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ABSTRACT

We discuss whether all resonances should be expected to occur in the partial-wave projection of the cross-channel Regge pole formula.

It has recently become clear [1),2),3)] that the Argand diagram plot of the s-channel partial-wave projection of the Regge pole fit to high s data contains near circles of the type usually associated with resonances. SCHMID [1)] has suggested that one can utilize this fact in doing simple bootstrap calculations.

Two assumptions appear to be necessary for such bootstrap calculations to be meaningful:

Assumption A

The "near circles" which occur in the partial-wave projections of cross-channel Regge pole expressions all correspond to resonances.

Assumption B

In strong interaction processes all resonances are "contained" (in this sense) in the partial-wave projection of the cross-channel Regge pole expression.

As already noted [2)], these assumptions do not appear particularly plausible, since they imply a consistency between s- and t- (or u-) channel quantum numbers which is surprising in a model which does not contain explicit unitarity. Some comments about Assumption A were made in Ref.2 ; here we want to consider the meaning and possible implications of Assumption B.

To do this we compare the two processes $\pi^- p \rightarrow \pi^0 n$ and $\pi^- p \rightarrow K^+ \Sigma^-$. These have the same s-channel quantum numbers and hence the same resonances (with residues that are of the same order of magnitude if SU(3) is a reasonable approximation). In previous types of dynamical theory this comes about through the unitarity equation which requires that, in general, coupled processes have poles at the same position. Following SCHMID [1)] we ignore the u-channel (baryon) Regge poles, so in the process $\pi^- p \rightarrow \pi^0 n$ we have only the ρ-pole.

Schmid has shown that this can satisfactorily reproduce the Δ trajectory. However, there are no known Regge poles for the second process, so if in the Assumption B only known trajectories are included, we predict a zero coupling of the Δ trajectory to the $K\Sigma$ system. This is surely false.

In passing, it is worth making the trivial observation that the example of $\pi^- p \to K^+\Sigma^-$ is one in which the "interference model" is better than the pure Regge pole model (using known trajectories) by a factor of infinity. Cases where the opposite is true can also be found - which serves to illustrate that the relative merits of the two models is not one of dogma, but depends on the circumstances.

To return to our problem. In order to save Assumption B we must clearly allow the introduction of new trajectories.*) Since we know of no charge 2, strangeness 1 mesons it is clear that in our example we must include a very low-lying trajectory. The fact that the trajectory must be low is strengthened by the absence of a forward peak in $\pi^- p \to K^+\Sigma^-$ at high energy. Indeed, since the kinematics of this process and $\pi^- p \to \pi^0 n$ are identical (if we take the SU(3) limit and ignore mass differences) it is clear by repeating SCHMID's calculation [1] that we can only obtain the same Δ trajectory if the new trajectory (which we call L) satisfies

$$\alpha_L(t) \simeq \alpha_\rho(t) - 2N \tag{1}$$

where N is an integer or zero. We have ruled out N equal to zero, so the most favourable case is N = 2. Thus Assumption B can only be valid if we postulate the existence of the L trajectory. If the prediction is valid then the R-meson would be well placed to be the lowest physical state or one of the non-strange SU(3) partners of the L trajectory.

Note, however, that in order to save Assumption B we have had to allow, in the Regge expression, trajectories which are two units in angular momentum below the ρ. Once having admitted such trajectories there is no reason why we should not go even

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*) We ignore here the possibility that the u-channel baryon Regge poles might give the required resonances. The elastic widths of any resonances obtained in this way will be small and decrease rapidly with energy compared to those obtained from ρ-exchange.

lower and it is then hard to see how Assumption B can lead to a complete bootstrap theory. Although, in the process we have considered, the L is the highest trajectory, its importance here shows that we should also include it, along with the K and K*, in $\pi^- p \rightarrow K^0 \Sigma^0$, say, where it is no longer the leading trajectory. Similarly, we should include the non-strange SU(3) partners of the L in $\pi N \rightarrow \pi N$ scattering.

Further, we note that Assumption B is, in this situation, incompatible with SU(3). To see this we compare the ratios of the residues of the Δ in $\pi^- p \rightarrow K^+ \Sigma^-$ and $\pi^- p \rightarrow \pi^0 n$ as we move along the trajectory. This is predicted, by our assumption, to behave as

$$\frac{G_{K^+\Sigma^-\Delta}(s)}{G_{\pi^0 n\Delta}(s)} \simeq \left(\frac{s}{s_0} \right)^{-2} \qquad (2)$$

where s_0 is the usual scale factor in the Regge pole model and $G_{K^+\Sigma^-\Delta}(s)$ is the coupling of $K^+\Sigma^-$ to the Δ Regge trajectory at energy equal to s. Since, according to SU(3), this ratio is constant $\left(\frac{1}{\sqrt{2}}\right)$ we have a clear contradiction.

To summarize, it appears to be hard to maintain Assumption B in a useful form in our example (others are not hard to find). The reason would seem to be clear: it is implied by this assumption that resonances in any given amplitude are caused by the "forces" in that amplitude, whereas in general this is false since particular resonances may well be due to "forces" in other coupled channels.

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