Tax Evasion, Social Customs and Optimal Auditing

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Abstract: The optimal audit policy is analysed for an independent revenue service when a social custom exists that rewards honest tax-paying. The implication of the existence of the social custom is that in equilibrium the income level of a taxpayer cannot always be inferred exactly from their report. The structure of the optimal audit policy is determined both for a fixed (report-invariant) audit probability and for when the audit probability is a function of the income report. For the constant probability of audit it is shown that an interior solution exists to the decision problem of the revenue service and comparative statics results are given. When the audit probability can vary, the audit function is proved to be a decreasing function of the income report which reaches zero at the highest income report of a tax evader. Increases in the fine for evading and in the tax rate raise the optimal audit probability.

Keywords: tax evasion, auditing, social customs

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1. Introduction

Tax evasion is a phenomenon that affects both developed and developing countries. Although estimates of the level of evasion are subject to considerable uncertainty, there can be no doubting that a significant part of national income is unrecorded in official statistics. It is therefore not surprising that tax evasion has become a topic of considerable research activity. In addition to attempts at the measurement of the extent of evasion, research has produced both models designed to explain the known facts of evasion and others that aim to provide guidance for the government in reacting to the existence of evasion. It is to the latter literature that the present paper aims to contribute.

An important avenue through which state intervention can affect the level of tax evasion is the policy of its revenue collection agency. Given that tax rates are set as part of a broader economic policy and that punishments for evasion are determined by the civil courts in relation to those for other offences[1], the auditing policy of the revenue service becomes the only dimension of choice. Essentially, the revenue service can be modelled as receiving income reports from the taxpayers and, on the basis of these, determining what proportion of reports should be audited at each income level[2]. The reaction of the taxpayers to these probabilities, in terms of the reports they submit, then determines the level of revenue, net of auditing costs, achieved by the revenue service. By analysing the effect of the auditing scheme on the equilibrium outcome the optimal, revenue maximising, scheme can be characterised.

Economic analysis of the determination of the optimal audit policy[3] has provided a number of significant results but all such analyses have been based on the standard Allingham and Sandmo (1972) model of tax evasion as an optimal portfolio decision under uncertainty. Although there is much to say in favour of this model as a preliminary insight into the tax evasion decision, its predictions are not entirely in agreement with the empirical and experimental evidence. For example, it fails to predict that aggregate evasion will rise as the tax rate increases (Clotfelter (1983)) or that many taxpayers do not evade even when they would accept a gamble equivalent to the tax evasion decision (Baldry (1986)). It also neglects the importance of social

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[1] An alternative view on this issue is given in Pestieau *et al* (1994).

[2] The revenue service may also try to influence perceptions about the chance of being caught by, for example, prosecuting a small number of "celebrity" cases which are guaranteed publicity. It may be possible to model this but the present paper works on the basis of objective probability assessments. For discussion of evidence on this, see Alm *et al* (1992).

[3] Such as Andreoni (1992), Cremer, Marchand and Pestieau (1990), Reinganum and Wilde (1986), and Sanchez and Sobel (1993).

factors as emphasised by the findings of Spicer and Lundstedt (1976)[4]. To overcome this, Gordon (1989) and Myles and Naylor (1995) have modified the Allingham-Sandmo framework by including social customs into the tax evasion decision. With this modification, a decision to evade brings both the possibility of a fine if caught evading but also the loss of the social custom utility. This specification is sufficiently flexible to lead to results consistent with observed evidence and captures in a simple, stylised, way the role of social interaction between taxpayers.

The purpose of the present paper is therefore to consider optimal auditing when the level of tax evasion is determined by the social custom model, thus embedding the auditing problem in a framework with greater explanatory power. The most important implication of the existence of the social custom is that in equilibrium the income level of a taxpayer cannot always be inferred exactly from their report since the honesty of the taxpayer cannot be observed by the revenue authority. A low reported income can come from either a low income, but honest, taxpayer or from a higher-income taxpayer who is engaging in evasion. This seems to concur much more closely with reality than the correct inference that follows from the revelation principle in the absence of the social custom. In addition, there may be some reports of high incomes that will only be filed by honest taxpayers and this will put a bias into the audit probabilities.

The analysis determines the structure of the optimal audit policy both for a fixed (report-invariant) audit probability and for audits which are a function of the report. The solution procedure involves first constructing a *social equilibrium* which captures the interdependency between taxpayers caused by the existence of the social custom. The optimal audit function is then determined through its effects upon the level of evasion in the social equilibrium. For the constant audit probability case, comparative statics analysis of the optimal solution show that, in contrast to the findings of Christiansen (1980) and Kolm (1973), in this framework more auditing is a complement to an increase in the fine paid on untaxed income rather than a substitute. The analysis of the general case shows that the optimal audit probability is a decreasing function of reported income, with the audit probability becoming zero at the highest income report made by a tax evader. The complementarity of the fine and audit probability is again shown to arise.

Section 2 introduces the form of the social custom, analyses individual choice behaviour and specifies the objective of the revenue service. A social equilibrium of verification and reporting is also defined. The characterisation of the optimal constant

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[4] Fuller discussion of these issues can be found in Cowell (1990) and Myles (1995).

audit probability and some comparative statics are given in Section 3. Much of the construction in this section also applies in the case of a variable probability of audit which is analysed in Section 4. The optimal audit function is characterised and a comparative statics analysis is conducted. Conclusions are presented in Section 5.

2. Taxpayers, revenue service and equilibrium

The basic structure of the model is that the revenue service chooses an audit function and taxpayers choose their level of evasion (which may be zero) or, equivalently, their reported level of income. The revenue service could calculate the level of evasion that a taxpayer of known characteristics would choose but, since the characteristics are not observed, must infer income from the observed report. The audit probability is chosen to maximise revenue less costs, given the beliefs about true incomes formed on the basis of the signals in the reports. Reports are chosen by taxpayers to maximise expected utility given the audit probabilities.

2.1 *The individual taxpayer*

The model of individual taxpayer behaviour combines the standard Allingham and Sandmo (1972) analysis of the evasion decision with a social custom that provides a return to honesty. The social custom is relevant for determining whether a taxpayer chooses to evade. It also introduces an interdependency between taxpayers that features in the determination of equilibrium.

Let the utility level for a taxpayer of income Y and facing the tax rate t, who chooses not to evade, be given by

$$U^{NE} = U(Y[1-t]) + bR(1-\mu), \tag{1}$$

where $b \geq 0$. The term $bR(1-\mu)$ is the utility of conforming with the group of tax payers (μ is the proportion of population evading tax), so that b can be interpreted as the importance the taxpayer attaches to the social custom. Each taxpayer is fully described by their $\{b, Y\}$ pair. The following assumption characterises the restrictions placed upon preferences.

Assumption 1
$U(\cdot)$ is smooth with $U'(\cdot) > 0$, $U''(\cdot) < 0$ and $\lim_{\psi \to 0} U(\psi) = \infty$. $R(\cdot)$ is smooth and $R'(\cdot) > 0$.

When the decision to evade tax is taken, the resulting level of utility is

$$U^E = \max_{\{I\}}[1-p(I)]U(Y-tI)+p(I)U(Y-tI-ft[Y-I]), \qquad (2)$$

where I is reported income, $p(I)$ the probability of detection given a report of level I, and f, $f>1$, the fine rate if caught. The maximiser of (2) is denoted by I^* and the maximised utility level by U^{E*}. It can be seen from (2) that the return from the social custom is lost when evasion is undertaken. This conforms to the framework adopted in the literature on social customs (see Akerlof (1980), Naylor (1989)) but differs from the model of Gordon (1989) in which the costs of evasion increase proportionately as evasion increases.

Define $W = Y - tI$ and $Z = Y - tI - ft[Y-I]$. If a taxpayer of income Y chooses to evade, the optimal level of reported income, $I^* \in (-\infty,\infty)$[5], must satisfy the first-order condition[6]

$$p'[U(Z)-U(W)]-[1-p]tU'(W)+[f-1]ptU'(Z)=0. \qquad (3)$$

It is clear from these conditions that for given Y the optimal choice I^*, and hence U^{E*}, is independent of b. So if an individual deviates from the social custom the extent to which they evade does not depend on the importance they attach to the social custom.

Whether an individual taxpayer evades depends upon the value of U^{NE} relative to U^{E*}. Evasion will take place if $U^{NE} < U^{E*}$ and the taxpayer will choose not to evade if $U^{NE} \geq U^{E*}$. The weak inequality on the latter expression indicates the implicit assumption that the taxpayer prefers to follow the social custom if there is no utility loss to doing so[7]. For given Y, t and $p(I)$, if there exists a value of μ^* satisfying

$$[1-p]U(W)+pU(Z)=U(X)+bR(1-\mu^*), \qquad (4)$$

where $X = Y[1-t]$, then, since the right-hand side is monotonically increasing in μ, taxpayer (b,Y) evades if $\mu > \mu^*$ but does not evade if $\mu \leq \mu^*$. Hence, given t, if (4) holds, there is a critical proportion, μ^*, such that the taxpayer begins to evade. If

$$[1-p]U(W)+pU(Z)>U(X)+bR(1-\mu), \qquad (5)$$

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[5] Restrictions on the range of I^* will be derived below. It should be noted that this permits income reports which are less than the lowest actual income level (which is assumed to be public knowledge). Although such reports are known to be false, auditing still requires the use of resources so they may not be audited with probability 1.

[6] A corner solution at $I^* = Y$ cannot occur because the value of U^{NE} is then at least as great since $b \geq 0$. Allowing the possibility of negative income reports removes corner solutions at the other extreme.

[7] When computing integrals below, the choice for this set of measure zero has no consequence.

at $\mu = 0$ then the taxpayer always chooses to evade. Alternatively, if the reverse inequality to (5) holds at $\mu = 1$ then such a taxpayer always chooses to pay tax.

2.2 *The population of taxpayers*

Moving to the economy as a whole, the population of taxpayers is described by a distribution function, $h(b,Y)$, of b and Y. This distribution function satisfies the restriction of Assumption 2.

Assumption 2

(i) $h(\cdot)$ is smooth:

(ii) $h(b,Y) > 0 \;\; \forall \{b,Y\} \in (0,B) \times (\underline{Y}, \overline{Y})$, $0 < B < \infty$, $-\infty < \underline{Y} < \overline{Y} < \infty$, $\overline{Y} > 0$,
$h(b,Y) = 0$ elsewhere.

(iii) $\int_{\underline{Y}}^{\overline{Y}} \int_0^B h(b,Y) db dY = 1$;

(iv) $\lim_{\Psi \to \underline{Y}} \int_0^B h(b,Y) db dY = 0$, $\lim_{\Psi \to \overline{Y}} \int_0^B h(b,Y) db dY = 0$.

In (ii) of Assumption 2, $\underline{Y}$ is the minimum level of income in the population and $\overline{Y}$ the maximum. The valuation placed on the social custom is restricted to be non-negative, so no-one actively despises it, but with a finite upper limit, B. Part (iii) is a normalisation and (iv) requires that the proportion of the population having a given level of income tends to zero as the boundaries of the income range are approached.

Exploiting the monotonicity of $R(\cdot)$ in μ, (4) can be solved to write $\mu^* = \Theta(b, Y, p(\cdot))$. Given the assumptions on $R(\cdot)$, it follows that $\Theta(\cdot)$ is continuously differentiable in b and that

$$\frac{d\mu^*}{db} = \frac{R}{bR'} > 0. \tag{6}$$

The strict inequality in (6) shows that $\mu = \Theta(b, Y, p(\cdot))$ can be solved to give

$$b = \beta(Y, \mu, p(\cdot)), \tag{7}$$

with $\beta(\cdot)$ differentiable and strictly increasing in μ. This has the interpretation that, given μ and $p(\cdot)$, all taxpayers described by pairs $\{b,Y\}$ that satisfy $b < \beta(Y, \mu, p(\cdot))$ evade tax.

Assuming that for those choosing to evade the second-order condition for the optimisation in (2) is satisfied, (3) can be solved to write $I = I(Y, p(\cdot))$. Accepting that $I(Y, p(\cdot))$ is strictly monotonic in Y for the present[8], $I = I(Y, p(\cdot))$ can be

[8] Monotonicity will be studied for alternative restrictions on the form of $p(\cdot)$ below.

inverted to give $Y = m(I, p(\cdot))$. The function $m(I, p(\cdot))$ determines the true income of a taxpayer who is evading as a function of reported income and the audit function.

Given a value of μ, the proportion of taxpayers who choose to evade is determined by those for whom $b < \beta(Y, \mu, p(\cdot))$. Denoting this proportion by $G(\mu)$, it follows that

$$G(\mu) = \int_{\underline{Y}}^{\overline{Y}} \int_0^{\beta(Y,\mu,p(\cdot))} h(b, Y) db dY . \tag{8}$$

2.3 *The revenue service*

The revenue service is concerned only with maximising tax revenue less the cost of auditing. It takes t and f as fixed[9]. Given an income report I, the revenue service infers an expected income level of the taxpayer making that report. Denote the expected level of income by $M(I)$. The cost of enforcing an audit probability of $p(I)$ for reports of level I is given by the cost function $c(p(I))$ which satisfies the conditions of Assumption 3.

Assumption 3

$c(\cdot)$ is smooth and $c(0) = 0$, $c'(0) = 0$, $c'(p) > 0$ for $p > 0$, $\lim_{p \to 1} c'(p) = \infty$.

With audit strategy $p(\cdot)$, expected revenue (less costs) is given by

$$R(p(\cdot)) = \int_{-\infty}^{\infty} [p(I)[tI + ft[M(I) - I]] + [1 - p(I)]tI - c(p(I))] g(I; p(\cdot)) dI , \tag{9}$$

where $g(\cdot)$ is the number of reports of income level I received when the audit function is given by $p(\cdot)$. The form of $g(\cdot)$ will be calculated below. The audit function $p(\cdot)$ is chosen to maximise (9) given the beliefs $M(\cdot)$ and the distribution $g(\cdot)$.

2.4 *Equilibrium*

The notion of equilibrium that will be employed is defined in two steps. In the first, the audit function is taken as given and a *social equilibrium* is defined. This equilibrium incorporates the social externalities between taxpayers arising from the existence of the social custom of conformity. The second step is to allow for optimisation of the audit function, taking into account the dependence of the social equilibrium upon the audit function. A social equilibrium with optimal audit function is termed a *social equilibrium of reporting and verification*.

A social equilibrium is now defined.

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[9] This is justified by viewing the revenue service as a distinct body from the courts which set the level of the punishment and the exchequer which determines the tax rate.

Definition 1
A *social equilibrium* is a level of evasion, $\hat{\mu}$, such that $G(\hat{\mu}) = \hat{\mu}$.

The interpretation of a social equilibrium is that the proportion evading is self-supporting. It is straightforward to prove that at least one level of evasion exists that is a social equilibrium. This result is summarised as Lemma 1.

Lemma 1
For any audit function $p(\cdot)$, there exists a social equilibrium.

Proof
As $\beta(\cdot)$ is continuous in μ, it can be seen from the definition in (8) that *G* defines a continuous map $G(\mu): [0,1] \rightarrow [0,1]$. By Brouwer's Theorem it has at least one fixed point and each fixed point satisfies the definition of equilibrium. ||

The definition of a social equilibrium of verification and reporting is now given in which both the audit function and the income reports are equilibrium strategies. It therefore incorporates the optimisation behaviour of the agents and the social interaction between taxpayers. The structure of such equilibria will be investigated in the following sections.

Definition 2
A *social equilibrium of reporting and verification* is an array $\{I, M(\cdot), p(\cdot), \hat{\mu}\}$ such that:

(i) the income report, *I*, from each taxpayer maximises their utility given the audit policy $p(\cdot)$ and level of evasion $\hat{\mu}$;

(ii) the beliefs of the revenue service satisfy $M(I) = E[Y|I]$, where the expectation is taken over the distribution of taxpayers making report *I*;

(iii) the audit policy $p(\cdot)$ maximises $R(p(\cdot))$ given the distribution of reports $g(\cdot)$ and beliefs $M(\cdot)$;

and

(iv) the level of evasion, $\hat{\mu}$, satisfies $G(\hat{\mu}) = \hat{\mu}$.

3. Restricted audit function

This section investigates the optimal audit policy under the restriction that the audit probability is chosen from the class of constant functions with the intention of characterising the optimal audit strategy and assessing the effects of variations in the underlying structure upon this. This restricted case is considered for its own interest and because many of the constructions undertaken in this simplified setting extend to the general case considered in the next section with only minor modification.

With attention restricted to the set of constant functions, a choice of audit strategy reduces to the announcement of an audit probability, p, which is independent of income. The first step in the characterisation is to determine the distribution of reports forthcoming from taxpayers and the beliefs of the revenue service. To do this, note that any announced audit probability will partition the population into those who evade and those who do not. The typical relation of reports is illustrated schematically in Figure 1; it is to be expected that the lowest reports will be provided by tax evaders and the highest by honest tax payers.

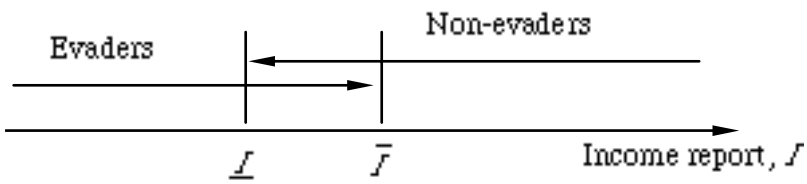

Figure 1: Distribution of income reports

In Figure 1, $\underline{I}$ denotes the minimum report filed by a non-evader so that reports below this level are filed only by those evading. Similarly, since $\bar{I}$ denoted the maximum report from an evader, any report above this must come from an honest taxpayer. If any report is received between these two levels[10], the revenue service remains uncertain whether it emanates from an honest taxpayer with a low income level or from an evader.

It is now necessary to verify that $I = I(Y, p(\cdot))$ is monotonic in Y. From (3) (or from (12)* of Yitzhaki (1974)) it follows that

$$\frac{dI}{dY} = \frac{[ft-1]R_A(Z) + R_A(W)}{t[f-1]R_A(Z) + tR_A(W)}, \tag{10}$$

where $R_A(M)$ is the coefficient of absolute risk aversion at income level M. For the second-order condition for the optimisation in (2) to be satisfied it is necessary that

[10] The possibility that the set of reports filed by the evaders may be disjoint from those filed by the non-evaders clearly arises for some audit probabilities. This is taken explicitly taken into account in the calculations below.

the denominator of (10) is positive. For $\frac{dI}{dY}$ to be positive it is then necessary that $[ft-1]R_A(Z)+R_A(W)$. This inequality is assumed to be satisfied[11]. It can also be seen from (3) that with $p' = 0$, Assumption 1 implies that for a taxpayer of income Y the declaration satisfies $I \geq K(p,Y) > -\infty$ since $\lim_{I\to-\infty} U'(W)=0$ and $\lim_{I\to-\infty} U'(Z)>0$. From the structure of the optimisation, $K(p,Y)$ is continuous in Y and hence has a minimum over the compact set $[\underline{Y},\overline{Y}]$. This minimum is denoted $K(p)$. Thus, for given p, the declaration must belong to the compact interval $[K(p),\overline{Y}]$.

The distribution of reports received by the revenue service is constructed from the distinct components consisting of the three intervals identified in Figure 1. On these intervals, the distribution of reports can be calculated as follows.

(a) For $I \leq \min\{\underline{I},\overline{I}\}$, $g(I;p)=\int_0^{\beta(m,\mu,p)} h(b,m(I))db$;

(b.i) If $\underline{I}<\overline{I}$, then for $I\in[\underline{I},\overline{I}]$, $g(I;p)=\int_0^{\beta(m,\mu,p)} h(b,m(I))db+\int_{\beta(m,\mu,p)}^{B} h(b,I)db$;

(b.ii) If $\underline{I}>\overline{I}$, then for $I\in[\underline{I},\overline{I}]$, $g(I;p)=0$;

(c) For $I>\max\{\underline{I},\overline{I}\}$ $g(I;p)=\int_{\beta(m,\mu,p)}^{B} h(b,I)db$;

(d) $g(I;p)=0$ elsewhere.

In a similar manner, the beliefs of the revenue service relating reports to expected income can now be constructed. As already noted in (ii) of Definition 2 these must be correct, in terms of expectations, at the equilibrium. When $\underline{I}<\overline{I}$ the expected income following a report $I\in[\underline{I},\overline{I}]$ is simply a weighted average of the incomes of evaders and non-evaders making that report. This is given as (β.i). Conversely, when $\underline{I}>\overline{I}$ no reports $I\in[\underline{I},\overline{I}]$ are actually received. To make the belief function continuous in such a case, the convention is adopted in (β.ii) that the expected income function is linearly extended from the highest income of an evader to the lowest income of a non-evader. This arbitrary convention does not affect any of the conclusions. The belief function is therefore as follows.

(α) For $I\leq\min\{\underline{I},\overline{I}\}, M(I;p)=m(I,p)$;

[11] As the coefficient of absolute risk aversion is positive, it is sufficient that either $ft > 1$ or that absolute risk aversion is non-decreasing in income.

(β.i) If $\underline{I} < \bar{I}$, for $I \in [\underline{I}, \bar{I}]$, $M(I;p) = \dfrac{m(I)\int_0^{\beta(m,\mu,p)} h(b,m(I))db + I\int_{\beta(I,\mu,p)}^{B} h(b,I)db}{\int_0^{\beta(m,\mu,p)} h(b,m(I))db + \int_{\beta(I,\mu,p)}^{B} h(b,I)db}$;

(β.ii) If $\underline{I} > \bar{I}$, for $I \in [\underline{I}, \bar{I}]$, $M(I;p) = \dfrac{[\underline{I} - I]m(\bar{I})}{[\underline{I} - \bar{I}]} + \dfrac{[I - \bar{I}]\underline{I}}{[\underline{I} - \bar{I}]}$;

(γ) For $I > \max\{\underline{I}, \bar{I}\}$, $M(I;p) = I$.

Lemma 1 proved that for given any choice of auditing strategy there exists a social equilibrium with respect to that auditing strategy. To show the existence of an equilibrium with reporting and verification, it must be demonstrated that there exists an optimal choice of p given the strategy of the taxpayers. To proceed with this assume that for all p, $1 - G'(\mu) \neq 0$ at the equilibrium value of μ associated with that p so the identity $G(\mu) = \mu$ can be solved to write the equilibrium proportion of evasion as $\mu = \mu(p)$. The analysis below requires that $\mu(p)$ is continuous but this is not restrictive. The only circumstances in which $\mu(p)$ can fail to be continuous are a) when the social equilibrium occurs at $\mu = 0$ or $\mu = 1$ and (i) $G'(0) > 1$ and (ii) there are some values of p for which $\mu = 0$ or $\mu = 1$ is not an equilibrium (see the analysis in Myles and Naylor (1995)), or b) $\mu \in (0,1)$ and $G'(\mu) = 1$. The latter of these has already been ruled out. Except for these exceptional cases, $\mu(p)$ will be continuous and is henceforth assumed to be so.

Employing (iv) of Assumption 2, the following lemma is immediate and is stated without proof.

Lemma 2
The distribution function $g(I;p)$ and the belief function $M(I;p)$ are continuous.

Using the facts that $\underline{I}(p) = \underline{Y}$ and $M(I;p) = I$ for $I \in [\max\{\underline{I}, \bar{I}\}, \bar{Y}]$, for the constant probability case, the objective of the revenue service can be written

$$\max_{\{p\}} R(p) = \int_{-\infty}^{\max\{\underline{Y},\bar{I}\}} [tI + pft[M(I;p) - I] - c(p)]g(I;p)dI$$

$$+ \int_{\max\{\underline{Y},\bar{I}\}}^{\infty} [tI - c(p)]g(I;p)dI . \qquad (11)$$

It should be noted that because of the assumption of Nash behaviour, the revenue service takes the reports of the taxpayers as fixed when it optimises. Effectively, this implies that $\bar{I}$, $g(I;p)$ and $M(I;p)$ are treated by the revenue service as constant.

The existence of a solution to the maximisation decision of the revenue service is demonstrated in the next lemma.

Lemma 3
There exists $p^* \in (0,1)$ such that p^* maximises $R(p)$ over $p \in [0,1]$.

Proof
The continuity of $R(p)$ and the compactness of the set from which p must be chosen ensure that a maximum exists. From Assumption 2, $p^* < 1$. To show that $p^* > 0$, note that

$$\frac{\partial R}{\partial p}|_{p=0} = \int_{-\infty}^{\max\{\underline{Y},\bar{I}\}} ft[M(I;p) - I]g(I;p)dI . \tag{12}$$

Since the set of tax evaders will be non-empty when $p = 0$, the integral in (12) will be over a set of positive measure for each of whom $M(I;p) - I > 0$. Therefore $\frac{\partial R}{\partial p}|_{p=0} > 0$ which proves the lemma. ||

The corollary follows directly.

Corollary 1
With a constant probability of audit, there exists a social equilibrium of verification and reporting.

Proof
Lemma 1 proved that a social equilibrium existed for any given p. Lemma 3 has now shown there exists an optimal choice of p. The aggregate reaction of the taxpayers is continuous in p and R is continuous in $M(I;p)$ and $g(I;p)$. The choice of the revenue service is therefore continuously dependent on the reaction of the taxpayers. Furthermore, the optimal probability is drawn from the compact set $[0,1]$. The lower limit of declarations, $K(p)$, is continuous in p and, over the compact set $[0,1]$, has a minimum $\underline{K}(p)$. The choice set of the taxpayers can then be restricted to the compact set $[\underline{K}(p), \bar{Y}]$. Under these conditions, the Nash equilibrium must exist. ||

Given the existence of an equilibrium, it is now possible to characterise the optimal audit probability and to investigate the effect of changes in the economic environment upon this. Differentiating (11) with respect to p, the first-order condition for the optimal choice of the revenue service is given by

$$\int_{-\infty}^{\max\{\underline{Y},\bar{I}\}} ft[M(I;p) - I]g(I;p)dI = c'(p), \tag{13}$$

where the normalisation assumption 2.iii has been employed to write $\int_{-\infty}^{\infty} c'(p)g(I;p)dI = c'(p)$. The interpretation of (13) is that the revenue service chooses the audit probability to balance the marginal return from increased auditing

of those already evading tax with the marginal cost. It is important to note that, as a consequence of the Nash behaviour, the revenue service does not take account of how a change in audit probability affects the reporting behaviour of those already evading tax nor of how the change in probability affects the decision to evade tax. Since an increase in p provides an incentive for evaders to increase their reported income and encourages marginal evaders to report their full income, the left-hand side of (13) understates the true benefit from increased auditing. As a result, the Nash equilibrium leads the revenue service to audit *less* than it would choose to do it were a Stackelberg leader.

The effects of parametric changes in the tax and fine rates upon the optimal audit probability are derived in the appendix assuming the optimal probability of audit satisfies the inequality $pf < 1$ so that there is some evasion. Under a set of reasonable assumptions, these suggest that the audit probability will increase if the fine rate increase but will decrease when the tax rate increases. The first of these findings is in direct contrast to the results of Kolm (1973) and Christiansen (1980) that show the two instruments are substitutes with the optimal policy consisting of an infinite punishment and a infinitesimal probability of detection. The reason for this differing conclusions is that in the Christiansen/Kolm analysis the objective is the minimisation of evasion by an authority that controls p and f. In contrast, in the present analysis p is chosen to maximise revenue with f taken as parametric and the revenue service, being concerned only with revenue maximisation, is not interested in minimising evasion as an end in itself. The tax rate result is explained by the observation that the increase in t raises the declaration of each tax evader (although it may add new tax evaders at the margin) and this is sufficient to induce a reduction in audit probability.

4. The optimal audit function

As may be expected, a number of additional difficulties arise once the constancy of the audit function is relaxed. Firstly, the expected utility function of the taxpayer need not be concave and continuity of the optimal decision may be lost. Secondly, the conditions guaranteeing an interior solution to the evasion decision need to be strengthened; from (3) it can be seen that the shape of the audit function, as well as its level, will be relevant. Finally, the analysis of the previous section was sufficiently general to be applicable whether the social equilibrium was interior or occurred at a corner and also allowed none or all taxpayers to be located at corner solutions. With

the extension to a general audit function it will not be possible to achieve this level of generality.

In response to similar difficulties, Reinganum and Wilde (1986) assumed linear utility, that the second-order condition for taxpayer maximisation was valid and that the equilibrium audit function was such that all taxpayers had an interior solution to the choice of evasion level. They did not demonstrate whether this interior equilibrium was dominated by audit functions that lead to corner solutions for all taxpayers. Of course, such a discrete comparison cannot be conducted with any degree of generality. Although restrictive, this is perhaps the best that can be achieved in the circumstances. In consequence, the optimal audit function is now constructed on the basis that the utility function is linear in income and that, in equilibrium, the taxpayers objective is concave with those who choose to evade being at an interior solution; that is, (3) holds as an equality for all taxpayers declaring $I < Y$.

Given these preliminaries, Lemma 4 follows directly from the assumption that (3) is an equality.

Lemma 4
For $I \in \left(-\infty, \bar{I}\right)$, $p'(I) < 0$.

Proof
If the evasion choice is to be interior, it follows from concavity that this requires $\frac{\partial U^E}{\partial I}\Big|_{I=Y} < 0$. When $I = Y$, $W = Z$. Hence from (3) $\frac{\partial U^E}{\partial I}\Big|_{I=Y} < 0 \equiv pf < 1$. Since this must hold for all I and the linearity of utility implies that $U'(W) = U'(Z)$ at an interior solution, (3) can only be satisfied if $p'(I) < 0$. ||

It should be noted that this direct argument is at the heart of the analysis of Reinganum and Wilde (1986) and that the decreasing audit probability follows directly from the assumption of an interior solution. Its consequences is that the revenue service chooses an audit function that is decreasing over the interval $\left(-\infty, \bar{I}\right)$. The motivation behind the choice of a decreasing audit probability is the incentive it gives the tax evader to increase their report in order to benefit from a reduced likelihood of detection.

Given Lemma 4, the monotonicity of $I = I(Y, p(I))$ in Y can be addressed by differentiation of (3) with respect to I and Y. Doing this shows that

$$\frac{dY}{dI} = \frac{S}{p'U'ft} > 0, \tag{14}$$

where $S < 0$ is the second-order condition for the optimisation. Therefore, the decreasing probability of audit ensures that higher income levels are connected with higher income reports. This monotonicity also has the implication that the constructions given for $g(I, p(\cdot))$ in (a)-(d) and $M(I, p(\cdot))$ in $(\alpha)-(\gamma)$ remain valid but with the replacement of p by the function $p(\cdot)$. The monotonicity result also shows that revenue is still given by (11) but again with the extension that p is a function of I.

To construct the form of the optimal audit function, evaluate revenue at the optimal $p(I)$ and consider a small variation $\delta p(I)$ in the optimal function. The assumed optimality of $p(I)$ implies that this variation should have no effect upon the level of revenue. Calculating the variation, under the assumption of Nash behaviour, shows that the latter statement is equivalent to

$$[ft[M - I] - c']g = 0, \text{ for } I \in \left(-\infty, \max\{\underline{I}, \bar{I}\}\right], \tag{15}$$

and

$$c'g = 0, \text{ for } I > \max\{\underline{I}, \bar{I}\}. \tag{16}$$

The form of (15) shows that the extra flexibility admitted by the allowing the probability of audit to vary is employed to equate the marginal return from auditing to the marginal cost at every report level. In contrast, condition (13) equates the marginal benefit averaged across income reports to the marginal cost.

From (15) and (16), the following theorem can be proved.

Theorem 1
For $I > \bar{I}$, $p(I) = 0$ and $\lim_{I \to \bar{I}} p(I) = 0$.

Proof
For $I > \max\{\underline{I}, \bar{I}\}$ the first statement of the theorem can be seen directly from Assumption 3 and (16). If $\underline{I} > \bar{I}$, since $g(I, p(\cdot)) = 0$ for $I \in \{\underline{I}, \bar{I}\}$, $p(I) = 0$ trivially satisfies (15) and any other choice of $p(\cdot)$ must lead to lower revenue. To prove the second part note that Assumption 2iv implies that the distribution satisfies $\lim_{\Psi \to \bar{Y}} \int_0^{\beta(Y,\mu,p(\cdot))} h(b, Y)db = 0$ and that at $\bar{I}$, $M = I$ for all $b > \beta(I, \mu, p(\cdot))$. Hence $\lim_{\Psi \to \bar{I}} ft[M - I]g = 0$, giving $c'g = 0$. The second part of the theorem then follows from Assumption 3. ||

Combining Lemma 5 and Theorem 1 shows that the audit function is a positive but decreasing function over the range $I \in \left(-\infty, \bar{I}\right)$ and is then 0 for $I \geq \bar{I}$. This audit function is efficient in the sense that no resources are spent auditing income reports that are known to originate from truthful revelation. However, if the revenue service could act as a Stackelberg leader, the reactions of the taxpayers (i.e. increased reports) would tend to raise the probability at each point and consequently the cut-off point. The equilibrium does ensure that all those evading tax stand a non-zero probability of being audited in contrast to the cut-off rule of Scotchmer (1987) where dishonest reports above the cut-off are not audited.

Turning now to the comparative statics of the optimal audit function, over the range where the function is positive, effects of variations in the underlying parameters can be found. These are stated as Lemma 5.

Lemma 5

The effect of an increase in the fine rate upon the detection probability is given by

$$\frac{dp}{df} = \frac{t[M - I]}{ftM_p - c''} > 0, \tag{17}$$

and the effect of an increase in the tax rate is

$$\frac{dp}{dt} = \frac{f[M - I]}{ftM_p - c''} > 0. \tag{18}$$

Proof

Directly from (15). ||

These results support the contentions discussed following (13). An increase in the fine rate raises the probability of audit as does an increase in the tax rate. In this model where the audit probability is determined by an independent revenue service, an increase in auditing is a complement to an increase in the fine rather than being a substitute.

5. Conclusions

The paper has considered the optimal audit policy for an independent revenue service in a social custom model of tax evasion. The fact that the revenue service could not always distinguish between evaders and non-evaders resulted in a signalling game of incomplete information. For the constant probability of audit it was shown that an interior solution existed to the decision problem of the revenue service and some

comparative statics results were given. These results, which were confirmed for the case of a general audit function, showed that the audit probability was a complementary instrument to the fine rate and the tax rate. This is in marked contrast to the outcome that arises when a central government agency controls all three instruments. When the audit probability could vary the audit function was proved to be a decreasing function of the income report, reaching zero at the highest income report of a tax evader. Given the additional complication caused by the existence of the social custom, it is surprising that this characterisation is stronger than that obtained without it.

Appendix

To develop the comparative statics of the optimal audit probability, it is necessary to employ (4) to analyse $b = \beta(Y, \mu, p, t, f)$, (3) to study the relation $Y = m(I, p, t, f)$, (8) to determine the equilibrium function $\mu = \mu(p, t, f)$ and then (a)-(d) and (α)-(γ) to construct $g(I; p, f, t)$ and $M(I; p, f, t)$. Given this information, it is then possible to compute the total derivative of (13).

From (4), total differentiation gives

$$\frac{db}{d\mu} \equiv \beta_\mu = \frac{bR'}{R} > 0, \tag{A1}$$

$$\frac{db}{dp} \equiv \beta_p = \frac{U(Z) - U(W)}{R} < 0, \tag{A2}$$

$$\frac{db}{df} \equiv \beta_f = \frac{-ptU'(Z)[Y - I]}{R} < 0, \tag{A3}$$

$$\frac{db}{dt} \equiv \beta_t = \frac{Y[U'(X) - pfU'(Z)]}{R}, \tag{A4}$$

and

$$\frac{db}{dY} \equiv \beta_Y = \frac{[1-t][pfU'(Z) - U'(X)]}{R}. \tag{A5}$$

Under the assumptions made to this point, neither β_t or β_Y is signed. However, the analysis of Myles and Naylor (1992) shows that the model predicts results in accord with empirical evidence when $pfU'(Z) - U'(X) > 0$. Adopting this restriction, $\beta_t < 0$ and $\beta_Y > 0$.

Employing the first-order condition (3) shows that

$$\frac{dY}{dI} \equiv m_I = -\frac{S}{[1-p]U'(W)[[ft-1]R_A(Z)+R_A(W)]} > 0, \tag{A6}$$

where S is the second-order condition for the optimisation,

$$\frac{dY}{df} \equiv m_f = \frac{[f-1]pU''(Z)[Y-1]}{[1-p]U'(W)[[ft-1]R_A(Z)+R_A(W)]} < 0, \tag{A7}$$

$$\frac{dY}{dp} \equiv m_p = -\frac{[[f-1]U'(Z)+U'(W)]}{[1-p]U'(W)[[ft-1]R_A(Z)+R_A(W)]} < 0, \tag{A8}$$

and

$$\frac{dY}{dt} \equiv m_t = I > 0. \tag{A9}$$

The equilibrium identity (8) then provides the results that

$$\frac{d\mu}{dp} \equiv \mu_p = \frac{\int_{\underline{Y}}^{\bar{Y}} h(\beta,Y)\beta_p dY}{1-\int_{\underline{Y}}^{\bar{Y}} h(\beta,Y)\beta_\mu dY} < 0, \tag{A10}$$

$$\frac{d\mu}{dt} \equiv \mu_t = \frac{\int_{\underline{Y}}^{\bar{Y}} h(\beta,Y)\beta_t dY}{1-\int_{\underline{Y}}^{\bar{Y}} h(\beta,Y)\beta_\mu dY} < 0, \tag{A11}$$

and

$$\frac{d\mu}{df} \equiv \mu_f = \frac{\int_{\underline{Y}}^{\bar{Y}} h(\beta,Y)\beta_f dY}{1-\int_{\underline{Y}}^{\bar{Y}} h(\beta,Y)\beta_\mu dY} < 0, \tag{A12}$$

where the signs follow from (A1) to (A4) and the stability requirement $1-\int_{\underline{Y}}^{\bar{Y}} h(\beta,Y)\beta_\mu dY > 0$.

In the comparative statics analysis of (13) two cases can arise depending on whether $\bar{I}$ is less than, or greater than, $\underline{Y}$. Only the case of $\bar{I} < \underline{Y}$ will be treated here; the analysis of the other is a simple extension. Since $\bar{I} < \underline{Y}$, $g(I;p,f,t)=0$ for $I \in (\bar{I},\underline{Y}]$, and for $I \in (-\infty,\bar{I}]$ it follows that $M(I;p,f,t)=m(I;p,f,t)$. The optimality condition (13) can then be written as

$$\int_{-\infty}^{\bar{I}(p,f,t)} ft[m(I;p,f,t)-I]g(I;p,f,t)dI - c'(p) = 0. \tag{A13}$$

Since a report of $\bar{I}$ must arise from a tax evader with an income of $\bar{Y}$, it follows from Assumption 2.iv that $g(\bar{I}; p, f, t) = 0$. Employing these restrictions, total differentiation of (A13) gives

$$\frac{dp}{dt} = -\frac{\int_{-\infty}^{\bar{I}} ftm_t dI + \int_{-\infty}^{\bar{I}} ft[m-I]g_t dI + \int_{-\infty}^{\bar{I}} f[m-I]gdI}{\int_{-\infty}^{\bar{I}} ftm_p dI + \int_{-\infty}^{\bar{I}} ft[m-I]g_p dI - c''}, \tag{A14}$$

and

$$\frac{dp}{df} = -\frac{\int_{-\infty}^{\bar{I}} ftm_f dI + \int_{-\infty}^{\bar{I}} ft[m-I]g_f dI + \int_{-\infty}^{\bar{I}} t[m-I]gdI}{\int_{-\infty}^{\bar{I}} ftm_p dI + \int_{-\infty}^{\bar{I}} ft[m-I]g_p dI - c''}. \tag{A15}$$

To sign these, note from (a) that

$$g_p = h(\beta, m)[\beta_m m_p + \beta_\mu \mu_p + \beta_p] + \int_0^B h_m m_p db, \tag{A16}$$

$$g_t = h(\beta, m)[\beta_m m_t + \beta_\mu \mu_t + \beta_t] + \int_0^B h_m m_t db, \tag{A17}$$

and

$$g_f = h(\beta, m)[\beta_m m_f + \beta_\mu \mu_f + \beta_f] + \int_0^B h_m m_f db. \tag{A18}$$

These expressions capture the two effects of a change: it changes the upper limit of the integral and changes the true level of income associated to each report. Although there is an expectation for an income distribution that $h_m < 0$, this need not necessarily hold in the present setting since the derivatives here are taken with b constant. To allow clear results to be derived, assume that $h_m = 0$. This implies from (A1)-(A5) and (A10)-(A12) that $g_p < 0$ and $g_f < 0$ but still leaves g_t unsigned.

Returning to (A14) and (A15) it can be seen that the denominator of both expressions is negative. In the denominator of (A15) the first two terms are negative, since an increased fine raises compliance, whilst the third is positive as it represents the increased income from the raising of the fine rate. Although the net effect cannot be restricted, there is a strong possibility that an increase in the fine rate will raise the probability of detection. In the denominator of (A14) the first and third terms are positive, with the sign of the second being uncertain. The expectation is therefore that an increase in the tax rate will reduce the optimal audit probability.

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