A Simple Statistical Energy Analysis Technique on Modeling Continuous Coupling Interfaces

Lin Ji[1]
School of Mechanical Engineering,
Shandong University,
Jinan 250061, China
e-mail: jilin@sdu.edu.cn

Zhenyu Huang
School of Electronic,
Information and Electrical Engineering,
Shanghai Jiao Tong University,
Shanghai 200240, China

In statistical energy analysis (SEA) modeling, it is desirable that the SEA coupling loss factors (CLFs) between two continuously connected subsystems can be estimated in a convenient way. A simple SEA modeling technique is recommended in that continuous coupling interfaces may be replaced by sets of discrete points, provided the points are spaced at an appropriate distance apart. Consequently, the simple CLF formulae derived from discretely-connected substructures can be applied for continuous coupling cases. Based on the numerical investigations on SEA modeling of two thin plates connected along a line, a point-spacing criterion is recommended by fitting the point- and line-connection data of the two plates. It shows that the point spacing depends on not only the wavelengths but also the wavelength ratio of the two coupled subsystems. [DOI: 10.1115/1.4025246]

Keywords: statistical energy analysis (SEA), coupling loss factor (CLF), power injection method (PIM), continuous coupling, discrete coupling

1 Introduction

Statistical energy analysis (SEA) is commonly used for higher frequency modeling of complex systems [1–3]. A key assumption of SEA is that the transmitted power between any two directly connected SEA subsystems is proportional to the energy difference between them, and the proportionality is determined by the coupling loss factors (CLFs) between the two subsystems. Therefore, a successful application of the SEA theory largely relies on the accurate and convenient estimations of CLFs [4–8]. However, simple formulas for CLFs are only available for limited coupling cases, e.g., structure to structure via discrete points, plate to plate with a line connection, plate to room/cavity via area couplings, room to room/cavity via volume couplings, which are mainly derived from the wave transmission bases [1]. For subsystems with generic couplings, modal analysis based methods are available, e.g., the widely used power injection method (PIM) [9–11]. Nevertheless, the mode-based techniques generally need certain levels of extra knowledge of the modes of the uncoupled subsystems except for the gross modal properties. It is therefore quite desirable that the SEA CLFs of two generically connected subsystems can be estimated by simple formulas as well.

In previous research [12], it was found that a line connection may be replaced by point connections by separating the points as a half (bending) wavelength apart from each other. Consequently, the simple CLF formulae derived from discretely-connected substructures can be used for continuous coupling interfaces. However, this criterion has implicitly assumed that the two subsystems are of the same wavelengths. For coupling cases with wavelength ratio not equal to 1, such a criterion may lose its accuracy.

The present research argues that the point-spacing should be determined by both the wavelengths and wavelength ratio of the two connected subsystems. As a good supplement to the previous research, a new point-spacing criterion is recommended based on the numerical investigations on two line-connected plates by fitting the point- and line-connection data of the two plates.

2 Model Descriptions

A plate-plate numerical model connected via a line is setup, as shown in Fig. 1. Plates a and b are both thin, rectangular with two opposite edges (along the length directions) simply supported and the other two (along the width directions) free. The nominal length and width of a are 0.9 m and 0.8 m, respectively, while those of b are 1.0 m and 0.8 m, respectively. The two plates are fully connected along their widths. The materials of both plates are chosen to be steel with Young's modulus of $2.1 \times 10^{11}\,\mathrm{N/m^2}$, density of $7.85 \times 10^3\,\mathrm{kg/m^3}$, Poisson ratio of 0.3, and damping loss factor 0.01. To simulate different wavelength ratios of the two plates, the thicknesses of plates a and b are allowed to be varying from 1 mm, 2 mm to 3 mm. For simplicity, only the bending motions of the two plates are considered during the CLF calculations.

3 Three Techniques for CLF Estimating

Coupling loss factors between plates a and b can then be calculated by the following three techniques.

3.1 Conventional CLF Formula Derived for Line-Couplings. A simple formula of CLFs for line-connected plates was derived in [1] by using wave approach and transmission coefficient, as (Eq. (10.2.1) of [1])

$$\eta_{\mathrm{ab}} \approx \frac{c_g^{(a)} L_c}{\omega A_a}\left(\frac{\tau_{\mathrm{ab},\infty}}{2-\tau_{\mathrm{ab},\infty}}\right) \approx \frac{L_a c_g^{(a)}}{2\omega A_a}\tau_{\mathrm{ab},\infty} \tag{1}$$

In the above equation, η_{ab} is the coupling loss factor between plates a and b, $c_g^{(a)}$, A_a, and L_c are the group speed, surface area, and length of coupling line of plate a, respectively, and $\tau_{\mathrm{ab},\infty}$ is the wave transmission coefficient between a and b. For uniform homogeneous thin plates, analytical solutions of $c_g^{(a)}$ and $\tau_{\mathrm{ab},\infty}$ are available [1] which can be combined with Eq. (1) to give

$$\eta_{\mathrm{ab}} \approx \frac{4L_c}{k_a A_a}\left(\frac{\mathrm{Re}\left\{Z_{\mathrm{line},\infty}^{(a)}\right\}\mathrm{Re}\left\{Z_{\mathrm{line},\infty}^{(b)}\right\}}{\left|Z_{\mathrm{line},\infty}^{(a)}+Z_{\mathrm{line},\infty}^{(b)}\right|^2}\right) \tag{2}$$

where k_a is the wavenumber of plate a, and $\mathrm{Z}_{\mathrm{line},\infty}$ represents the characteristic line impedance of the plate.

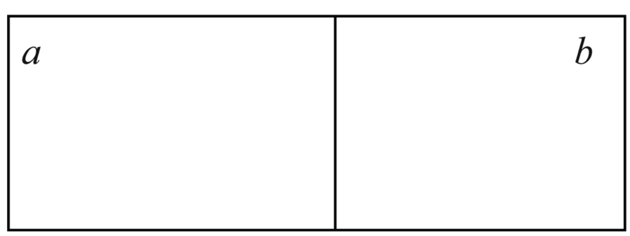

Fig. 1 Two line-connected plates

[1]Corresponding author.

Contributed by the Design Engineering Division of ASME for publication in the Journal of Vibration and Acoustics. Manuscript received October 25, 2012; final manuscript received July 30, 2013; published online September 24, 2013. Assoc. Editor: Thomas J. Royston.

The coupling loss factor η_{ba} can be evaluated by using the reciprocity relation of $n_a\eta_{ab} = n_b\eta_{ba}$ [1], where $n_{a,b}$ are the modal densities of plates a and b.

3.2 Conventional CLF Formula Derived From Point-Couplings. For a single point connection between subsystems a and b, the CLF expression was derived in [1] (by substituting Eq. (10.1.3) into Eq. (10.1.6) in [1]), as

$$\eta_{\mathrm{ab}} \approx \frac{1}{2\pi\omega n_a}\left(\frac{4\mathrm{Re}\left\{Z^{(a)}_{\mathrm{point},\infty}\right\}\mathrm{Re}\left\{Z^{(b)}_{\mathrm{point},\infty}\right\}}{\left|Z^{(a)}_{\mathrm{point},\infty} + Z^{(b)}_{\mathrm{point},\infty}\right|^2}\right) \quad (3)$$

where $Z_{\mathrm{point},\infty}$ represent the characteristic point impedance of the subsystem. When a and b are connected via N discrete points (assuming each point acting independently from others [1]), η_{ab} can be simply estimated as

$$\eta_{\mathrm{ab}} \approx \frac{N}{2\pi\omega n_a}\left(\frac{4\mathrm{Re}\left\{Z^{(a)}_{\mathrm{point},\infty}\right\}\mathrm{Re}\left\{Z^{(b)}_{\mathrm{point},\infty}\right\}}{\left|Z^{(a)}_{\mathrm{point},\infty} + Z^{(b)}_{\mathrm{point},\infty}\right|^2}\right) \quad (4)$$

Note that, on deriving of Eq. (4), the coherences among the coupling points have been ignored which implicitly requires the coupling points to be spaced at least one-wavelength apart from each other [1].

3.3 Power Injection Method. The principle of PIM methods has been widely employed to estimate the CLFs between two generically connected subsystems in situ [11]. It generally requires the calculation (or measurement) of the power input into every single subsystem and that of the energy level of every subsystem. By inverting the corresponding energy matrix, the SEA coupling loss factors can then be determined. Because the SEA equations are set up in sense of frequency- and space-averages, PIM typically requires more than three excitation and five response locations to be taken [11] so that the space-averaged subsystem energies and power input levels can be well captured.

In general, the CLF calculations based on the above three techniques are only valid for the frequency range where both plates are of high modal density.

4 Results and Discussions

Three plate-plate coupling models with different thickness-ratios (and hence the wavelength ratios) are considered: (1) $h_a = h_b = 2$ mm, (2) $h_a = 3$ mm and $h_b = 1$ mm, and (3) $h_a = 1$ mm and $h_b = 3$ mm. Therefore, the subsystem wavelengths $\lambda_a = \lambda_b = \lambda$ for case 1, while $\lambda_a = \lambda_b = \lambda$ and $\lambda = \lambda_a < \lambda_b$ for cases 2 and 3, respectively.

The corresponding η_{ab} are then calculated by using Eq. (2), Eq. (4), and the PIM technique, respectively. It should be noted that on calculating η_{ab} by Eq. (4) different numbers of interface points are employed by letting the points are separated by different distances along the coupling line. Here, the point-spacing Δ used are λ, $\lambda/2$, $(2/3)\lambda$, and $(1/4)\lambda$, respectively. Clearly, the number of N in Eq. (4) is a frequency-dependent variable. In the PIM calculations, five excitation locations and seven response locations on each plate model are chosen. The space-averaged energy responses of a and b as well as the power input levels of each plate are then determined based on an exact subsystem modal approach. Meanwhile, in order to minimize the spatial variation and hence to obtain the CLFs between the two plates more accurately, an ensemble of 64 plate-plate samples are generated, of which the modal densities and total masses of a and b are fixed, but the exact natural frequencies and mode shapes may differ from plate to plate. This is achieved by varying the ratio of the length and width of a (also for b) while keeping the surface area

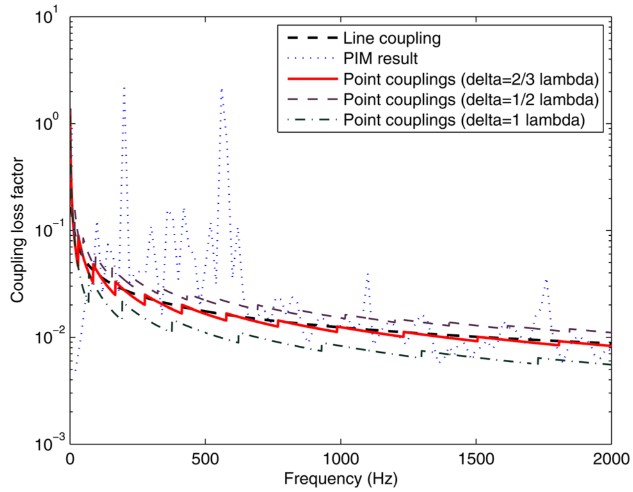

Fig. 2 Coupling loss factor η_{ab} between plates a and b when $h_a = h_b = 2$ mm, in which, line-coupling (Eq. (2), thick dashed line); PIM result (thin dotted line); point couplings (Eq. (4)) when $\Delta = (2/3)\lambda$ (thick real line), $\Delta = (1/2)\lambda$ (thin dashed line) and $\Delta = \lambda$ (thin dashed-dotted line)

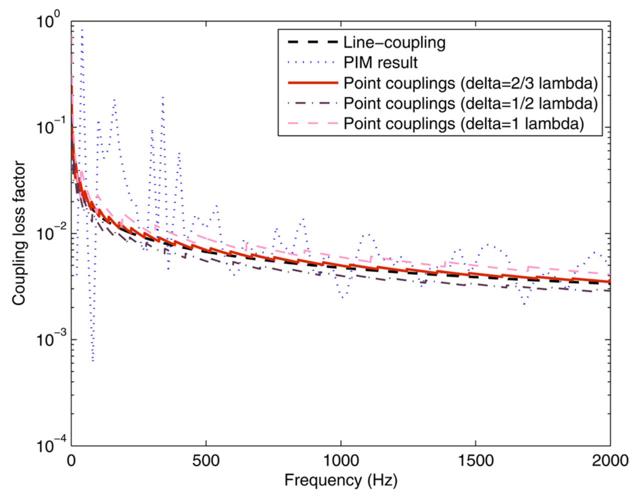

Fig. 3 Coupling loss factor η_{ab} between plates a and b when $h_a = 3$ mm and $h_b = 1$ mm, in which, line-coupling (Eq. (2), thick dashed line); PIM result (thin dotted line); point couplings (Eq. (4)) when $\Delta = (2/3)\lambda$ (thick real line), $\Delta = (1/2)\lambda$ (thin dashed line) and $\Delta = \lambda$ (thin dashed-dotted line)

fixed (±5% of the nominal value). η_{ab} can finally be estimated from the space- and ensemble-averaged energy responses and power inputs of the two plates.

The results of η_{ab} calculated by the above three techniques are compared in Figs. 2–4 for cases 1, 2, and 3, respectively. It is seen that the results agree fairly well when the sets of points are separated by a distance of $\Delta = \frac{2}{3}\lambda$ in Figs. 2 and 3, while $\Delta = \frac{2}{5}\lambda$ in Fig. 4. The good agreements in Figs. 2–4 thus strongly suggest that the coupling loss factors between two continuously connected SEA subsystems can be estimated simply by the formulas derived for multipoint couplings, provided the sets of discrete points are separated at a certain distance apart.

In fact, such a new criterion can be deduced from more numerical investigations as [1] the point-spacing Δ is generally within a range of $(\lambda/2, \lambda)$ in case of $\lambda_a \approx \lambda_b \approx \lambda$, of which $\Delta \approx \frac{2}{3}\lambda$ may be a more appropriate option than the previously suggested $\Delta \approx \frac{1}{2}\lambda$ in Refs. [12]; and Ref. [2] in case of $\lambda_a > \lambda_b = \lambda$, $\Delta \in (\lambda/2, \lambda)$, and $\Delta \in (\lambda/4, \lambda/2)$ are recommended for estimating η_{ab} and η_{ba}, respectively, of which $\Delta \approx \frac{2}{3}\lambda$ and $\Delta = \frac{2}{5}\lambda$ may be taken as widely

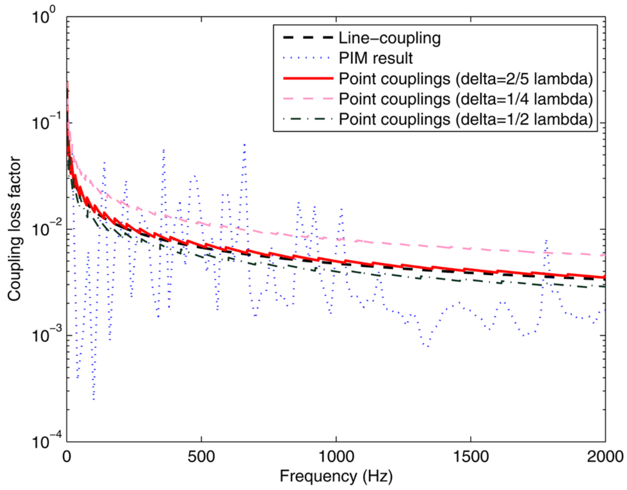

Fig. 4 Coupling loss factor η_{ab} between plates *a* and *b* when $\lambda_a = 1$ mm and $\lambda_b = 3$ mm, in which, line-coupling (Eq. (2), thick dashed line); PIM result (thin dotted line); point couplings (Eq. (4)) when $\Delta = (2/5)\lambda$ (thick real line), $\Delta = (1/4)\lambda$ (thin dashed line) and $\Delta = (1/2)\lambda$ (thin dashed-dotted line)

applicable options. Although it is deduced based on the numerical investigations of two line-connected plates, the criterion has a great potential to be extended for more general continuous coupling interfaces. More research that is relevant is underway.

5 Conclusions

With a clear aim to predict the SEA parameters of two continuously-connected subsystems in a simple and convenient way, numerical investigations are made on estimating the coupling loss factors between two line-connected plates by replacing the line interface with a number of discrete points. A new point-spacing criterion is thus recommended as a good supplement to the existing SEA modeling. It suggests that the appropriate point-spacing Δ depends on not only the wavelengths, but also the wavelength ratio of the two connected subsystems.

References

[1] Lyon, R. H., and DeJong, R. G., 1995, *Theory and Application of Statistical Energy Analysis*, Butterworth, London.

[2] Woodhouse, J., 1981, "An Introduction to Statistical Energy Analysis of Structural Vibration," J. Appl. Acoust., **14**, pp. 455–469.

[3] Fahy, F. J., 1994, "Statistical Energy Analysis: A Critical Review," Philos. Trans. R. Soc. A: Math. Phys. Eng. Sci., **346**, pp. 431–447.

[4] Finnveden, S., 2011, "A Quantitative Criterion Validating Coupling Power Proportionality in Statistical Energy Analysis," J. Sound Vib., **330**, pp. 87–109.

[5] Langley, R. S., 1990, "A Derivation of the Coupling Loss Factor Used in Statistical Energy Analysis," J. Sound Vib., **141**, pp. 207–219.

[6] Maxit, L., and Guyader, J. -L., 2001, "Estimation of SEA Coupling Loss Factors Using a Dual Formulation and FEM Modal Information Part 1: Theory; Part 2: Numerical Applications," J. Sound Vib., **239**, pp. 907–948.

[7] Mace, B. R., 2005, "Statistical Energy Analysis: Coupling Loss Factors, Indirect Coupling and System Modes," J. Sound Vib., **279**, pp. 141–170.

[8] Langley, R., 2008, "Recent Advances and Remaining Challenges in the Statistical Energy Analysis of Dynamic Systems," Proceedings of the 7th European Conference on Structural Dynamics, Southampton, UK, July 7–9.

[9] Simmons, C., 1991, "Structure-Borne Sound Transmission Through Plate Junctions and Estimates of SEA Coupling Loss Factors Using the Finite Element Method," J. Sound Vib., **144**, pp. 215–227.

[10] Fahy, F. J., and Ruivo, H. M., 1997, "Determination of Statistical Energy Analysis Loss Factors by Means of an Input Power Modulation Technique," J. Sound Vib., **203**, pp. 763–779.

[11] DeLanghe, K., and Sas, P., 1996, "Statistical Analysis of the Power Injection Method," J. Acoust. Soc. Am., **100**, pp. 294–303.

[12] Craik, R. J. M., and Smith, R. J., 2000, "Sound Transmission Through Lightweight Parallel Plates, Part II: Structural-Borne Sound," Appl. Acoust., **61**, pp. 247–269.