

To ensure stable operation of the system, the eigenvalue  $\lambda$  must fall between  $-1$  and  $1$ . In particular, the first period-doubling will occurs when  $\lambda=1$  or  $\lambda=-1$ .

When  $t_n \geq T$ , the system will locate in the critical state between stable and unstable. Thus, we will only consider the case when  $t_n < T$ . In this case, for the boost converter, the equivalent criterion with no bifurcation is

$$|\lambda| = \frac{m_2 - K(m_1 + m_2)}{m_1} < 1. \quad (10)$$

The inductor current sampled feedback gain can then be obtained as follows

$$K > \frac{m_2 - m_1}{m_1 + m_2} = K_c \quad (11)$$

where  $K_c$  is the critical feedback gain. Equation (11) shows that if an appropriate feedback chaos control gain  $K$  is designed, the current-mode boost converter shown in Fig. 1 can be stable without chaos while the system locates in the case of  $m_2/m_1 > 1$ . This result indicates that the chaos control strategy presented in this paper can be applied to chaos controlling of the switching DC-DC converter.

If we are interested in the inner current loop dynamics near steady state, we may write

$$K_c = \frac{v - 2E}{v} = 2D - 1 \quad (12)$$

where  $D$  is the duty ratio defined in the usual way, and the voltage transfer ratio of input to output is  $v/E = 1/(1-D)$ . Hence, the critical duty ratio can be derived as

$$D_c = 0.5(1 + K) \quad (13)$$

From (13) we can see that when  $K = 0$ , i.e. when boost converter operates without the inductor current sampled feedback control, the critical duty ratio is  $D_c = 0.5$ , which agrees with well-known stable operation criterion that the duty ratio must be smaller than  $0.5$  in order to maintain a stable period-1 operation<sup>[7]</sup>.

From (13), we can also find that when inductor current sampled feedback control is introduced in the boost converter, the critical duty ratio will be  $0.5K$  larger than the critical duty ratio of traditional current-mode boost converter without such feedback control, which means that the stable operation range of the boost converter becomes bigger.

Fig. 4 shows the bifurcation diagram with the following circuit parameters:  $I_{ref} = 3$  A,  $E = 10$  V,  $v = 30$  V,  $L = 1.5$  mH,  $T = 100$   $\mu$ F,  $0 \leq K \leq 0.5$ . From Fig. 4, we can know that the boost converter goes into stable operation range when  $K_c$  is bigger than  $0.33$ , it is consistent with the value calculated from (12). When  $K_c = 0.33$ , the critical duty ratio is  $0.67$ .

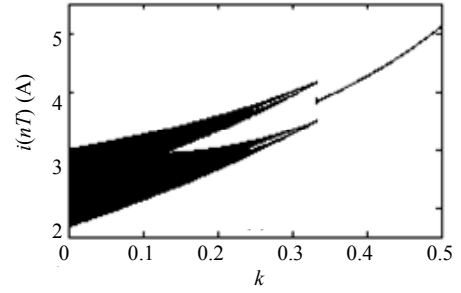


Fig. 4. Bifurcation diagram with  $K$ .

## 4. Simulation Verification

In our simulation verification, we investigate the effect on the chaotic behavior of the current-mode boost converter when using the inductor current sampled feedback control method. The simulation is performed by using the same circuit parameters as Fig. 2 (b).

Fig. 5 shows the capacitor voltage bifurcation diagram for  $K=0 \sim 0.5$ . When  $K$  is about  $0.4$ , the first period-doubling bifurcation of the system appears, the critical feedback gain is then  $K_c = 0.4$ . From (12), we can obtain the steady state output voltage as  $v = 2E/(1 - K_c) = 33.33$  V, which is close to the simulation result of  $v(nT) = 33.86$  V from Fig. 5.

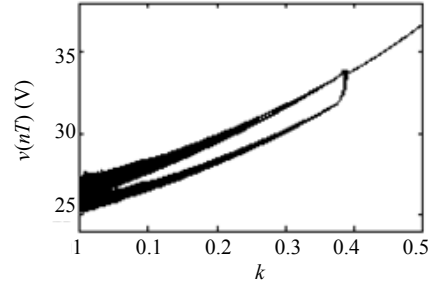


Fig. 5. Bifurcation diagram for the same parameters as Fig. 2(b).

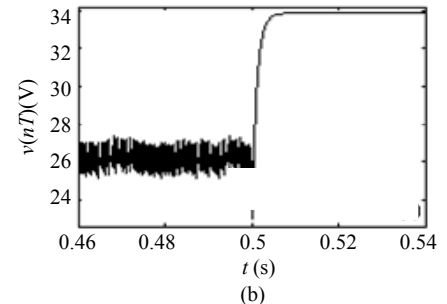
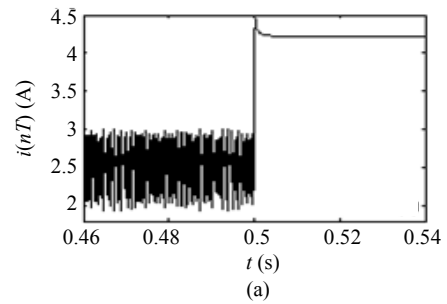


Fig. 6. Simulation waveforms of the boost converter: (a) inductor current and (b) capacitor voltage.

Fig. 6 (a) and (b) show the simulating waveforms of the inductor current and capacitor voltage when the additional control works at  $t = 0.5\text{s}$  for  $K = 0.4$ . It is observed that the controlled system can be successfully stabilized by applying the inductor current sampled feedback control. The results obtained indicate that this method can be used to control chaotic behavior in the boost converter.

## 5. Conclusions

Switching power converter circuits are strong nonlinear circuits with rich nonlinear phenomena. They exhibit bifurcation and chaotic behavior under some conditions. Hence, the control of nonlinear phenomena in DC-DC converters is of great significance. The inductor current sampled feedback control method proposed in this paper can directly control chaos in the boost converter. This control strategy can also be easily applied to the control of other power converters.

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**Bo-Cheng Bao** was born in Jiangsu Province, China, in 1965. He received his B.S. and M.S. degrees from University of Electronics Science and Technology of China (UESTC), Chengdu, in 1986 and 1989, respectively, both in electronic engineering. He is currently pursuing the Ph.D. degree with the Department of Electronic Engineering, Nanjing University of Science and Technology, Nanjing. His research interests include bifurcation and chaos, analysis and simulation in power electronic circuits.

**Jian-Ping Xu** was born in Guizhou Province, China, in 1963. He received his B.S. and Ph.D. degrees from University of Electronics Science and Technology of China (UESTC), Chengdu, in 1984 and 1989, respectively, both in electronic engineering. He is now a professor with the School of Electrical Engineering, Southwest Jiaotong University, Chengdu, China. His research interests include modeling, analysis and simulation of power electronics, novel control technique of power electronics systems.

**Yan Liang** was born in Hebei Province, China, in 1979. She received the B.S and M.S degrees from Nanjing University of Science and Technology (NJUST), Nanjing, in 2001 and 2004. She is currently pursuing the Ph.D. degree with the Department of Electronic Engineering, NJUST. Her research interests include wireless communication and signal processing.