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Two-Phase Pulsed Flow of a Non Newtonian Fluid in Pipe with Elastic Wall

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Abstract

The object of this study is to show certain specific effects caused by rheological and elastic parameters in a pulsating flow of non Newtonian two-fluid models through elastic tube. A numerical finite difference method using Crank Nicholson scheme is employed to determine pressure and velocity profiles. This study is considered as a step in two-phase flow of blood in small vessels.

Keywords: two-phase pulsed flow, non Newtonian fluid, pipe with elastic wall

1. Introduction

Fluid flows in pipes of small diameter, modeling the flow of blood, in the arteries were studied so far with assumptions of various types:

Some authors use two-phase models in rigid pipes, with a constant viscosity for the core [1], others consider also two-phase models in rigid pipes but assume a variable viscosity for the core [2, 3]. Finally some authors examine blood flow in the microcirculation as a pulsed two-phase flow of non-Newtonian fluid [5].

It is proposed in this work, to study a pulsed non-Newtonian fluid flow in a two-phase medium in an axisymmetric elastic pipe with diameter equal to 100 microns.

The rheological behavior of the wall is described by a relationship between radius and internal pressure. The expression of this relationship varies depending on the model used to describe the behavior of the vessel wall [6].

We have considered here the wall is a homogeneous isotropic and elastic medium. This study aims at modeling flow of blood in the arterioles or venues.

2. Theoretical part

The flow in question comprises a central core of radius a(z,t) rich in globules, surrounded by a Newtonian plasma layer, dynamic viscosity contrast denoted respectively μC and μp . We suppose that μp is constant and that μC follows the rheological Ostwald law:

$$\mu_c = kS^{\frac{n-1}{2}} \tag{1}$$

With: $S=2\ .tr\ (D^2)$

n: is behavior index
K: the consistency of the fluid
D: the deformation rate tensor

The conduct concerned is an axisymmetric tube with radius R (z,t), length L and axis $\vec{z}$ ($\vec{z}$ is the longitudinal axis).

The study is performed with respect to a cylindrical coordinate system (r, θ, z). The $\vec{z}$ axis coincides with the axis of the pipe.
The velocity field $\vec{V}$ is defined by its components:

$$V_r = u(r, z, t)$$
$$V_\theta = 0$$
$$V_Z = w(r, z, t)$$

We assume that the core- layer interface plasma has the same profile as the wall of the pipe [7]. Its radius is given by:

$$a(z, t) = \gamma R(z, t) \quad (2)$$

Or γ is a constant that takes 0, 9. In fact the plasma layer has a width equal to the eighth or tenth of the diameter of the vessel (8).

The rheological behavior of the tube is introduced in the form of a pressure-radius relationship.

The establishment of such relationship is very complex. It has been the subject of many studies. A review of the main work related was published by Taylor and Gerrard [6]. It shows the diversity proposed in the literature based on the assumptions that lead to these relations.

This relationship can be simple if we suppose a thin isotropic and elastic wall. It can become complex and involve many parameters which are difficult to determine accurately, if considering a viscoelastic wall, and large deformations. We shall content ourselves use a pressure-radius relation, deduced from the theory of linear elasticity in small deformation for an isotropic, homogeneous, incompressible wall, and whose longitudinal displacement is negligible compared to the radial displacement, this relationship is:

$$P - P_{ext} = \frac{2Eh}{3R_0}\left[1 - \frac{{R_0}^2}{R^2}\right] \quad (3)$$

with:
P: is the internal pressure
P_{ext} : the external pressure
β: Coefficient of elasticity
E: Young's modulus
h: the thickness of the wall
R: the radius of the pipe pressure
R_0: the radius of the pipe break

This relation is frequently used in the field of cardiovascular biomechanics [9, 11].

The experimental determination of the coefficient β of the wall for arterioles or venues is very delicate (small diameter vessels react strongly at the slightest mechanical stimulation).

Considerably different results depending on the other for the in vitro carotid Hayashi [11] gives the value E_L=1 ,48.10^7 dynes /cm^2 whereas Hudetz [12] obtained $E_{L=}$ 3 ,1.10^7 dynes /cm^2.

These are mainly due deference to the measurement methods as well as experimental conditions.

The equations resulting from the conversation of momentum and continuity of writing fluid in the absence of gravitational forces volume are:

$$\rho\frac{d\vec{V}}{dt} = \overrightarrow{d\imath V}\Sigma \tag{4}$$

$$\overrightarrow{d\imath V} = 0 \tag{5}$$

With: $\Sigma = -\mathrm{PI} + 2\eta\mathrm{D}$

Or: Σ is the stress tensor, I the identity tensor and ρ the density as assumed the same for each phase due to the effects of sedimentation.

Projected on the radial axes $\vec{r}$ and axial $\vec{z}$, equations (4) and (5) are written for the two phases:

$$-\frac{\partial P}{\partial r} + \eta\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial z\partial r}\right) + \frac{\partial \eta}{\partial z}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) + \frac{2\eta}{r}\frac{\partial u}{\partial r} + 2\frac{\partial}{\partial r}\left(\eta\frac{\partial u}{\partial r}\right) = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right)$$

$$-\frac{\partial P}{\partial z} + \eta\left(\frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 u}{\partial r\partial z}\right) + \frac{\partial \eta}{\partial r}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) + \frac{\eta}{r}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) + 2\frac{\partial}{\partial z}\left(\eta\frac{\partial w}{\partial z}\right) = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \tag{6}$$

In which $\eta = \eta_P\ constant$ in the plasma layer.

And $\eta = \mu_c = kS^{\frac{n-1}{2}}$ in the core

With:

$$S = 2\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{u}{r}\right)^2\right] + \left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right]^2$$

2 .1 Dimensionless variables and simplifying equations

To highlight the simplifications given various orders of magnitudes involved the problem is transcribed into dimensionless form and we set:

$u^* = \frac{uL}{R_0 w_0}$; $w^* = \frac{w}{w_0}$; $r^* = \frac{r}{R_0}$; $z^* = \frac{z}{L}$; $P^* = \frac{P{R_0}^2}{\eta_0 w_0 L}$;

$t^* = t\omega$; $\Re_e = \frac{\rho w_0 R_0}{\eta_p}$; $\alpha^2 = \frac{\rho\omega {R_0}^2}{\eta_p}$; $\varepsilon = \frac{R_0}{L} = \frac{u_0}{w_0} \ll 1$

Or:

$\Re_e$: is Reynolds number, which characterizes the inertia effects comparing to viscosity effects

α^2 Womersley parameter which is proportional to the ratio of diffusion time in the radial direction of flow $\frac{\rho {R_0}^2}{\eta_p}$, the characteristic period of pulsation $\frac{2\pi}{w}$, consequently it characterizes the relative importance of unsteadiness and the effects of viscosity effects, and ω pulsation of the phenomenon.

The system (6) is written in dimensionless form:

$$-\frac{\partial P^*}{\partial r^*} + \varepsilon^2\left[\frac{2}{r^*}\frac{\partial}{\partial r^*}\left(r^*\eta^*\frac{\partial u^*}{\partial r^*}\right) + \frac{\partial}{\partial z^*}\left(\eta^*\frac{\partial w^*}{\partial r^*}\right)\right] + \varepsilon^4\frac{\partial}{\partial z^*}\left(\eta^*\frac{\partial u^*}{\partial z^*}\right)$$
$$= \varepsilon^2\alpha^2\frac{\partial u^*}{\partial t^*} + \varepsilon^3\Re_e\left[u^*\frac{\partial u^*}{\partial r^*} + w^*\frac{\partial u^*}{\partial z^*}\right]$$
$$-\frac{\partial P^*}{\partial z^*} + \varepsilon^2\left[\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\eta^*\frac{\partial u^*}{\partial z^*}\right) + 2\frac{\partial}{\partial z^*}\left(\eta^*\frac{\partial w^*}{\partial z^*}\right)\right] + \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\eta^*\frac{\partial w^*}{\partial r^*}\right)$$

$$= \alpha^2\frac{\partial w^*}{\partial t^*} + \varepsilon \ \Re_e\left[u^*\frac{\partial w^*}{\partial r^*} + w^*\frac{\partial w^*}{\partial z^*}\right] \qquad (7)$$

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}(r^*u^*) + \frac{\partial w^*}{\partial z^*} = 0$$

In which: $\eta^* = 1$ in the plasma layer and $\eta^* = k^*(S^*)^{\frac{n-1}{2}}$ in the core.

Where:

$$(S^*)^{\frac{n-1}{2}} = \left\{\varepsilon^4\left(\frac{\partial u^*}{\partial z^*}\right)^2 + 2\varepsilon^2\left[\left(\frac{\partial u^*}{\partial r^*}\right)^2 + \left(\frac{\partial w^*}{\partial z^*}\right)^2 + \left(\frac{u^*}{r^*}\right)^2 + \frac{\partial u^*}{\partial z^*}\frac{\partial w^*}{\partial r^*}\right] + \left(\frac{\partial w^*}{\partial r^*}\right)^2\right\}^{\frac{n-1}{2}}$$

And $k^* = \frac{k}{\eta_p}\left(\frac{w_0}{R_0}\right)^{n-1}$

By neglecting terms on ε^2 or more, the equations of motion and continuity within the nucleus and plasma become:

$$\begin{cases} \frac{\partial P^*}{\partial r^*} = 0 \\ -\frac{\partial P^*}{\partial z^*} + \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\eta^*\frac{\partial w^*}{\partial r^*}\right) = \alpha^2\frac{\partial w^*}{\partial t^*} \\ \frac{1}{r^*}\frac{\partial}{\partial r^*}(r^*u^*) + \frac{\partial w^*}{\partial z^*} = 0 \end{cases} \quad (8)$$

In which $\eta^* = 1$ in the plasma layer
And
$\eta^* = \frac{\eta_C}{\eta_P} = k^*\left|\frac{\partial w^*}{\partial r^*}\right|^{n-1}$ in the core.

As in microcirculation the radii of the ducts are small, and convective acceleration terms are negligible.

After simplification, the equations are rewritten in dimensional forms:

$$\begin{cases} \frac{\partial P}{\partial r} = 0 \\ -\frac{\partial P}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \quad \eta \quad \frac{\partial w}{\partial r}\right) = \rho\frac{\partial w}{\partial t} \\ \frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \end{cases} \quad (9)$$

In which $\eta = \eta_p$ in the plasma layer
$\eta = \eta_c = k \left|\frac{\partial w}{\partial r}\right|^{n-1}$ in the core

The first equation of the system (9) indicates that the pressure is uniform in each section of conduit.

In the following expressions, index c will be allocated to amounts for the central region and the index p to those related to plasma.

The conditions associated with this problem limits are:
$U_c(0, z, t) = 0$ Symmetrical flow about the axis $\vec{z}$

$$\frac{\partial w_c}{\partial r}(0, z, t) = 0$$

$U_c(a(z,t), z, t) = U_p(a(z,t), z, t)$ Equal speeds at the interface core-plasma
$W_c(a(z,t), z, t) = W_p(a(z,t), z, t)$
$\eta_c\frac{\partial w_c}{\partial r}(a(z,t), z, t) = \eta_p\frac{\partial w_p}{\partial r}(a(z,t), z, t)$
Of interracial equality constraints

$$U_p(R(z,t), z, t) = \frac{\partial R}{\partial t}$$

$W_p(R(z,t), z, t) = 0$ Adhesion to the wall
The data of the problem are the pressures at the inlet and outlet of the tube:

$$P(0, t) = P_1 - P_2 cos\omega t$$
$$P(L, t) = P_3 - P_4 cos\omega t$$

In microcirculation, in the short length pipe allows to neglect the phase difference between the pressures at the ends

2.2 Solving the problem

The application of the method « Integral Equation » system (9) provides a partial differential equation of second order pressure P(z,t), parabolic type which is given by:

$$-\frac{R}{2}\frac{\partial^2 P}{\partial z^2}-\frac{\partial R}{\partial z}\frac{\partial P}{\partial z}+\eta_p\left[\frac{1}{R}\frac{\partial R}{\partial z}\frac{\partial w_p}{\partial r}(R)\right]+\frac{\partial}{\partial z}\left(\frac{\partial w_p}{\partial r}(R)\right)+\frac{\rho}{R}\frac{\partial}{\partial t}\left(R\frac{\partial R}{\partial r}\right)=0$$

This equation is solved using a numerical finite difference method using an implicit scheme type Crank-Nicholson. We obtain a linear system of equation, defined by a matrix tridiagonal which is solved using the method of dual scanning Choleski [15].

The field of study is two-dimensional (z,t). If j and k are respectively the index of space in the axial direction of the tube and the time index, convergence tests and frequency are applied to the pressure.

At each point of the mesh, the pressure P (j,k) must check:

$sup\left|\frac{P_{j,k}^m+P_{j,k}^{m+1}}{P_{j,k}^m}\right|<\varepsilon_1$ whatever the point considered

The calculation must be repeated several times to establish the periodicity of the solution. This periodicity is tested by imposing:

$$sup\left|\frac{P_{j,k}^m+P_{j,k+T}^m}{P_{j,k+T}^m}\right|<\varepsilon_2$$

$\varepsilon_1 and \varepsilon_2$ being amounts that are fixed in advance and which must be chosen small and the number m of cycle calculations.

2.3- Initial profile and program data

The initial profile can be of any problem provided it satisfies the boundary conditions. However, to reduce the calculation cycle, we chose a profile that is close enough to the real profile. We adopted as the initial velocity profile on a two-phase permanent flow of Newtonian fluid in a cylindrical pipe of radius R_0 and with the core radius $a_0=\gamma R_0$

Program data taken from the literature, [5], are data on hydrodynamic conditions and parameters characterizing the rheological behavior of the fluid and the pipe:

- The density of the fluid concerned, ρ, is equal to 1.06g/cm^3=1060Kg/m^3
- Plasma viscosity,η_p, assumed constant, is equal to 1.2cp.
- The conduct in question is of length L=0.1cm radius at rest $R_0=50\mu m$ and the wall thickness is h=10^{-3}m .
- The value of the geometric parameterε, is therefore equal to 0.05.
- The average speed is $W_0=1cm/s$ and the Reynolds number is then 0.44.

- The number of Womersley α^2 Its value is equal to 0.014 and a frequency of 1 Hz.
-The external pressure, considered constant, is equal to 1330dynes/cm².
- The pressure at the inlet of the pipe is the result of a continuous component $P_1 = 35000\ dynes/cm^2$ and a sinusoidal component$P_2 = 350\ dynes/cm^2$.
-The pressure at the outlet of the conduit has the continuous component $P_1 = 34500\ dynes/cm^2$ and amplitude of the sinusoidal component$P_4 = 345\ dynes/cm^2$.

3. Numerical results

a- influence of the behavior index n

Figure 1 shows the variations of the axial velocity profile in function of the behavior index. Increased pseudoplasticity resulting in an increase in n leads to increased amplitudes speeds. This increase in velocity is due to a decrease in the apparent viscosity decreases when K and geometric conditions keep the same values. We can translate this result a reduction in resistance to flow when the pseudo plasticity increases.

The same phenomenon is observed for the radial velocities, recent increase when n decreases figure 2.

These results are qualitatively similar to those of Theodorou [16], Gueraoui and al [19] who has studied the unsteady flow of non-Newtonian fluids in a sectional narrowing and those of Zeggwagh [2] which studied the unsteady flow of non-Newtonian fluids in a rigid tapered pipe.

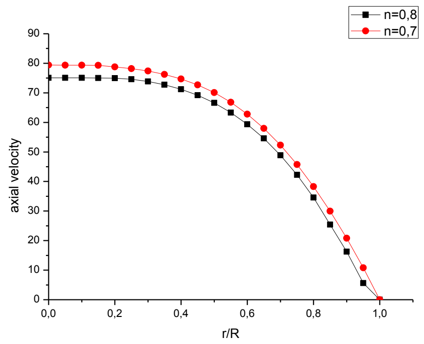

Figure 1: axial vel**ocity** profile as function of dimensionless radial variable for two different index n

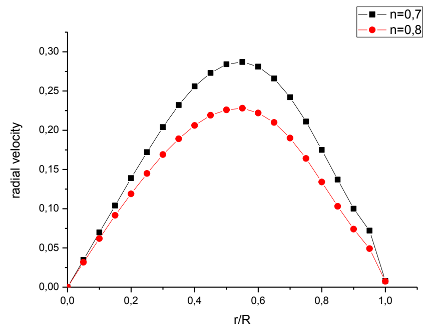

Figure 2: radial velocity profile as function of dimensionless radial variable for two different index n

b- Influence of consistency K

Figures 3 and 4 illustrate the influence of consistent, K, on the flow. The influence of this second rheological parameter is similar to that of the viscosity in the Newtonian case. There is a decrease in axial values and a more or less pronounced flattening of profiles. This is due to the increase in the internal friction between the various layers of the coaxial fluid when K increases.

The variation of these two rheological parameters, n and K has no influence on the values of the axial velocity in the plasma layer, as can be expected.

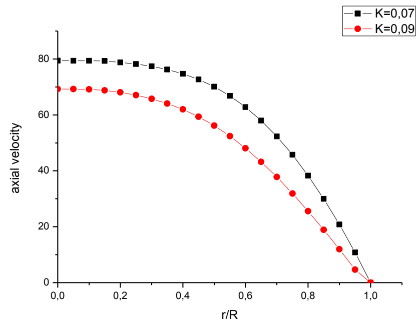

Figure 3: axial velocity profile as function of dimensionless radial variable for two different consistency k

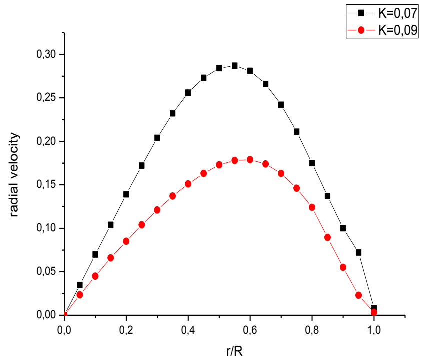

Figure 4: radial velocity profile as function of dimensionless radial variable for two different consistency k

c- Influence of phase

Figure 5 shows the profiles of axial velocities at the instants t=0, T/4, T/2. There is gradual decrease of the axial velocity at any point and can provide a minimum at T/2 then primer a progressively increased to t=T.

We find, as some authors have already done [2], [17], that the velocity profiles are in phase with the pressure gradient, which indicates that the viscous forces are dominant compared to inertial forces.

The low variation of velocity profiles is due to the fact that the amplitudes of the oscillatory components P_2 *and* P_4 low before components continuousP_1 *and* P_3.

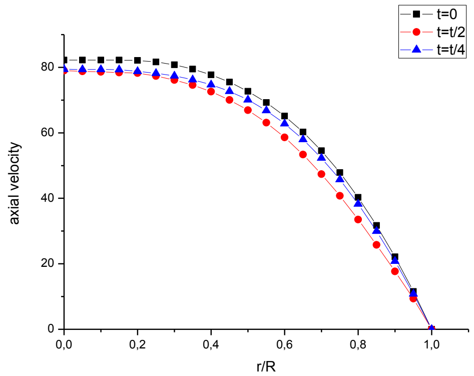

Figure 5: axial velocity profile as function of dimensionless radial variable for three different times

d- Influence of the coefficient of elasticity

In Figure 6, was brought changes the pressure distribution a function of time in a given section (z=L/2) for two values of elasticity coefficient β.

It is observed, the influence of the latter on the pressure distribution is negligible. This is due to the low value of the oscillatory component of the pressure.

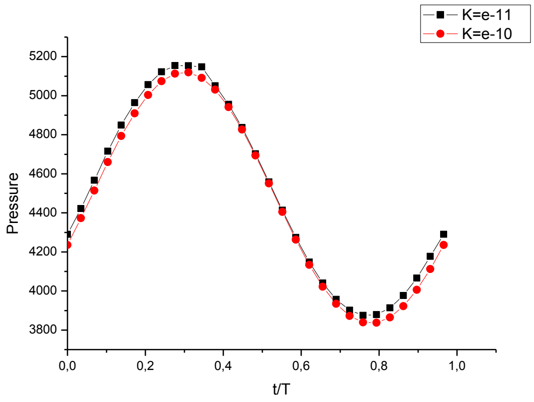

Figure 6: pressure profile as function of dimensionless time variable for two different coefficient of elasticity

4. Conclusion

The objective we have set for ourselves consisted in proposing a model of two-phase flow of non-Newtonian fluid in conduct with elastic wall. This study focuses on applications that can be made in hemodynamic more particularly in the microcirculatory system.

Theoretical and numerical studies using finite difference method implicit scheme were used to determine the velocity field, the radius profile of the pipe, and the pressure distribution in a pipe with elastic wall.

We studied the influence of rheological parameters of the fluid (K consistency and behavior index n), from hydrodynamic, the flow parameters and the coefficient of elasticity of the wall β. One can able to discern the importance of these parameters on the flow in a part of the microcirculatory system.

But, we must note that many more research is needed to deal satisfactorily a subject of such importance. Indeed, we limited ourselves to the consideration of an elastic and impermeable pipe wall.

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