The Unsteady Potential Flow in an Axially Variable Annulus and Its Effect on the Dynamics of the Oscillating Rigid Center-Body

Dan Mateescu
Associate Professor.

Michael P. Paidoussis
Professor and Chairman.
Fellow ASME

Department of Mechanical Engineering,
McGill University,
Montreal, Quebec, Canada H3A 2K6

This paper presents an analytical investigation of the unsteady potential flow in a narrow annular passage formed by a motionless rigid duct and an oscillating rigid center-body, both of axially variable cross section, in order to determine the fluid-dynamic forces exerted on the center-body. Based on this theory, a first-approximation solution as well as a more accurate solution are derived for the unsteady incompressible fluid flow. The stability of the center-body is investigated, in terms of the aerodynamic (or hydrodynamic) coefficients of damping, stiffness and inertia (virtual mass), as determined by this theory. The influence of various system parameters on stability is discussed.

1 Introduction

The dynamics and stability of flexible cylindrical pipes conveying fluid have been studied very extensively, partly as a means to solving real flow-induced vibration problems in industrial installations, and partly because of the inherent interest in the dynamical behavior of this class of fundamental problems in Applied Mechanics [1]. In the first set of studies, a historical review of which may be found in reference [2], the pipe was considered to be a tubular beam. Later, the case of thin-walled pipes, considered to be thin cylindrical shells, was also analyzed; a literature review in this case is given in reference [3].

The dynamics of cylindrical bodies in axial flow has also received considerable attention, e.g., [4–6], the impetus in this case coming from the presence of such systems in nuclear reactors and in aeronautical and underwater applications – vide [7] for a review of this topic.

However, the study of dynamics of coaxial cylindrical bodies, with flow in the annular space in between, has received considerably less attention, despite the fact that such geometries are common in many engineering systems (e.g., in PWR and AGR nuclear reactors, in heat exchangers, certain types of valves, and large jet pumps) and that they have been shown to be particularly prone to a host of vibration-induced problems [1]. Some of the few studies on this topic were limited to cylindrical beams in cylindrical conduits, e.g., [8, 9], utilizing analytical tools not particularly suitable for very narrow annuli (which are of particular interest here), and very recently to coaxial cylindrical shells [3, 10]. The problem of a nonuniform body of revolution in annular flow has been given most attention in the United Kingdom, in conjunction with flow-induced vibration of the fuel stringers of AGR-type nuclear reactors, experienced while refueling [11–15]. The early work by Miller and Kennison [16, 17] and a recent review by Mulcahy [18] also ought to be mentioned.

Hobson [14] considered the problem of a rigid, flexibly mounted cylindrical structure with a rounded leading edge ("nose") in a cylindrical conduit or a diffuser. Based on the assumption of a very narrow annular clearance and neglecting the radial variation of the fluid velocity, interesting solutions for the fluid forces on the center-body were obtained, which were found to be in good accord with experiment. This work [14, 15] is of particular interest, because it attempts to solve a similar type of problem as that addressed in the present paper – although by a completely different approach and method of solution.

The present paper, which is the first part of a larger research program undertaken by the authors in this field, is devoted to the study of flow-induced vibrations of a flexibly mounted rigid body of revolution in a duct or nozzle, where the annular clearance is small and where both center-body and duct have specified axial variations of their cross sections. Despite the assumption of a small annular clearance with respect to the center-body radius, radial variations in the unsteady annular flow are nevertheless taken into account.

In order to determine the generalized unsteady fluid-dynamic (aerodynamic or hydrodynamic) forces, the present analysis of the unsteady annular flow will deal with the case of incompressible and inviscid fluids. A subsequent paper will be devoted to the analysis of the effect of fluid viscosity on the flow-induced vibrations of the center-body. In fact, a preliminary investigation of the influence of viscosity [19] indicates that it is not very important and that its effect on the dynamics of the center-body is stabilizing. Hence, the analysis presented here is a valid first approximation for the problem at hand.

Contributed by the Fluids Engineering Division and presented at the Symposium on Flow Induced Vibrations, Winter Annual Meeting, New Orleans, LA, December 5-10, 1984. Manuscript received by the Fluids Engineering Division, July 19, 1984.

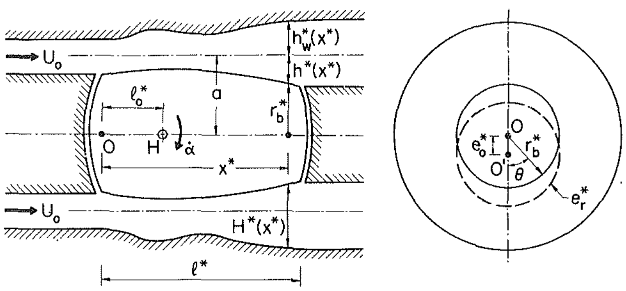

Fig. 1 Geometry of the oscillating body of revolution inside the duct of variable cross section

In the first part of the paper, the equations of unsteady annular flow are considered and solved by a method based on a local linearization scheme to determine the pressures and the fluid-dynamic forces acting on the center-body. This method of solution is applied to the usual case, where the length of the oscillating center-body is larger than, or comparable with, the mean annular radius; however, an extension of the method to the case of smaller center-body length is also presented, in Section 7.

With the fluid-dynamic forces acting on the body determined, the stability of the system is considered next, with the aim of establishing the conditions under which fluid-elastic instabilities are possible. The influence on flow-induced vibration of some geometrical parameters is also investigated, e.g., the effect of the shape of the annular passage (convergent or divergent), the position of the point of support of the body, and so on.

2 Problem Formulation

The system under consideration is shown in Fig. 1. It consists of a rigid center-body of revolution, hinged at point H and lying in a duct, so that it is subjected to an annular flow, as shown. The center-body is considered to perform oscillatory motion about the hinge-point, under the action of motion-dependent fluid forces, and restoring and retarding forces of a rotational spring and a dashpot (not shown). The circular cross sections of both center-body and duct are assumed to be axially variable, such that the annular clearance has a specified axial variation over the length of the center-body.

Two fixed cylindrical bodies are considered to be situated upstream and downstream of the oscillating center-body, and the corresponding annular clearance over these portions is considered to remain constant.

The following notation will be used in formulating the analytical model:

l^* = length of oscillating center-body
l_0^* = length OH, defining the position of the hinge, as shown in Fig. 1
a = mean radius of the annular passage
x^*, r^*, θ = cylindrical coordinate system
$r_b^*(x^*) = a - h^*(x^*)$ = center-body radius at location x^*
$r_w^*(x^*) = a + h_w^*(x^*)$ = duct-wall radius at location x^*
$H^*(x^*) = h^*(x^*) + h_w^*(x^*)$ = overall annular clearance at location x^*
$H_0^* = H^*(0), H_1^* = H^*(l^*)$ = annular clearances upstream and downstream of the oscillating center-body, respectively
$\alpha(t)$ = angle of rotation of center-body axis, in oscillatory motion about hinge-point
$\dot{\alpha}(t) = d\alpha/dt$ = angular velocity of center-body about hinge-point
Ω = circular frequency of center-body oscillation
$e_0^*(x^*,t) = (x^* - l_0^*)\alpha(t)$ = lateral displacement of center-body axis

The starred dimensional quantities have been utilzied here, so that their dimensionless counterparts, which are more widely used, may be defined later in simpler form; e.g., $x = x^*/a$.

At the upstream end of the system, where the annular clearance is constant, the fluid flow is assumed to be uniform and steady, with velocity U_0. Furthermore, assuming incompressible fluid flow, the velocity potential in cylindrical coordinates is

$$\frac{\partial^2 \Phi}{\partial x^{*2}} + \frac{\partial^2 \Phi}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial \Phi}{\partial r^*} + \frac{1}{r^{*2}}\frac{\partial^2 \Phi}{\partial \theta^2} = 0, \qquad (1)$$

subject to the boundary conditions

$$\frac{\partial \Phi}{\partial r^*}\bigg|_{r^*=r_b^*} = \frac{\partial e_r^*}{\partial t} + \left[\frac{\partial \Phi}{\partial x^*}\right]_{r^*=r_b^*}\left\{\frac{\partial e_r^*}{\partial x^*} - \frac{\partial h^*}{\partial x^*}\right\} + \left[\frac{1}{r^*}\frac{\partial \Phi}{\partial \theta}\right]_{r^*=r_b^*}\frac{1}{r^*}\frac{\partial e_r^*}{\partial \theta} \qquad (2a)$$

$$\frac{\partial \Phi}{\partial r^*}\bigg|_{r^*=r_w^*} = \left[\frac{\partial \Phi}{\partial x^*}\right]_{r^*=r_w^*}\frac{dh_w^*}{dx^*}, \quad \frac{\partial \Phi}{\partial x^*}\bigg|_{x^*\to-\infty} = U_0, \qquad (2b)$$

where $e_r{}^*(x^*, \theta, t)$ represents the radial displacement of the oscillating center-body surface,

$$e_r^*(x^*,\theta,t) = e_0^*(x^*,t)\cos\theta = (x^* - l_0^*)\alpha(t)\cos\theta \qquad (3)$$

in which the oscillatory motion of the center-body may be expressed as follows:

$$\alpha(t) = \alpha_0 \exp(i\Omega t) \qquad (4)$$

Introducing the dimensionless quantities

$$x = x^*/a,\ r = r^*/a,\ \tau = U_0 t/a,\ \omega = \Omega a/U_0, \qquad (5)$$

$$e_r = e_r^*/a,\ h = h^*/a,\ h_w = h_w^*/a,\ H(x) = h(x) + h_w(x), \qquad (6)$$

and separating the fluid flow into a steady axisymmetric component (denoted with the subscript "s") and an unsteady one (without subscript), so that

$$\Phi(x^*, r^*\theta, t) = U_0 a[\phi_s(x,r) + \phi(x,r,\theta,\tau)] \qquad (7)$$

equation (1) becomes

$$\frac{\partial^2 \phi_s}{\partial x^{*2}} + \frac{\partial^2 \phi_s}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_s}{\partial r} = 0 \qquad (8)$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0 \qquad (9)$$

subject to the boundary conditions

$$\frac{\partial \phi_s}{\partial r}\bigg|_{r=1-h} = -\left[\frac{\partial \phi_s}{\partial x} + \frac{\partial \phi}{\partial x}\right]_{r=1-h}\frac{\partial h}{\partial x}, \qquad (10a)$$

$$\frac{\partial \phi_s}{\partial r}\bigg|_{r=1+h_w} = \left[\frac{\partial \phi_s}{\partial x} + \frac{\partial \phi}{\partial x}\right]_{r=1+h_w}\frac{\partial h_w}{\partial x}, \quad \frac{\partial \phi_s}{\partial x}\bigg|_{x\to-\infty} = 1, \qquad (10b)$$

$$\frac{\partial \phi}{\partial r}\bigg|_{r=1-h} = \frac{\partial e_r}{\partial \tau} + \left[\frac{\partial \phi_s}{\partial x} + \frac{\partial \phi}{\partial x}\right]_{r=1-h}\frac{\partial e_r}{\partial x} + \left[\frac{1}{r}\frac{\partial \phi}{\partial \theta}\right]_{r=1-h}\frac{1}{r}\frac{\partial e_r}{\partial \theta}, \qquad (11a)$$

$$\frac{\partial \phi}{\partial r}\bigg|_{r=1+h_w} = 0, \quad \frac{\partial \phi}{\partial x}\bigg|_{x\to-\infty} = 0. \qquad (11b)$$

Denoting by $p_s(x, r)$ and $p(x, r, \theta, \tau)$ the perturbation pressures corresponding to the axisymmetric steady fluid flow and the unsteady fluid motion, respectively, and applying the Bernoulli-Lagrange equation, one obtains

$$p_s(x, r)+p(x,r,\theta,\tau)= -\tfrac{1}{2}\rho U_0^2\left\{2\,\frac{\partial\phi}{\partial\tau}+(\nabla\phi_s+\nabla\phi)^2-1\right\}. \quad (12)$$

Obviously, the generalized unsteady fluid-dynamic forces acting on the oscillating center-body do not depend on the perturbation pressure $p_s(x, r)$, because it is axially symmetric and time-independent; hence, the axisymmetric steady flow corresponding to the potential $\phi_s(x, r)$ will not be discussed in the subsequent analysis.

3 Linearization Procedure

In order to obtain an analytical solution of the problem, equations (10)–(12) have to be linearized, through the use of the small disturbance assumption.

In this connection, it is noted that the usual procedure of linearization with respect to the undisturbed fluid velocity U_0 (or, in nondimensional form, with respect to $\bar{U}_0 = 1$) is not sufficiently accurate for the case of narrow annuli, because the mean axial velocity $U^*(x^*)$ at location x^* could be substantially different from U_0, i.e.,

$$U^*(x^*)=\frac{1}{\pi(h_w^*+h^*)(2a+h_w^*-h^*)}\int_{a-h^*}^{a+h_w^*}\int_0^{2\pi}\frac{\partial\Phi}{\partial x^*}\,r^*dr^*d\theta. \quad (13)$$

For this reason, a procedure of local linearization will be used, with respect to the local dimensionless mean axial flow velocity, $U(x) = U^*(x^*)/U_0$, which may be obtained by applying the equation of continuity in integral form, so that

$$U(x)=H_0/H(x) \text{ where } H_0=H(0)=H^*(0)/a. \quad (14)$$

At the same time, defining the nondimensional variable

$$z=(r^*-a)/a=r-1, \quad (15)$$

the assumption of a narrow annular passage ($z << 1$) is introduced in the potential flow equations, (8) and (9).

Taking into consideration the form of boundary conditions (11 *a*, *b*), it is convenient to introduce the reduced motion potentials $\hat{\phi}_0(x, z)$ and $\hat{\phi}_1(x, z)$, defined by the relation

$$\phi(x, z, \theta, \tau)=\left\{\frac{d\alpha}{d\tau}\,\hat{\phi}_1(x,z)+\alpha(\tau)\hat{\phi}_0(x,z)\right\}\cos\theta, \quad (16)$$

where

$$\alpha(\tau)=\alpha_0\exp(i\omega\tau). \quad (17)$$

In this manner, the problem may be formulated by the following linearized equations:

$$\frac{\partial^2\hat{\phi}_j}{\partial x^2}+\frac{\partial^2\hat{\phi}_j}{\partial z^2}+\frac{\partial\hat{\phi}_j}{\partial z}-\hat{\phi}_j=0,\quad j=0,1, \quad (18)$$

subject to the boundary conditions

$$\left.\frac{\partial\hat{\phi}_0}{\partial z}\right|_{z=-h}=\frac{H_0}{H(x)},\quad \left.\frac{\partial\hat{\phi}_0}{\partial z}\right|_{z=h_w}=0, \quad (19)$$

$$\left.\frac{\partial\hat{\phi}_1}{\partial z}\right|_{z=-h}=x-l_0,\quad \left.\frac{\partial\hat{\phi}_1}{\partial z}\right|_{z=h_w}=0, \quad (20)$$

where $l = l^*/a$ and $l_0 = l_0^*/a$.

The perturbation pressure equation, in linearized form, now becomes

$$-\frac{1}{\rho U_0^2}\,p(x,z,\theta,\tau)=\left\{\frac{d^2\alpha}{d\tau^2}\,\hat{\phi}_1(x,z)+\frac{d\alpha}{d\tau}\left[\hat{\phi}_0(x,z)+U(x)\,\frac{\partial\hat{\phi}_1}{\partial x}\right]+\alpha(\tau)U(x)\,\frac{\partial\hat{\phi}_0}{\partial x}\right\}\cos\theta. \quad (21)$$

4 Equation of Motion of the Center-Body

The equation of motion of the oscillating center-body may be written as

$$J_H\ddot{\alpha}+C\dot{\alpha}+K\alpha=M, \quad (22)$$

where J_H is the moment of inertia of the body about the hinge axis, K and C are the mechanical spring stiffness and damping; M represents the moment of pressure forces about the hinge axis,

$$M=-a^3\int_0^l\int_0^{2\pi}(x-l_0)p(x,-h,\theta,\tau)S(x)\cos\theta\,dx\,d\theta. \quad (23)$$

where

$$S(x)=\frac{1-h(x)}{[1+h'^2(x)]^{1/2}}\left\{1-\frac{1-h(x)}{x-l_0}\,h'(x)\right\}\simeq\frac{1-h(x)}{[1+h'^2(x)]^{1/2}}$$

accounts for the local slope of the body surface; $h(x)$ has been defined in equation (6) and $h'(x) = dh/dx$.

Introducing the dimensionless parameters

$$\sigma=\frac{\pi\rho a^5}{J_H},\ k_0^2=\frac{K\,a^2}{J_H U_0^2},\ k_1=\frac{C\,a}{J_H U_0}, \quad (24)$$

the foregoing equation of motion may be written in nondimensional form as

$$(1+\sigma q_2)\frac{d^2\alpha}{d\tau^2}+(k_1+\sigma q_1)\frac{d\alpha}{d\tau}+(k_0^2+\sigma q_0)\alpha(\tau)=0, \quad (25)$$

where q_0 and q_1 represent the dimensionless coefficients of the fluid-dynamic stiffness and damping, respectively, and q_2 is the nondimensional added or apparent (virtual) mass; more specifically,

$$q_0=-\int_0^l(x-l_0)S(x)U(x)\,\frac{\partial\hat{\phi}_0}{\partial x}(x,-h)dx, \quad (26a)$$

$$q_1=-\int_0^l(x-l_0)S(x)\left\{\hat{\phi}_0(x,-h)+U(x)\,\frac{\partial\hat{\phi}_1}{\partial x}(x,-h)\right\}dx, \quad (26b)$$

$$q_2=-\int_0^l(x-l_0)S(x)\hat{\phi}_1(x,-h)dx. \quad (26c)$$

As previously mentioned, these coefficients, which in fact represent the generalized unsteady fluid-dynamic forces, in nondimensional form and with inverted sign, do not depend on the steady flow potential, $\phi_s(x, r)$, because of the axial symmetry of the flow passage. For this reason, in this analysis, only the reduced motion potentials $\phi_0(x,z)$ and $\hat{\phi}_1(x,z)$, will be examined, wherein are included all unsteady effects.

5 First-Approximation Solution

A further simplification of the problem in the case of smooth axial variation of the narrow annular clearance may be introduced, based on the slender body assumption. In this case, the first term of equation (18) may be neglected, so that it simplifies to

$$\frac{\partial^2\hat{\phi}_j}{\partial z^2}+\frac{\partial\hat{\phi}_j}{\partial z}-\hat{\phi}_j=0,\quad j=0,1. \quad (27)$$

This clearly has a solution of the form

$$\hat{\phi}_j(z;x)=A_j(x)\exp(\beta_1 z)+B_j(x)\exp(\beta_2 z), \quad (28)$$

where

$$\beta_1=\tfrac{1}{2}(\beta-1),\ \beta_2=-\tfrac{1}{2}(\beta+1),\ \beta=\sqrt{5} \quad (29)$$

The coefficients A_j and B_j are smooth functions of x and may be determined from boundary conditions (19) and (20). Thus, for example, in the case of a cylindrical center-body and a variable cross-section duct, i.e., $H(x) = h_w(x)$, the solutions for the reduced potentials $\hat{\phi}_j$ and their axial derivatives $\partial\hat{\phi}_j/\partial x$ on the oscillating center-body may be expressed in the form

$$\hat{\phi}_0(-h;x) = U(x)G(H), \quad (30a)$$

$$\frac{\partial \hat{\phi}_0}{\partial x}(-h;x) = U(x)G'(H) + U'(x)G(H), \quad (30b)$$

$$\hat{\phi}_1(-h;x) = (x-l_0)G(H), \quad (31a)$$

$$\frac{\partial \hat{\phi}_1}{\partial x}(-h;x) = (x-l_0)G'(H) + G(H), \quad (31b)$$

$$G(H) = -\frac{\beta}{\exp(\beta H) - 1} - ½(\beta - 1),$$

$$G'(H) = \beta^2 \frac{\exp[\beta H]}{[\exp(\beta H) - 1]^2}. \quad (32)$$

Introducing these results into equations ($26a$–c), the determination of the nondimensional fluid-dynamic coefficients of stiffness, damping and virtual mass is reduced to simple quadratures. Numerical examples of these solutions will be presented in Section 8.

6 A More Accurate Method of Solution

It is expected that for higher frequency oscillations, the first approximation solution developed in Section 5 is not sufficiently accurate. This is because the virtual mass coefficient and the coefficient of fluid-dynamic damping, which depend on the reduced potential $\hat{\phi}_1$, cannot be accurately reproduced if the slender body approximation is utilized, even for very smooth or null axial variation of the annular clearance.

In this case, the complete form of partial differential equation (18) must be considered. A solution of the following form is substituted therein,

$$\hat{\phi}_j(x,z) = \sum_k f_{jk}(x) F_{jk}(z), \quad (33)$$

which yields the ordinary differential equations

$$f''_{jk}(x) - \gamma_k^2 f_{jk}(x) = 0, \quad (34a)$$

$$F''_{jk}(z) + F'_{jk}(z) - (1-\gamma_k^2) F_{jk}(z) = 0; \quad (34b)$$

these admit the general solutions

$$f_{jk}(x) = A_{jk} \exp(\gamma_k x) + B_{jk} \exp(-\gamma_k x), \quad (35a)$$

$$F_{jk}(z) = a_{jk} \exp(\beta_{1k} z) + b_{jk} \exp(-\beta_{2k} z), \quad (35b)$$

where

$$\beta_{1k} = ½(\beta_k - 1),\ \beta_{2k} = ½(\beta_k + 1),\ \beta_k = \sqrt{5 - 4\gamma_k^2}. \quad (36)$$

Assuming imaginary values for the constants $\gamma_k (\gamma_k = ic_k)$ the expressions $U(x) = H_0/H(x)$ and $x - l_0$, which appear in boundary conditions (19) and (20), may be expanded in the form of truncated Fourier expansions

$$U(x) = \frac{H_0}{H(x)} = \sum_{k \in N^*} D_{0k} \cos c_k x + \sum_{k \in N} E_{0k} \sin c_k x, \quad (37a)$$

$$x - l_0 = \sum_{k \in N^*} D_{1k} \cos c_k x + \sum_{k \in N} E_{1k} \sin c_k x, \quad (37b)$$

where

$$N = \{1,2,3,\ldots,n,\ldots\},\ N^* = \{0,1,2,\ldots,n,\ldots\}. \quad (38)$$

Introducing these expressions in (19) and (20), for example, in the case of a cylindrical center-body and a variable cross-section duct, i.e., $H(x) = h_w(x)$, the solutions

$$\hat{\phi}_j(x,-h) = \sum_{k \in N^*} D_{jk} G_k(H) \cos c_k x + \sum_{k \in N} E_{jk} G_k(H) \sin c_k x, \quad (39)$$

may be obtained, where

$$G_k(H) = -\frac{4\beta_k}{\beta_k^2 - 1} \frac{1}{\exp(\beta_k H) - 1} - \frac{2}{\beta_k + 1},\ \beta_k = \sqrt{5 + 4c_k^2}. \quad (40)$$

The determination of the nondimensional fluid-dynamic coefficients of stiffness, damping and virtual mass has been thus reduced again to simple quadratures.

A comparison between the solutions obtained with this more accurate method and with the approximate method developed earlier is presented in Section 8.

7 Extension of the Method for the Case of Short Center-Bodies

The foregoing solutions have been derived on the assumption that the length of the oscillating center-body is larger than or comparable with the mean annular radius, and hence that it is much greater than the width of the narrow annular passage. In fact, this is the usual situation in the majority of concrete applications involving narrow annular passages. Under such conditions, small variations in the pressure distribution at the two ends of the center-body (resulting from differences in the upstream and downstream boundary conditions of the unsteady fluid flow) have very little effect on the fluid-dynamic coefficients of stiffness, damping and virtual mass.

Nevertheless, the case where the length of the oscillating body is smaller than its radius, and hence not very large with respect to the narrow annular clearance, can also be treated by the method of Section 6. Special precautions have to be taken in this case, however, in order to obtain solutions satisfying accurately all the boundary conditions.

The boundary conditions (19) and (20) on the oscillating center-body have to be expressed in convenient truncated expansions (with real or imaginary values for the constants γ_k) of the form

$$\left.\frac{\partial \hat{\phi}_0}{\partial z}\right|_{z=-h} = \frac{H_0}{H(x)} \mathcal{H}(x)\mathcal{H}(l-x)$$

$$= \sum_{k \in N^*} \{D_{0k} \exp(\gamma_k x) + E_{0k} \exp(-\gamma_k x)\}, \quad (41)$$

$$\left.\frac{\partial \hat{\phi}_1}{\partial z}\right|_{z=-h} = (x - l_0)\mathcal{H}(x)\mathcal{H}(l-x)$$

$$= \sum_{k \in N^*} \{D_{1k} \exp(\gamma_k x) + E_{1k} \exp(-\gamma_k x)\}, \quad (42)$$

where $\mathcal{H}(x)$ denotes the Heaviside generalized function

$$\mathcal{H}(X) = \begin{cases} 0 \text{ for } X < 0, \\ 1 \text{ for } X > 0. \end{cases} \quad (44)$$

8 Stability of Flow-Induced Vibrations of the Center-Body in a Variable Cross-Section Duct

The dynamics of the oscillating center-body is governed by the equation of motion (25), where the nondimensional coefficients of fluid-dynamic stiffness q_0, damping q_1, and virtual mass q_2, as well as all the other parameters, have been defined in Section 4.

All essential characteristics of vibration of the center-body in the annular flow, e.g., the frequency, amplitude and stability of the vibrations, depend critically on the fluid-dynamic coupling terms (fluid-dynamic stiffness, damping and virtual mass), as well as on the relative magnitude of these terms vis-à-vis the mechanical stiffness and damping

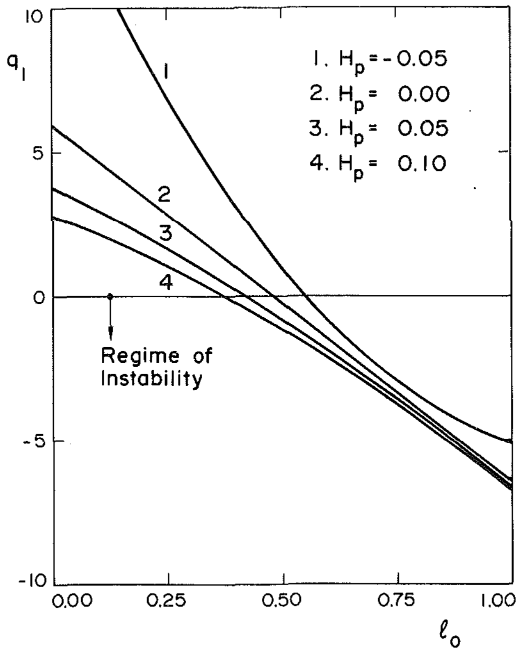

Fig. 2 Typical variation of the nondimensional coefficient of fluid-dynamic damping, by the analysis of Section 6; $H_0 = 0.1$

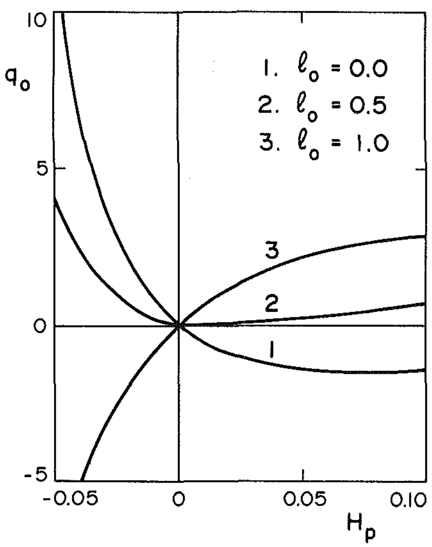

Fig. 3 Typical variation of the nondimensional coefficient of fluid-dynamic stiffness, by the analysis of Section 6; $H_0 = 0.1$

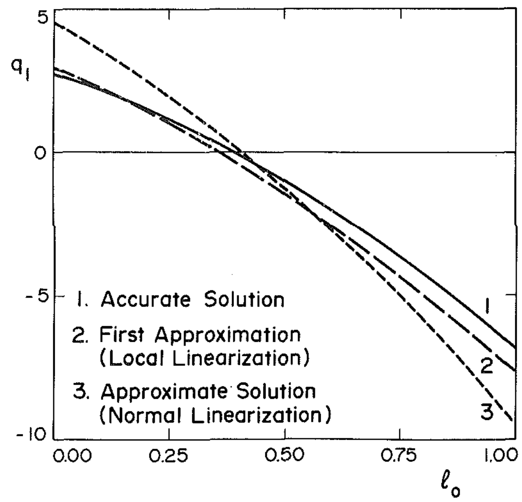

Fig. 4 Nondimensional coefficient of fluid-dynamic damping ($H_0 = 0.1$. $H_p = 0.1$); comparison between: 1 Accurate solution (by the analysis of Section 6); 2 First approximation solution based on local linearization (by the analysis of Section 5); 3 Approximate solution based on normal linearization (by an analysis similar to that of Section 5).

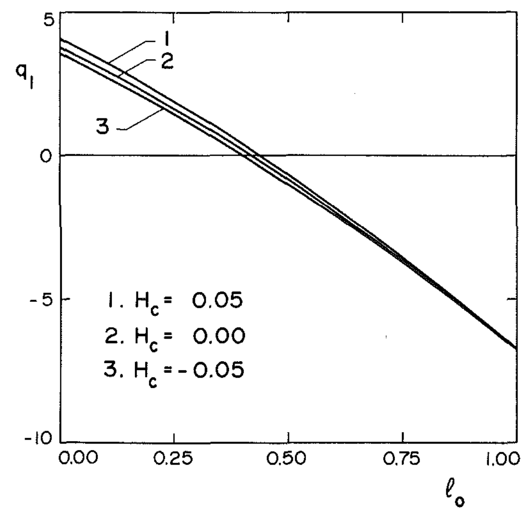

Fig. 5 Influence of the wall curvature parameter H_c on the nondimensional coefficient of aerodynamic damping ($H_0 = 0.1$; $H_p = 0.05$)

parameters. A very important role is played by the dimensionless parameter σ, defined by equation (24), and involving the fluid density, the mean radius of the annular passage and the moment of inertia of the oscillating center-body.

For the sake of simplicity – and without any loss of generality – the case of vanishingly small mechanical stiffness and damping coefficients ($k_1 \simeq k_0 \simeq 0$) will be considered first. In this case, amplified (unstable, in the linear sense) vibrations will occur, provided that the fluid-dynamic stiffness coefficient is positive and the corresponding damping coefficient negative (i.e., $q_0 > 0$, $q_1 < 0$).

The case of negative fluid-dynamic stiffness is of course also possible – which could lead to divergence, if $|\sigma q_0| > k_0^2$ – but will not be examined here in great detail, again for simplicity, and because it is considered that, for a real system, the condition $k_0^2 + \sigma q_0 > 0$ is likely to be satisfied, in any case.

Of course, the condition of an overall negative damping *must* be satisfied if amplified vibrations are to materialize, so that, generally, the condition $k_1 + \sigma q_1 < 0$ must be satisfied. It is noted that this condition is more restrictive than the one specified in the foregoing for $k_1 \simeq 0$. Nevertheless, if the damping characteristics of a system are known, so that k_1 may be determined, the results to be presented may be used as they stand, simply by linearly shifting the condition of amplified vibrations from $q_1 \leq 0$ to $q_1 \leq -k_1/\sigma$.

There is a great variety of possible geometrical systems which could be analyzed. However, to illustrate the effect of some of the system parameters and to show the type of results which can be obtained by means of the analytical models developed here, calculations are confined to one particular type of system: a cylindrical center-body, hinged at a variable dimensionless distance $l_0 = l_0^*/a$ (see Fig. 1), in a duct of variable cross section; the resulting clearance of the annular passage may be linear or parabolic, as defined by the relationships

$$H(x) = H_0 + H_p x, \; H_p = (H_1 - H_0)/l, \tag{44}$$

$$H(x) = H_0 + H_p x + H_c x^2, \tag{45}$$

respectively.

The variation of fluid-dynamic damping coefficient q_1, with l_0 is shown in Fig. 2, for four values of H_p, as defined by equation (44). It is seen that in all cases, if l_0 is sufficiently large, i.e., as the hinge-point is moved downstream, q_1 eventually becomes negative, indicating the possibility of amplified vibrations. It is of interest that for $H_p = 0$, i.e., for a cylindrical duct, if the hinge-point is downstream of the mid-length of the center-body, approximately, amplified vibrations are predicted to occur.

It is seen, furthermore, that a linearly divergent (diffuser-shaped) duct, corresponding to $H_p > 0$, destabilizes the system vis-à-vis a cylindrical duct – so that it is necessary to place the hinge-point further upstream to ensure stability. The opposite is true for a convergent duct shape, which has a stabilizing influence on the system.

The dependence of the fluid-dynamic stiffness coefficient, q_0, on H_p is illustrated in Fig. 3 for three values of l_0. It is seen that for $H_p > 0$, i.e., a diffuser-shaped duct, it is possible to have a slightly negative q_0, provided that the hinge-point is located at the foremost end of the body ($l_0 = 0$); however, *this does not correspond to an instability*, because for this set of parameters $q_1 > 0$. On the other hand, for a convergent duct and $l_0 = 1.0$, q_0 can be strongly negative in a region of $q_1 < 0$, indicating the possibility of divergence.

A comparison between the two variants of the analytical model, i.e., between the first-approximation (slender-body) solution of Section 5 and the more accurate soluton of Section 6, is made in Fig. 4. These two solutions were obtained by the method of local linearization, developed in Section 3; results, based on the normal (as opposed to local) linearization scheme, with respect to the undisturbed flow velocity U_0, are also shown. It is important to note that significant differences in the predicted q_1 arise, specifically if the inferior "normal linearization" scheme were to be employed (curve 3), instead of that based on local linearization; moreover, predictions of instability, based on the former, are nonconservative.

Considering parabolic variations of annular width, as given by equation (45), the results of Fig. 5 indicate that the effect on q_1 is not very strong. Nevertheless, in the case of a divergent annular passage, a concave shape, as viewed from the annulus-side of the system, tends to destabilize the system, as compared to a linear diffuser-shaped one; while the opposite is true for a convex shape.

At this point, it would be useful to present a concrete example, so as to illustrate the utilization of the results of Figs. 2–5 and to discuss stability in terms of the annular flow velocity, U_0. Consider, therefore, a cylindrical center-body of radius $a = 0.05$ m, in a divergent duct defined by $H_0 = 0.10$ and $H_p = 0.05$. Consider further that the center-body is supported at its midpoint ($l_0 = 0.5$), in such manner that its natural frequency is $\omega_n = 60$ rad/s and the mechanical damping factor $\zeta = C/(2\omega_n J_H) = 0.005$. Furthermore, assuming that the fluid is water, let σ, defined in (24), take on the reasonable value of $\sigma = 0.2$ (to which corresponds a certain J_H for given a and ρ).

Now, from Fig. 3 it is seen that the fluid-dynamic stiffness coefficient q_0 is positive (which precludes the possibility of divergence), and from Fig. 2 it is found that the fluid-dynamic damping coefficient is negative, $q_1 = -0.79$, indicating that the system could be susceptible to amplified oscillation. The instability threshold, defined by $k_1 + \sigma q_1 = 0$ in equation (25), may be rewritten in dimensional terms as $U_{0\,\mathrm{cr}} = 2 \times \zeta\omega_n a/(-q_1 \sigma)$. Utilizing the numerical values given previously, it is found that the system develops amplified oscillation for $U_0 \geq 0.19$ m/s, which is a very small flow velocity indeed; of course for $l_0 > 0.5$ the critical U_0 would be lower. On the other hand, if the center-body were located in a convergent duct ($H_0 = 0.10$, $H_p = -0.05$), then $q_1 > 0$ for $l_0 = 0.5$, indicating that the system would be unconditionally stable for all values of U_0 and ζ. In this case, increasing U_0 would generate additional fluid-dynamic damping which will increase the stability of the system.

9 Conclusions

An analytical theory for studying the unsteady potential flow in an axially variable narrow annular passage has been developed in the first part of this paper. Based on a new and more accurate method of local linearization of the problem, this theory has permitted to determine analytically, for the first time, a first-approximation solution, as well as a more accurate one.

To the best of the authors' knowledge, this theory for the unsteady annular flow is original and is presented for the first time in this paper.

The stability of flow-induced vibrations of the center-body has been studied with the aid of the fluid-dynamic forces obtained by means of the foregoing analyses. It was found that the most influential parameter on the stability of flow-induced vibrations is the position of the center-body hinge. Amplification of flow-induced vibrations occurs when the hinge-point is situated toward the downstream end of the center-body, behind a critical position at which the fluid-dynamic damping coefficient vanishes.

The critical position of the hinge axis is substantially influenced by axial variations of the annular clearance. A divergent annular passage will move upstream the critical position of the hinge, having a destabilizing effect, while a convergent annular passage will do the opposite, having a stabilizing effect.

The curvature of the duct wall and the ratio of annular clearance to mean radius of the center-body have smaller effects on the stability of flow-induced vibrations.

Currently, work is under way to extend the theoretical model in order to take into account the influence of viscosity (vide some preliminary results in reference [19]) and to test the theoretical results experimentally.

Acknowledgments

The authors gratefully acknowledge the support given to this research by the Natural Sciences and Engineering Research Council of Canada.

References

1 Paidoussis, M. P., "Flow-Induced Vibrations in Nuclear Reactors and Heat Exchangers: Practical Experiences and State of Knowledge," in *Practical Experiences with Flow Induced Vibrations*, eds., Naudascher, E., and Rockwell, D., Springer-Verlag, Berlin, 1980, pp. 1–81.

2 Paidoussis, M. P., and Issid, N. T., "Dynamic Stability of Pipes Conveying Fluid," *Journal of Sound and Vibration*, Vol. 33, 1974, pp. 267–294.

3 Paidoussis, M. P., Chan, S. P., and Misra, A. K., "Dynamics and Stability of Coaxial Cylindrical Shells Containing Flowing Fluid," *Journal of Sound and Vibration*, Vol. 97, 1984, pp. 201–235.

4 Paidoussis, M. P., "Dynamics of Flexible Slender Cylinders in Axial Flow. Part 1: Theory; Part 2: Experiments," *Journal of Fluid Mechanics*, Vol. 26, 1966, pp. 717-736, 737-751.

5 Paidoussis, M. P., "Dynamics of Cylindrical Structures Subjected to Axial Flow," *Journal of Sound and Vibration*, Vol. 29, 1973, pp. 365–385.

6 Dowell, E. H., and Widnall, S. E., "Generalized Aerodynamic Forces on an Oscillating Cylindrical Shell: Subsonic and Supersonic Flow," *AIAA Journal*, Vol. 4, 1966, pp. 607–610.

7 Paidoussis, M. P., "Fluidelastic Vibration of Cylinder Arrays in Axial and Crossflow: State of the Art," *Journal of Sound and Vibration*, Vol. 76, 1981, pp. 329–360.

8 Paidoussis, M. P., and Pettigrew, M. J., "Dynamics of Flexible Cylinders in Axisymmetrically Confined Flow," ASME *Journal of Applied Mechanics*, Vol. 46, 1979, pp. 37–44.

9 Paidoussis, M. P., and Ostoja-Starzewski, M., "Dynamics of a Flexible Cylinder in Subsonic Axial Flow," *AIAA Journal*, Vol. 19, 1981, pp. 1467–1475.

10 Paidoussis, M. P., Misra, A. K., and Chan, S. P., "Dynamics and Stability of Coaxial Cylindrical Shells Conveying Viscous Fluid," Symposium on

Flow-Induced Vibrations; Vol. 4: Vibration Induced by Axial and Annular Flows, eds., Paidoussis, M. P., and Au-Yang, M. K., pp. 71–86; Dec. 1984, ASME, New York, ASME *Journal of Applied Mechanics*, Vol. 52, 1985, pp. 389–396.

11 Parkin, M. W., "A Review of Basic Studies and Modelling Applied to Annular Flow," *Proceedings UKAEA/NPL International Symposium on Vibration Problems in Industry*, Keswick, U.K., 1973, Paper No. 628.

12 Roberts, G. P., Hallam, M. G., Dean, R. B., and Parkin, M. W., "A Theoretical Study of Hartlepool AGR Stringer Vibration during Refueling at Power," *Proceedings 3rd International Conference in Vibration in Nuclear Plant*, Keswick, U.K., pp. 624–648.

13 Parkin, M. W., France, E. R., and Boley, W. E., "Flow Instability Due to a Diameter Reduction of Limited Length in a Long Annular Passage," ASME *Journal of Vibraiton, Acoustics, Stress, and Reliability in Design*, Vol. 105, 1983, pp. 355–360.

14 Hobson, D. E., "Fluid-Elastic Instabilities Caused by Flow in an Annulus," *Proceedings 3rd Keswick International Conference on Vibration in Nuclear Plant*, Keswick U.K., pp. 440–463.

15 Hobson, D. E., et al., A Series of Confidential C.E.G.B. Memoranda, Marchwood Engineering Laboratories, U.K., 1980–1983, made available to the authors.

16 Miller, D. R., and Kennison, R. G., "Theoretical Analysis of Flow-Induced Vibration of a Blade Suspended in Channel Flow," ASME Paper No. 66-WA/NE-1, 1966.

17 Miller, D. R., "Generation of Positive and Negative Damping With Flow Restrictor in Axial Flow," *Proceedings of the Conference on Flow-Induced Vibrations in Reactor System Components*, Argonne National Laboratory Report ANL-7685, 1970, pp. 304–307.

18 Mulcahy, T. M., "A Review of Leakage-Flow Induced Vibrations of Reactor Components," Argonne National Laboratory Report ANL-83-43, 1983.

19 Mateescu, D., and Paidoussis, M. P., "Annular-Flow-Induced Vibration of an Axially Variable Body of Revolution in a Duct of Variable Cross Section," Symposium on Flow-Induced Vibrations; Vol. 4: Vibration Induced by Axial and Annular Flows, eds., Paidoussis, M. P., and Au-Yang, M. K., Dec. 1984, ASME, New York, p. 53–69.