A stochastic hybrid algorithm for multi-depot and multi-product routing problem with heterogeneous vehicles

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Abstract. A mathematical model and heuristic method for solving multi-depot and multi-product vehicle routing problem (MD-MPVRP) with heterogeneous vehicles have been proposed in this article. Customers can order eclectic products and depots are supposed to deliver customers' orders before the lead time, using vehicles with diverse capacities, costs and velocities. Hence, mathematical model of multi-depot vehicle routing problem has been developed to mirror these conditions. This model is aimed at minimizing the serving distances which culminates in a reduction in prices and also serving time. As the problem is so complex and also solving would be too time-taking, a heuristic method has been offered. The heuristic method, at first, generates an initial solution through a three-step procedure which encompasses grouping, routing and vehicle selection, scheduling and packaging. Then it improves the solution by means of simulated annealing. We have considered the efficiency of offered algorithm by comparing its solutions with the optimum solutions and also during a case study.
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1. Introduction

Vehicle routing problem (VRP) is a general name for optimization problems in which some vehicles are serving the customers. The vehicles leave a depot, serve network customers and then return to same depot. In other words solving the VRP is to design a collection of routes for a fleet of vehicles, in which the depot is the start line and also the end point. Each customer must be met among one of these routes. The purpose for VRP is to minimize the entire prices of services.

VRP has been developed in so many different ways such as: periodic vehicle routing problem (PVRP) in which customers are served in a time more than one day. Pickup and delivery vehicle routing problem (VRPPD) in which customers have both possibilities of receiving and sending products. Time windows vehicle routing problem (VRPTW) in which vehicles must arrive to customers before the latest permitted entrance time and also arriving before the latest permitted entrance time will confront some penalties. A mutual point in all freshly mentioned problems is that they are based on only one depot. So they can be categorized in single depot vehicle routing Problems. Single depot VRPs have been mentioned a lot but in comparison, multi-depot vehicle routing problems (MDVRP) are less studied. Since for reserving or distributing products in supply chain or big cities, usually more than a depot is used so the MDVRP has many uses in real world. Each multi-depot can be denominated in three parts: the first part shows that which customers are being served by a specific depot. This is also called customers grouping. The second part is the routing; determining the routes. In fact, in this part all appropriate routes will be defined. And the last part is also determining the priority of serving which puts the customers in order. This kind of VRP is evidently more complicated than the single-depot ones. Multi-depot vehicle routing problem is NP-hard Which means that an efficient algorithm for solving the problem to optimality is unavailable.

During the last years different kinds of single-depot vehicle routing problems have been mentioned in so many essays but researches on MDVRP are very scant compared to that of classical VRP. Chao et al have drawn an improving heuristic method for MDVRP which improves upon best known solutions. The heuristic method of tabu searches for MDVRP was proposed by Renaud et al. (1996) Through some examples, they compared their suggested method among the others. Cordeau et al (1997) also presented general heuristic method which solves periodic vehicle routing problems and also travelling salesman problems. Su C.T. (1999) represented a dynamic vehicle controlling and scheduling system for solving MDVRP. All control decisions are spotted based on the time and states of the real system such as locating, amount of demands and the lead time. Salhi, S., & Nagy, G. (1999) dealt with a cluster insertion heuristic for single and multiple depot vehicle routing problems with backhauling. Thangiah, S. R., & Salhi, S. (2001) explained the Genetic clustering: An adaptive heuristic for the multi-depot vehicle routing problem. Wu et al (2002) studied the multi-depot location-routing problems (MDLRP) which is a development for MDVRP. MDLRP decomposed to locating problems and vehicle routing problems and then they were solved sequentially by simulated annealing. Giosa et al (2002) surveyed MDVRP with time window which was a kind of development for MDVRP. The authors designed six heuristic methods for devoting the customers to depots and also comparing them. Wasner M. and Zapfel G. (2004) studied on MDVRP for network planning of parcel service and described an integrated model for it. Nagy G. and Salhi S. (2005) represented some heuristic solving methods for single-depot problems. These methods can be developed to approach multi-depot pickups and deliveries. Haghani A. and Jung S. (2005) represented a genetic algorithm to solve dynamic VRP with time-dependent travel times. Pisinger D. and Ropke S.(2007) introduced a heuristic method which could solve various vehicle routing problems. Crevier et al (2007) made an expanded MDVRP model in which vehicles had possibility to reload at inter-depots on their way. Bae et al (2007) used a revised genetic algorithm for solving MDVRP. They used three heuristic methods for finding their algorithm's initial solution in order to optimize their provision's costs also with a regard to vehicles capacity and time deadlines. Ho et al (2008) developed two synthetic genetic algorithms for MDVRP. The main diversity between these two is about initial solutions. In multi-product vehicle routing problem fields Fallahi, prins and calvo (2008) worked on the conventional VRP in a case that customers have possibility to order different kinds of product which is more similar to reality. In this model customers' demands for a specific kind of product is responded by one vehicle. Consequently different kinds of product would be received by different vehicles. They have used synthetic genetic algorithm and tabu searching method. Kuo et al (2009) developed an optimized solving method for specifying products and vehicle routing problem. In that study they regarded the time of travel depended on vehicle velocity then showed the efficiency of represented method among the existing ones in a case study. Chen et al (2009) suggested a nonlinear mathematical model which supposed production scheduling and vehicle routing with time windows for perishable food products. The model purposefully used to maximize the expected profits. In this model retailers would probably demand and also food products would lose their quality by the time elapsing. Producer's profit is not absolute and it depends on cost and amount of trading products. Optimum production, production start line and vehicle routes could be determined coincidentally in this model. The investigators depicted a solving algorithm for this complex problem and drew its efficiency through some examples. Mirabi and Fatemi Ghomi (2010) developed an efficient stochastic hybrid heuristics for the multi-depot vehicle routing problem. Uncertain demand in conjunction with undetermined distribution in VRP recently received attention by Farhang Moghaddam and Seyedhosseini (2010) They used hybrid algorithm in order to solve these kind of problem. A.M. Benjamin and J.E. (2010) Beasley also represented metaheuristics for waste collection vehicle routing problem with time windows, driver rest period and multiple disposal facilities. Using the algorithm of branch-and-cut-and price algorithm, bettinneli, ceselli and Righini (2010) represented an accurate method for solving MDVRP with heterogeneous vehicles and time windows. They considered the method by numerical tests. Wu, Ho and Szeto (2011) actually have developed an artificial bee colony algorithm for the capacitated vehicle routing problem. This heuristic method is swarm-based heuristic which imitates the foraging behavior of honey bee swarm. They evaluated the proposed heuristic on two sets of standard benchmark instances and also compared with original artificial bee colony heuristic. Baladacci, Mingozzi and Roberti (2011) represented recent exact algorithms for solving the vehicle routing problem under capacity and time window constraints. C.K.Y Lin (2011) studied a vehicle routing problem with pickup and delivery time windows, and coordination of two types of transportable resources. A heavy resource (in this case a van) may carry both Delivery items and one or more units of the lighter resource (foot couriers) on its single-or multi-route assignment. Foot couriers can pick up and deliver items independently or travel with a van on its Out bound and/or return leg. Coordination between resources can save time and cut costs. Gulczynsky, Golden and Wasil (2011) addressed an integer programming-based heuristic, new test problems, and computational results for the multi-depot split delivery vehicle routing problem. They applied their heuristic to 30 instances to determine the reduction in distance traveled that can be achieved by allowing split deliveries among vehicles based at the same depot and vehicles based at different depots. And they also generated new test instances with high-quality, visually estimated solutions and report results on these instances.

According to significance of routing problem, in this article a mathematical model has been developed for MDRVP actually in a case that more than one product is supposed to be distributed and also the capacities of vehicles are variable. Then a heuristic method has been introduced to solve the model and finally initial solutions have been improved through simulated annealing.

This paper is organized as follows:

Section 2 discusses the principles of the mathematical model for MDVRP with heterogeneous vehicles. Section 3 describes the heuristic method and way of encoding the answers. Section4 improves the solutions based on simulated annealing. Section 5 illustrates the efficiency and effectiveness of suggested method during some numerical instances and also a case study. And finally the conclusion will be represented in the section 6.

2. Formulating the MD-MPVRP with heterogeneous vehicles

In this section, mathematical model of multi-depot and multi-product vehicle routing problem with heterogeneous vehicle, which is tended to minimize the distributions prices, has been carried out. The solutions are delivery routes from depot to customers, which determine the serving priority of customers for each vehicle. Customer demands and also the needed time for passing the routes are definite. Vehicles are heterogeneous which means they are different in capacity. And each vehicle starts serving from a specific depot and returns to the same one at the end. For each customer, all demands are supplied by a specific vehicle.

2.1. Defining the parameters and variables

$i = 1, 2, \dots, m$: The set of m depots

$j = m+1, m+2, \dots, m+n$: The set of n customers (n= number of the whole customers)

$k = 1, 2, \dots, K$: The set of k existing vehicles (each vehicle has particular capacity)

$r = 1, 2, \dots, R$?: The set of r routes

$a = 1, 2, \dots, A$?: The set of distributable products

Q_k : The capacity of the vehicle k

P_a : The volume of product in the packaged shape

d_{ja} : The demand of customer j for product a

V_{ia} : The capacity of depot i for supplying the product a

t_i^k : Products' collecting and provision time for vehicle k in depot i

t_{ij} : Traveling time between node i and node j

T_j : Offloading time for customer j

F_j ?: The lead time for customer j

C^k_{ij} ?: The price for serving customer j by depot i and vehicle k

U_{ir}^k : Starting time for vehicle k which starts moving from depot i and goes through route r

W_j^k : The time when vehicle k arrives to customer j.

X_{ijr}^k : 1, if node i immediately precedes node j on route r; otherwise 0.

Z_{ij} : 1, if customer j is assigned to depot i; otherwise 0.

燃$_{ir}$: Auxiliary variable for sub-tour elimination constrains in route r.

2.2. The mathematical model

$$\text{Min } Z = \sum_{i=1}^{n+m}\sum_{j=1}^{n+m}\sum_{k=1}^{K}\sum_{r=1}^{R} C_{ij}^k . X_{ijr}^k \qquad (1)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{r=1}^{R} X_{ijr}^k = 1 \qquad (2)$$
$$j = m+1, m+2, \dots, m+n$$

$$\left[\sum_{j=m+1}^{m+n}\sum_{a=1}^{A} d_{ja}.P_a\right]\left[\sum_{i=1}^{m+n} X_{ijr}^k\right] \le Q_k \qquad (3)$$
$$k = 1,2,\dots,K \quad r = 1,2,\dots,R$$

$$Y_{ir} - Y_{jr} + N.X_{ijr}^k \le N-1$$
$$k = 1,2,\dots,K \quad r = 1,2,\dots,R \qquad (4)$$
$$i = 1,2,\dots,m \quad j = m+1, m+2,\dots,m+n$$

$$\sum_{i=1}^{n+m} X_{ijr}^k - \sum_{j=1}^{n+m} X_{ijr}^k = 0 \qquad (5)$$
$$k = 1,2,\dots,K \quad r = 1,2,\dots,R$$

$$\sum_{j=m}^{n+m} X_{0jr}^k = 1 \qquad (6)$$
$$k = 1,2,\dots,K \quad r = 1,2,\dots,R$$

$$\sum_{i=1}^{m} X_{i0r}^{k} = 1 \quad k = 1,2,...,K \quad r = 1,2,...,R \tag{7}$$

$$\sum_{i=1}^{n+m}\sum_{j=1}^{n+m} X_{ijr}^{k} \le 1 \quad k = 1,2,...,K \quad r = 1,2,...,R \tag{8}$$

$$\sum_{i=1}^{n+m}\sum_{a=1}^{A} d_{ja}.Z_{ij} \le V_{ia} \quad j = m+1, m+2,...,m+n \quad r = 1,2,...,R \tag{9}$$

$$\sum_{g=1}^{m+n}\left(X_{igr}^{k} + X_{gjr}^{k}\right) - Z_{ij} \le 1 \quad i = 1,2,...,m \quad j = m+1, m+2,...,m+n \quad k = 1,2,...,K \quad r = 1,2,...,R \tag{10}$$

$$X_{jir}^{k}\left(W_j^k + t_{ij} + T_j - W_i^k\right) \le 0 \quad k = 1,2,...,K \quad r = 1,2,...,R \tag{11}$$

$$W_j^k + t_{ij} + T_j - W_i^k \le \left(1 - X_{jir}^{k}\right).M \quad k = 1,2,...,K \quad r = 1,2,...,R \tag{12}$$

$$t_i^k + t_{ij} - W_j^k \le (1 - X_{ijr}^{k}).M \quad k = 1,2,...,K \quad r = 1,2,...,R \tag{13}$$

$$W_j^k \le F_j.\sum_{i=1}^{m} X_{ijr}^{k} \quad j = m+1, m+2,...,m+n \quad k = 1,2,...,K \quad r = 1,2,...,R \tag{14}$$

$$U_{ir}^{k} \ge t_i^k \quad i = 1,2,...,m \quad k = 1,2,...,K \quad r = 1,2,...,R \tag{15}$$

$$X_{ijr}^{k} \in \{0,1\} \quad i = 1,2,...,m \quad j = m+1, m+2,...,m+n \quad k = 1,2,...,K \quad r = 1,2,...,R \tag{16}$$

$$Z_{ij} \in \{0,1\} \quad i = 1,2,...,m \quad j = m+1, m+2,...,m+n \tag{17}$$

$$Y_{ir} \ge 0 \quad i = 1,2,...,m \quad r = 1,2,...,R \tag{18}$$

$$W_j^k \ge 0 \ , \ U_{ir}^{k} \ge 0 \quad i = 1,2,...,m \quad j = m+1, m+2,...,m+n \quad k = 1,2,...,K \quad r = 1,2,...,R \tag{19}$$

Eq. 1 addresses the objective function which is defined as minimization of the entire prices for distribution. Eq. 2 allocates each customer to a specific vehicle and also a distributing route. Eq. 3 indicates that the total volume of products which are assigned to a specific vehicle cannot exceed the vehicle's capacity. Eq. 4 eliminates sub-tours in each route. Eq. 5 causes loop formation. That means each vehicle should return to a depot from which it has started. Eq. 6 shows that each route begins from a depot. Eq. 7 states that each route ends at a depot. Equation (8) ensures that each route is being served by particular vehicle. Equation (9) makes the maximum product volume which is coming from a special depot, not exceed the depots capacity. Equation (10) depicts that a specific customer can be assigned to a depot only if there is a route between them. Equation (11) explains that for a specific customer, arrival time shall not be less than that for prior customer plus offloading durations and also passing time between them. This equation is not linear and equation (12) is the linear form of it. M is also a gigantic positive number which equals Wkj + tij + Tj at least. Because of linearity, equation (12) has been used in this model. Equation (13) illustrates that product delivering time is more than the duration of collecting and providing demands plus the time for passing between depot and the customer. In this equation M is a huge positive number which at least equals tij + tkj. Equation (14) forces the delivering time to be less than the maximum lead time. And equation (15) demonstrates that each vehicle evidently should starts moving after collecting and providing demands or accurately at the same time. And finally, positive values for decision variables are declared in equations (16), (17), (18) and (19).

3. The stochastic hybrid heuristic algorithm

As it was mentioned before, the vehicle routing problem is NP-hard in big scales.[1,2,3] So a heuristic algorithm has been offered in this article in order to solve the problem in a short time and actually achieve to an apt solution for it. In suggested method, it is assumed that all freshly mentioned parameters and also local coordinates of all depots and customers are certain and determined numbers. The introduced stochastic hybrid heuristic algorithm comes in four steps. The first one is customer grouping or in other words is to assign each customer to a specific depot. The second one is routing and vehicle selection which consists on determining the routes and type of vehicle in each depot. The third step is scheduling and packaging in which priority of customers and products packaging would be represented. And in the last step the initial solution has been improved by simulated annealing. More explanations have been provided as following:

3.1. Grouping

Denomination into some sub-problems is one the methods used to simplify the intricate problems. In offered heuristic method, firstly, through the customer grouping, the multi-depot vehicle routing problem is decomposed to some single-depot routing problems. As it is assumed that customers and depots are all located in a same town and also all the routes are perpendicular so coordinal distances are used in order to classify the customers. Coordinal distances are suitable for vertical routes. In simulation of the town as a graph, depots and customers would be some nodes in it. These nodes are related to each other and possess the two dimension coordinates as (x, y).

$D(A, j)$: The coordinal distance between depot A and customer j.

$D(B, j)$: The coordinal distance between depot B and customer j.

$$D(A, j) = |x_A - x_j| + |y_A - y_j| \quad (20)$$

$$D(B, j) = |x_B - x_j| + |y_B - y_j| \quad (21)$$

$$\begin{cases} 1)\, D(A, j) \succ D(B, j) \\ 2)\, D(A, j) = D(B, j) \\ 3)\, D(A, j) \prec D(B, j) \end{cases} \quad (22)$$

Suppose that there are two depots; A and B. The distance of customers is calculated from these two. Then each customer will be assigned to the closer depot. In case of equality in distance from both depots, the customer will be assigned to one of the depots randomly. During this step, it needs to be mentioned that every customer can just be allocated to one depot.

3.2. Routing and vehicle selection

In this section, the serving routes and also the type of vehicles used in each route can be determined for customers of each depot. In other words, during this step it is exactly determined that a specific customer from which route and also by which vehicle should be served. Vehicles differ in capacity and also their costs. Vehicles' prices vary type to type and all vehicles of a specific type costs consistently. Product transporting prices can be calculated for per unit of products' volume regarding to type of vehicles and their constant costs. The price calculation has been shown as follows:

φ_k : The nominal costs of transporting by vehicle k for per unit of products volume.

Q_k : The capacity of vehicle k.

ς_k : Constant costs for vehicle k.

$$\varphi_k = \frac{\varsigma_k}{Q_k} \quad (23)$$

The Clarke and Wright method has been developed in order to determine the routes and vehicles type. At first, for each customer of a depot, a specific route is determined. Then these routes will be combined to each other according to feasible savings and also problem limitations. Initially, the vehicle which has the minimum φ_k is assigned to all routes. In the case that more than one vehicle possess the minimum φ_k, the vehicle with the most capacity will be selected. The procedure of saving matrix construction is shown as following:

S_{jw} : The amount of savings through combination of routes which nodes j and w are their start point or end point.

C_{jw} : The distance between j and w. ($C_{jw} = C_{wj}$)

C_{dj} : The distance between depot d and node j.

C_{dw} : The distance between depot d and node w.

$$S_{jw} = C_{dj} + C_{dw} - C_{jw} \quad (24)$$

The positive S_{jw} s will be arranged in descending order and then by starting from the greatest S_{jw}, related routes will be combined . If the new route satisfies the problem limitations and form no sub-tour, it would be acceptable. Otherwise, the combination will not be triggered. After determining the routes, the whole demands volume in route r will be compared with capacity of all kinds of vehicles in order to the last selection of vehicles. According to demands volume, If was able to choose more than one type of vehicle, for the most economic vehicle selection the φ'_k is calculated as following and the lowest φ'_k would be assigned to route r.

φ'_k : The real costs of transporting by vehicle k on route r, for per unit of products volume.

Q_r : The entire demand volume in route r

ς_k : The constant costs for vehicle k

$$\varphi'_k = \frac{\varsigma_k}{Q_r} \quad (25)$$

3.3. Scheduling and packaging

Till now, it has been illustrated that from which depot and through which route and also which kind of vehicle, a specific customer should be served. Each route can encompass more than one customer so the priority is so important in serving. On the other hand, customer may recommend various products so each request must be clear from the others. In order to clarify the priorities in each route, the latest serving start is used as a criterion. The latest serving start would be figured as follows:

LS_j : The latest serving start for customer j

$D(j)$: The coordinal distance between customer j and its depot

V_k : The average velocity of vehicle k which serves customer j

F_j : The lead time for customer j

$$LS_j = F_j - \frac{D(j)}{V_k} \tag{26}$$

In order to schedule for each route, the LS_j s computed for all customers in that route. Thereafter, these LS_j s will be arranged increasingly. Serving begins from the least LS_j and will be continued in sequence. When the LS_j equals for some customers, the priority will be determined according to nearing to the last served customer. In this case, the one which is closer to last served customer is the next customer who will be mentioned. The requests of each customer will be collected as a package of demanded products and then be uploaded according to priorities. Product packs can be in diverse sizes. And also the LIFO system must be mentioned for uploading. It means that the last uploaded pack refers to the first customer.

3.3.1. Encoding the solutions

Coding the solutions is carried out in order to exhibit the routes. Route exhibition means to determine the depot and also type of vehicle which serves the customer and also to perceive the priority of the customer. The solution is coded as a three-dimension chromosome. For instance, the (i,j,q) Gene introduces a customer who is served by depot i and vehicle k and his/her priority in serving is q.

Assume two depots and two types of vehicle which serve six customers who are shown as numbers 3…9. In Fig. 1 a coded solution has been shown for this problem. To describe this solution it can be said that depot 1 has three routes. One of them is the route which is passed by vehicle 1. This route starts from depot 1 and customer 8 is being served through this route. This route ends at depot 1. The two other routes of depot 1 are passed by vehicle 2. Firstly, the vehicle meets customer 3. Then customer 5 and 7 will be met in order and after that the vehicle returns to depot. Secondly, customer 9 will be met among another route and after all, the vehicle returns to depot again. Depot 2 has one route which is passed by vehicle 2. First, customer 4 and then customer 6 will be visited during this route.

3.4. Improvement by means of Simulated annealing

SA of Aarts and Korst (1985) and Van Laarhoven and Aarts, (1987) a heuristic search method based on ideas drawn from statistical physics, has been found to be effective in many combinatorial optimization problems. The algorithm begins with a randomly generated initial point (the trial solution).This initial solution is the "current" solution. A neighbor (an adjacent point) of this current solution is then generated, following some predetermined neighbor-generating method. If the neighbor is found to be better than the current point, it (the neighbor) is unconditionally accepted as the new current point. On the other hand, if the neighbor is found to be worse, it is not rejected outright, but accepted with a certain probability. The algorithm proceeds by iterating a certain number of times over the transition from the current point to the adjacent point (which becomes the next current point). At the beginning of an SA run, the probability of accepting a worse point is kept high (there by reducing the chance of the SA algorithm getting trapped in a local optimum). As the number of iteration increases, this probability is reduced according to a specific policy. It is customary to use a control parameter, called the temperature (analogous to the temperature of the physical process), to alter the probability of acceptance/rejection. Usually, the temperature is started at a high value and is gradually brought down according to a schedule known as the annealing (cooling) schedule. This annealing schedule determines how the probability of accepting a worse point decreases with time (iterations).

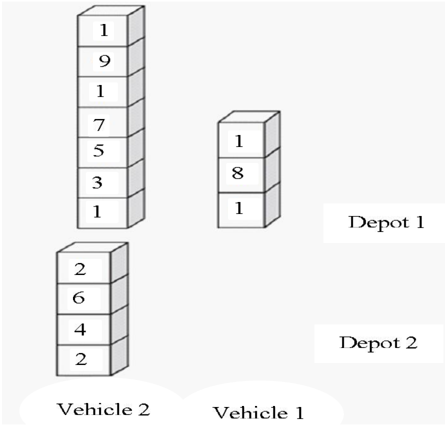

Fig. 1. Displaying the routes with three dimension chromosome

This heuristic applies SA to improve up on the best solution obtained at any step of the algorithm. The improvement algorithm is described as follows:

1. For i=1–N do

(a) Initialize max-iterations, temp-start. Set count=1, θ=temp-start. Let the best solution obtained in the initialization step be called the current solution X_c. Compute objective (X_c).

(b) Randomly generate a neighboring solution using either the interchange neighborhood: forward insertion neighborhood or back ward insertion neighborhood (these neighborhoods are explained below) Let the neighboring solution be called the adjacent solution X_a.

If the possibility of adjacent solution is satisfied the algorithm would be continued otherwise another adjacent solution is triggered. Possibility of adjacent solution would be satisfied as follows:

"Assume the swap of customer j, located on route 1, and customer w which is located on route 2.

H_j : The total demand of customer j in volume unit.

H_w : The total demand of customer w in volume unit.

G_{r_1} : The free capacity of vehicle k which is serving the route r_1. It is in volume unit and can be determined from this equation: $G_{r_1} = Q_k - Q_{r_1}$

G_{r_2} : The free capacity of vehicle k' which is serving the route r_2. It is in volume unit and can be determined from this equation: $G_{r_2} = Q_{k'} - Q_{r_2}$

$$H_j \le H_w + G_{r_2} \qquad (27)$$

$$H_w \le H_j + G_{r_1} \qquad (28)$$

Swaps which satisfy Eqs (27),(28) satisfy the possibility of solution too and they are permissible. Then compute objective (X_a).

(c) If Objective (X_a) < Objective (X_c) Then set $X_c = X_a$;

Else

Set Δ= Objective (X_a) - Objective (X_c);

Set θ=temp-start / log (1+count);

With probability $e^{-\Delta/\theta}$ set $X_c = X_a$.

(d) Increment count by1;

If count < max-iterations, go to step (b).

2. Use the output of the current solution as the final solution. The annealing schedule used in step1(c) of the above algorithm is due to Hajek. The inter change neighborhood, by far the most popular scheme, is extremely simple: swap two randomly chosen customers in the sequence. Two other neighborhoods are something that Gupta and Smith have introduced. In forward insertion neighborhood a customer is relocated further forward in the sequence and in back ward insertion neighborhood a customer is relocated further back ward in the sequence.

Caution: The limitation of demands which have delay must be mentioned too. The maximum number of demands which have delay is inserted to the program as one parameter, in order to consider the possibility of solutions among this criterion. The mentioned parameter can be altered in every new run of program.

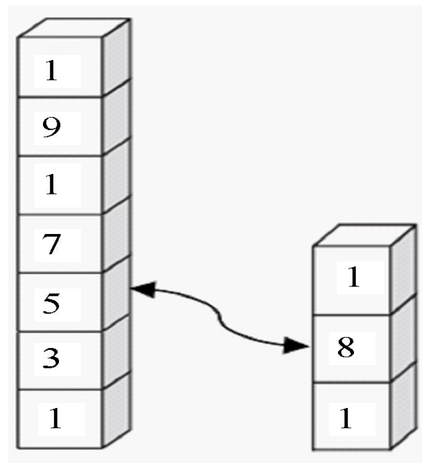

Fig. 2. An interchange between two routes of a depot

4. Computational experiments

There is no standard problem for MD-MPVRP to compare the solutions of heuristic method with its solutions. So twenty four randomly generated examples were considered in order to indicate the efficiency of heuristic method among the current method, by comparing the solutions of MATLAB with Lingo12's. The randomly generated examples had 2 to 5 depots, 2 or 3 vehicles, 5 or 10 products and the number of customers was between 2 and 300. Dimensions of the program have been showed as I.J.K.A.

"I" shows the number of depots, "J" shows the number of customers, "K" addresses how many kinds the vehicles are and "A" also do as well for products. Table [2] shows the whole transporting distance and also elapsed runtime.

As it's shown in table [2], for problems with virtually small proportions the suggested method achieves the optimum solutions or near to optimum ones, in a shorter time. And actually for gigantic-dimension problems in which achieving the optimum solution is either impossible or too time-taking, the heuristic method in MATLAB reaches the solutions at least 24 times sooner than the Lingo reaches. The total transportation distance and also solving time, reported in the table, are the average amount of 30 repeats for each problem. It is clear that the maximum deviation of represented solutions from the optimum ones is less than 7.7 percent.

Table 2. Comparing the offered algorithm's results with Lingo 12's.

Problem's proportions	Elapsed runtime (s)		Total transportation distance (kilometers)		Comparing the solving time and solutions (%)	
I.J.K.A	Optimum solution	Offered algorithm	Optimum solution	Offered algorithm	Reduction in solving time	Deviation from the optimum solution
2*8*2*5	9	0.089	47	47	99.1	0
2*10*2*5	14	0.197	105	105	98.5	0
2*15*2*5	58	0.578	164	164	99.0	0
2*20*3*10	2495	2.017	185	185	99.9	0
2*25*3*10	4143	2.304	241	243	99.9	0.8
3*15*2*5	261	0.970	194	203	99.6	4.6
3*20*2*5	524	1.260	310	324	99.7	4.5
3*25*3*10	9391	4.875	352	369	99.9	4.8
3*30*3*10	13297	5.364	376	401	99.9	6.6
3*35*3*10	16281	6.362	418	428	99.9	2.3
4*20*2*5	1068	2.519	331	354	99.8	6.9
4*25*2*5	2186	2.716	397	412	99.9	3.8
4*30*2*5	2574	2.713	482	508	99.9	4.9
4*35*2*5	4091	2.690	646	668	99.9	3.4
4*40*3*10	18072	32.461	723	742	99.8	2.6
4*45*3*10	23867	74.578	796	814	99.8	2.2
5*40*3*10	21974	79.934	948	967	99.7	2.0
5*45*3*10	28644	96.432	1193	1285	99.7	7.7
5*50*3*10	-	254.359	-	1485	-	-
5*75*3*10	-	396.384	-	1842	-	-
5*100*3*10	-	482.173	-	2354	-	-
5*150*3*10	-	839.463	-	2781	-	-
5*200*3*10	-	1173.401	-	3379	-	-
5*300*3*10	-	2662.037	-	4962	-	-
average	-	-	-	-	99.7	3.1

5. Case study

The KALEH factory is one the most immense producer of dairy products in IRAN, which produce various kinds of products. In this research, scheduling and vehicle routing of this factory has been studied in YAZD city in order to display the efficiency and effectiveness of the proposed method. The factory has three sale centers which serve 198 retailers. The local position of depots and customers is specified in the city. The Fig. 3 shows the city's map and actually a specific section of it in which position of customers has been shown by some signs. In this research, central square of city was selected as coordinates (0,0) then coordinates of depots and customers were defined according to it. Serving is presented by three different kinds of vehicle. In order to carry the products, some standard baskets have been determined in the factory. The vehicles' volume has been evaluated according to number of baskets which fulfill the vehicle. Visitors would collect customers' demands before the serving starts. In current method, first, all demands of a route are collected and then uploaded. But in this stochastic hybrid heuristic method, each customer's demands are assumed as a package and these packages are uploaded according to LIFO system.

The advised method was coded by MATLAB software and then the program was performed by a computer which had these features: Dou CPU 2.53 GHz, Core 2 for processor and RAM 4 MB. The elapsed runtime never exceeded 45 minutes. Solutions of heuristic method have been compared to existing information for current method in table [1]. As it's shown the introduced method has resulted in an improvement for entire transporting distances and also caused a reduction in number of delays during the serving. In addition, lessening the maximum serving time, the heuristic method makes the transportations finish earlier.

Table1. Comparing the offered algorithm's results with existing condition in the case study

Total number of customers which have delay	The most time-taking route for serving (kilometers)	Total transportation Distance (kilometers)	
-	293	407	Offered algorithm
38	389	486	Existing condition

Fig. 3. The map of YAZD and a section which embraces location of customers

6. Conclusion

Routing and scheduling of deliveries are two crucial operational decisions in distribution management. Better routing and scheduling can result in shorter delivery distance, or time, and thus, higher level of efficiency and lower delivery cost can be achieved. Reduction in prices and also serving time would be overtly more gratifying for customers. For industrial centers, Internal and external travels comprise a grave part of the prices. In this research, we have tried to represent a mathematical model and stochastic hybrid heuristic algorithm and for shortening the travelled distances in MD-MPVRP. In this method, different kinds of vehicle with eclectic capacity, velocity and costs were used for serving the customers who could order various types of product. Solution, generated by the first three steps of the algorithm, was improved by simulated annealing. The efficiency of proposed method considered during some computational experiments and also a case study. In the computational experiment the average deviation of solutions from the optimum solutions is about 3.1 percent and the average elapsed runtime is 99.7 percent sooner than the average one in Lingo. Inducing this method in KALEH factory, located on YAZD, caused 16.2 percent reduction in serving prices and also there were no delay any more. This method can be developed in future studies by determining time periods for serving and also embedding fuzzy parameters.

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