Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2015, Article ID 797542, 8 pages
http://dx.doi.org/10.1155/2015/797542

Research Article

Mathematical Analysis for a Discrete Predator-Prey Model with Time Delay and Holling II Functional Response

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Received 31 May 2014; Revised 19 August 2014; Accepted 28 August 2014

Academic Editor: Zbigniew Leśniak

This paper is concerned with a discrete predator-prey model with Holling II functional response and delays. Applying Gaines and Mawhin's continuation theorem of coincidence degree theory and the method of Lyapunov function, we obtain some sufficient conditions for the existence global asymptotic stability of positive periodic solutions of the model.

1. Introduction

In recent years, numerous studies have been carried out on predator-prey interactions using Lotka-Volterra type functional response [1]. Considering the simplification of assumptions on prey searching, prey consumption, and environmental complexity, Holling suggested three different kinds of functional response to model more realistic predator-prey interactions than what is possible with the standard Lotka-Volterra type response [1, 2]. Many predator-prey systems with Holling type II functional response have been investigated. In particular, the periodic solutions are of great interest. During the past decades, a large number of excellent results have been reported for a lot of different predator-prey models with Holling type II functional response. For example, Ko and Ryu [3] investigated the qualitative analysis of a predator-prey model with Holling type II functional response incorporating a prey refuge. Zhou and Shi [4] considered the existence, bifurcation, and stability of positive stationary solutions of a diffusive Leslie-Gower predator-prey model with Holling type II functional responses. Liu and Yan [5] dealt with the positive periodic solutions for a neutral delay ratio-dependent predator-prey model with a Holling type II functional response. For more related work, one can see [6–25]. Dunkel [26] pointed out that feedback control item in predator-prey models depends on the population number for certain time past and also depends on the average of the population number for a period of time past. Motivated by the viewpoint, we proposed the following predator-prey model with Holling II functional response and distributed delays:

$$\begin{aligned} \frac{dx_1}{dt} &= x_1(t)\left[r_1(t) - \int_{-\infty}^{t} k_1(s-t)\,x_1(s)\,ds\right] \\ &\quad - x_1(t)\int_{-\infty}^{t} \frac{k_2(s-t)\,x_2(s)}{1+mx_2(t)}ds, \\ \frac{dx_2}{dt} &= x_2(t)\left[-r_2(t) - \int_{-\infty}^{t} k_3(s-t)\,x_2(s)\,ds\right] \\ &\quad + x_2(t)\int_{-\infty}^{t} \frac{k_4(s-t)\,x_1(s)}{1+mx_1(t)}ds, \end{aligned} \tag{1}$$

where $x_i(t)$ $(i = 1, 2)$ stands for the prey and predator density at time t. For the biological meaning of model (1), one can see [27].

As pointed out in [28–35], discrete time models are more appropriate to describe the dynamics relationship among populations than continuous ones when the populations have nonoverlapping generations. What is more, we can also get more accurate numerical simulation results from the discrete-time systems. Thus it is reasonable and interesting to investigate discrete-time systems governed by difference equations. Following [33, 36], we obtain the discrete form of system (1) as follows:

$$\begin{aligned} x_1(k+1) &= x_1(k) \\ &\times \exp\left\{\left[r_1(k) - \sum_{l=0}^{+\infty} k_1(-l)\, x_1(k-l) \right.\right. \\ &\left.\left. - \sum_{l=0}^{+\infty} \frac{k_2(-l)\, x_2(k-l)}{1+m x_2(k-l)}\right]\right\}, \\ x_2(k+1) &= x_2(k) \\ &\times \exp\left\{\left[-r_2(k) - \sum_{l=0}^{+\infty} k_3(-l)\, x_2(k-l) \right.\right. \\ &\left.\left. + \sum_{l=0}^{+\infty} \frac{k_4(-l)\, x_1(k-l)}{1+m x_1(k)}\right]\right\}, \end{aligned} \tag{2}$$

which is a discrete time analogue of system (1), where $k = 0, 1, 2, \ldots$, $x_i(k)$ $(i = 1, 2)$ stands for the prey and predator density at time k, $r_i(k), k_j(k)$ $(i = 1, 2,\ j = 1, 2, 3, 4)$ are strictly positive sequences, and m is a positive constant.

In order to obtain our main results, we assume that

(H1) $r_i : Z \to R^+$ is positive ω-periodic; that is, $r_i(k + \omega) = r_i(k)$ $(i = 1, 2)$ for any $k \in Z$, where ω, a fixed positive integer, denotes the common period of the parameters in system (2);

(H2) the following inequalities are satisfied:

$$0 < \sum_{l=0}^{+\infty} k_i(-l) < +\infty \quad (i = 1, 2, 3, 4). \tag{3}$$

The principle aim of this paper is to discuss the effect of the periodicity of the ecological and environmental parameters on the dynamics of discrete time predator-prey model with Holling II functional response and distributed delays.

The paper is organized as follows. In Section 2, applying the coincidence degree and the related continuation theorem, a series of sufficient conditions to ensure the existence of positive solutions of difference equations are given. In Section 3, by means of the method of Lyapunov function, a set of sufficient conditions for the global asymptotic stability of the model are established. Some numerical simulations are given to illustrate the theoretical results in Section 4.

2. Existence of Positive Periodic Solutions

Throughout the paper, we always use the notations below:

$$I_\omega := \{0, 1, 2, \ldots, \omega - 1\}, \qquad \overline{f} := \frac{1}{\omega} \sum_{k=0}^{\omega-1} f(k), \tag{4}$$

where $f(k)$ is an ω-periodic sequence of real numbers defined for $k \in Z$. In order to explore the existence of positive periodic solutions of (2) and for the reader's convenience, we will first summarize below a few concepts and results without proof, borrowing from [37].

Let X, Y be normed vector spaces, let $L : \operatorname{Dom} L \subset X \to Y$ be a linear mapping, and let $N : X \to Y$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim\operatorname{Ker} L = \operatorname{codim}\operatorname{Im} L < +\infty$ and $\operatorname{Im} L$ is closed in Y. If L is a Fredholm mapping of index zero and there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$ such that $\operatorname{Im} P = \operatorname{Ker} L, \operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q)$, it follows that $L \mid \operatorname{Dom} L \cap \operatorname{Ker} P : (I - P)X \to \operatorname{Im} L$ is invertible. We denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_P(I-Q)N : \overline{\Omega} \to X$ is compact. Since $\operatorname{Im} Q$ is isomorphic to $\operatorname{Ker} L$, there exists an isomorphism $J : \operatorname{Im} Q \to \operatorname{Ker} L$.

Lemma 1 ([37] continuation theorem). *Let L be a Fredholm mapping of index zero and let N be L-compact on $\overline{\Omega}$. Suppose*

(a) *for each $\lambda \in (0, 1)$ every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$;*

(b) *$QNx \neq 0$ for each $x \in \operatorname{Ker} L \bigcap \partial\Omega$, and $\deg\{JQN, \Omega \bigcap \operatorname{Ker} L, 0\} \neq 0$.*

Then the equation $Lx = Nx$ has at least one solution lying in $\operatorname{Dom} L \bigcap \overline{\Omega}$.

Lemma 2 (see [33]). *Let $g : Z \to R$ be ω-periodic; that is, $g(k + \omega) = g(k)$; then for any fixed $k_1, k_2 \in I_\omega$ and any $k \in Z$, one has*

$$\begin{aligned} g(k) &\le g(k_1) + \sum_{s=0}^{\omega-1} \left| g(s+1) - g(s) \right|, \\ g(k) &\ge g(k_2) - \sum_{s=0}^{\omega-1} \left| g(s+1) - g(s) \right|. \end{aligned} \tag{5}$$

Lemma 3. *$(\hat{x}_1(k), \hat{x}_2(k))$ is an ω-periodic solution of (2) with strictly positive components if and only if $(\ln(\hat{x}_1(k)), \ln(\hat{x}_2(k)))$ is an ω-periodic solution of*

$$\begin{aligned} x_1(k+1) - x_1(k) &= r_1(k) - \sum_{l=0}^{+\infty} k_1(-l) \exp\left(x_1(k-l)\right) \\ &\quad - \sum_{l=0}^{+\infty} \frac{k_2(-l)\, x_2(k-l)}{1 + m x_2(k-l)}, \\ x_2(k+1) - x_2(k) &= -r_2(k) - \sum_{l=0}^{+\infty} k_3(-l) \exp\left(x_2(k-l)\right) \\ &\quad + \sum_{l=0}^{+\infty} \frac{k_4(-l) \exp\left(x_1(k-l)\right)}{1 + m \exp\left(x_1(k)\right)}. \end{aligned} \tag{6}$$

The proofs of Lemma 3 are trivial, so we omitted the details here.

Define

$$l_2 = \left\{z = \left\{z(k) : z(k) \in R^2, k \in Z\right\}\right\}. \tag{7}$$

For $a = (a_1, a_2)^T \in R^2$, define $|a| = \max\{|a_1|, |a_2|\}$. Let $l^\omega \subset l_2$ denote the subspace of all ω-periodic sequences equipped with the usual supremum norm $\|\cdot\|$, that is, $\|z\| = \max_{k\in I_\omega}|z(k)|$, for any $z = \{z(k) : k \in Z\} \in l^\omega$. It is easy to show that l_ω is a finite-dimensional Banach space.

Let

$$\begin{aligned} l_0^\omega &= \left\{z = \{z(k)\} \in l^\omega : \sum_{k=0}^{\omega-1} z(k) = 0\right\}, \\ l_c^\omega &= \left\{z = \{z(k)\} \in l^\omega : z(k) = h \in R^2, k \in Z\right\}, \end{aligned} \tag{8}$$

and then it follows that l_0^ω and l_c^ω are both closed linear subspaces of l^ω and

$$l^\omega = l_0^\omega + l_c^\omega, \quad \dim l_c^\omega = 2. \tag{9}$$

Next, we will be ready to establish our result.

Theorem 4. *Suppose that (H1), (H2), and (H3)* $\overline{r}_1 > (1/m)\sum_{l=0}^{+\infty} k_2(-l)$ *hold. Then system (2) has at least an* ω*-periodic solution with positive components.*

Proof. Let $X = Y = l^\omega$,

$$\begin{aligned} (Lz)(k) &= z(k+1) - z(k) = \begin{bmatrix} x_1(k+1) - x_1(k) \\ x_2(k+1) - x_2(k) \end{bmatrix}, \\ (Nz)(k) &= \begin{bmatrix} f_1(k) \\ f_2(k) \end{bmatrix}, \end{aligned} \tag{10}$$

where $z \in X, k \in Z$ and

$$\begin{aligned} f_1(k) &= r_1(k) - \sum_{l=0}^{+\infty} k_1(-l)\exp(x_1(k-l)) \\ &\quad - \sum_{l=0}^{+\infty} \frac{k_2(-l)\exp(x_2(k-l))}{1 + m\exp(x_2(k-l))}, \\ f_2(k) &= -r_2(k) - \sum_{l=0}^{+\infty} k_3(-l)\exp(x_2(k-l)) \\ &\quad + \sum_{l=0}^{+\infty} \frac{k_4(-l)\exp(x_1(k-l))}{1 + m\exp(x_1(k))}. \end{aligned} \tag{11}$$

Then it is trivial to see that L is a bounded linear operator and

$$\begin{gathered} \operatorname{Ker} L = l_c^\omega, \qquad \operatorname{Im} L = l_0^\omega, \\ \dim\operatorname{Ker} L = 2 = \operatorname{codim}\ \operatorname{Im}\ L. \end{gathered} \tag{12}$$

It follows that L is a Fredholm mapping of index zero. Define

$$\begin{aligned} Py &= \frac{1}{\omega}\sum_{s=0}^{\omega-1} y(s), \quad y \in X, \\ Qz &= \frac{1}{\omega}\sum_{s=0}^{\omega-1} z(s), \quad z \in Y. \end{aligned} \tag{13}$$

It is not difficult to show that P and Q are continuous projectors such that

$$\operatorname{Im} P = \operatorname{Ker} L, \qquad \operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q). \tag{14}$$

Furthermore, the generalized inverse (to L) $K_P : \operatorname{Im} L \to \operatorname{Ker} P \bigcap \operatorname{Dom} L$ exists and is given by

$$K_P(z) = \sum_{s=0}^{\omega-1} z(s) - \frac{1}{\omega}\sum_{s=0}^{\omega-1}(\omega - s) z(s). \tag{15}$$

Obviously, QN and $K_P(I-Q)N$ are continuous. Since X is a finite-dimensional Banach space, it is not difficult to show that $\overline{K_P(I-Q)N(\overline{\Omega})}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\overline{\Omega})$ is bounded. Thus, N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset X$.

Now we are at the point to search for an appropriate open, bounded subset Ω for the application of the continuation theorem. Corresponding to the operator equation $Lz = \lambda Nz, \lambda \in (0, 1)$, we have

$$\begin{aligned} x_1(k+1) - x_1(k) &= \lambda\left[r_1(k) - \sum_{l=0}^{+\infty} k_1(-l)\exp(x_1(k-l)) \right. \\ &\quad \left. - \sum_{l=0}^{+\infty} \frac{k_2(-l)\exp(x_2(k-l))}{1 + m\exp(x_2(k-l))}\right], \\ x_2(k+1) - x_2(k) &= \lambda\left[-r_2(k) - \sum_{l=0}^{+\infty} k_3(-l)\exp(x_2(k-l)) \right. \\ &\quad \left. + \sum_{l=0}^{+\infty} \frac{k_4(-l)\exp(x_1(k-l))}{1 + m\exp(x_1(k))}\right]. \end{aligned} \tag{16}$$

Suppose that $z(k) = (x_1(k), x_2(k))^T \in X$ is an arbitrary solution of system (16) for a certain $\lambda \in (0, 1)$; summing both sides of (16) from 0 to $\omega - 1$ with respect to k, respectively, we obtain

$$\begin{aligned} &\sum_{k=0}^{\omega-1}\left[\sum_{l=0}^{+\infty} k_1(-l)\exp(x_1(k-l)) + \sum_{l=0}^{+\infty} \frac{k_2(-l)\exp(x_2(k-l))}{1 + m\exp(x_2(k-l))}\right] = \overline{r}_1\omega, \\ &\sum_{k=0}^{\omega-1}\left[\sum_{l=0}^{+\infty} k_3(-l)\exp(x_2(k-l)) - \sum_{l=0}^{+\infty} \frac{k_4(-l)\exp(x_1(k-l))}{1 + m\exp(x_1(k))}\right] = \overline{r}_2\omega. \end{aligned} \tag{17}$$

It follows from (16) and (17) that

$$\sum_{k=0}^{\omega-1}\left|x_1(k+1)-x_1(k)\right| \le 2\bar{r}_1\omega,$$
$$\sum_{k=0}^{\omega-1}\left|x_2(k+1)-x_2(k)\right| \le 2\bar{r}_2\omega. \tag{18}$$

In view of the hypothesis that $z = \{z(k)\} \in X$, there exist $\xi_i, \eta_i \in I_\omega$ such that

$$x_i(\xi_i) = \min_{k\in I_\omega}\{x_i(k)\}, \quad x_i(\eta_i) = \max_{k\in I_\omega}\{x_i(k)\} \quad (i = 1,2). \tag{19}$$

By (17), we have

$$\sum_{l=0}^{+\infty} k_1(-l)\exp(x_1(\xi_1)) \le \sum_{l=0}^{+\infty} k_1(-l)\exp(x_1(k-l)) < \bar{r}_1\omega,$$
$$\sum_{l=0}^{+\infty} k_3(-l)\exp(x_2(\eta_2)) \ge \sum_{l=0}^{+\infty} k_3(-l)\exp(x_2(k-l)) > \bar{r}_2\omega,$$
$$\sum_{l=0}^{+\infty} k_1(-l)\exp(x_1(\eta_1))\omega + \frac{1}{m}\sum_{l=0}^{+\infty} k_2(-l)\omega \ge \bar{r}_1\omega,$$
$$\sum_{l=0}^{+\infty} k_3(-l)\exp(x_2(\xi_2)) - \frac{1}{m}\sum_{l=0}^{+\infty} k_4(-l) \le \bar{r}_2\omega. \tag{20}$$

Thus

$$x_1(\xi_1) < \ln\left[\frac{\bar{r}_1}{\sum_{l=0}^{+\infty} k_1(-l)}\right] := \alpha_1,$$
$$x_2(\eta_2) > \ln\left[\frac{\bar{r}_2}{\sum_{l=0}^{+\infty} k_3(-l)}\right] := \beta_2,$$
$$x_1(\eta_1) > \ln\left[\frac{\bar{r}_1 - (1/m)\sum_{l=0}^{+\infty} k_2(-l)}{\sum_{l=0}^{+\infty} k_1(-l)}\right] := \alpha_2,$$
$$x_2(\xi_2) < \ln\left[\frac{\bar{r}_2 + (1/m)\sum_{l=0}^{+\infty} k_4(-l)}{\sum_{l=0}^{+\infty} k_3(-l)}\right] := \beta_1. \tag{21}$$

It follows from (18), (21), and Lemma 2 that

$$x_1(k) \le x_1(\xi_1) + \sum_{s=0}^{\omega-1}\left|x_1(s+1)-x_1(s)\right| \le \alpha_1 + 2\bar{r}_1\omega := \Theta_1,$$
$$x_1(k) \ge x_1(\eta_1) - \sum_{s=0}^{\omega-1}\left|x_1(s+1)-x_1(s)\right| \ge \alpha_2 - 2\bar{r}_1\omega := \Theta_2,$$
$$x_2(k) \le x_2(\xi_2) + \sum_{s=0}^{\omega-1}\left|x_2(s+1)-x_2(s)\right| \le \beta_1 + 2\bar{r}_2\omega := \Theta_3,$$
$$x_2(k) \ge x_2(\eta_2) - \sum_{s=0}^{\omega-1}\left|x_2(s+1)-x_2(s)\right| \ge \beta_2 - 2\bar{r}_2\omega := \Theta_4. \tag{22}$$

In view of (22), we derive

$$\max_{k\in I_\omega}\{x_1(k)\} \le \max\{|\Theta_1|, |\Theta_2|\} := \Upsilon_1,$$
$$\max_{k\in I_\omega}\{x_2(k)\} \le \max\{|\Theta_4|, |\Theta_5|\} := \Upsilon_2. \tag{23}$$

Obviously, Υ_i $(i = 1,2)$ are independent of $\lambda \in (0,1)$. Take $M = \max\{\Upsilon_1, \Upsilon_2\} + M_0$, where M_0 is taken sufficiently large such that $\max\{|\ln(x_1^*)|, |\ln(x_2^*)|\} < M_0$, where $(x_1^*, x_2^*)^T$ is the unique positive solution of (6). Now we have proved that any solution $z = \{z(k)\} = \{(x_1(k), x_2(k))^T\}$ of (16) in X satisfies $\|z\| < M, k \in Z$.

Let $\Omega := \{z = \{z(k)\} \in X : \|z\| < M\}$; then it is easy to see that Ω is an open, bounded set in X and verifies requirement (a) of Lemma 1. When $z \in \partial\Omega \cap \operatorname{Ker} L, z = \{(x_1, x_2)^T\}$ is a constant vector in R^2 with $\|z\| = \max\{|x_1|, |x_2|\} = M$. Then

$$QNz = \begin{bmatrix}\chi_1\\ \chi_2\end{bmatrix} \ne 0, \tag{24}$$

where

$$\chi_1 = \bar{r}_1 - \sum_{l=0}^{+\infty} k_1(-l)\exp(x_1) - \sum_{k=0}^{\omega-1}\sum_{l=0}^{+\infty}\frac{k_2(-l)\exp(x_2)}{1+m\exp(x_2)},$$
$$\chi_2 = -\bar{r}_2 - \sum_{l=0}^{+\infty} k_3(-l)\exp(x_2) + \sum_{k=0}^{\omega-1}\sum_{l=0}^{+\infty}\frac{k_4(-l)\exp(x_1)}{1+m\exp(x_1)}. \tag{25}$$

Now let us consider homotopic $\phi(x_1, x_2, \mu) = \mu QNz + (1-\mu)Gz, \mu \in [0,1]$, where

$$Gz = \begin{pmatrix}\bar{r}_1 - \sum_{l=0}^{+\infty} k_1(-l)\exp(x_1)\\ -\bar{r}_2 - \sum_{l=0}^{+\infty} k_3(-l)\exp(x_2)\end{pmatrix}. \tag{26}$$

Letting J be the identity mapping and by direct calculation, we get

$$\begin{aligned}
&\deg\left[JQN(x_1,x_2)^T;\ \Omega\bigcap\ker L;\ 0\right]\\
&=\deg\left[QN(x_1,x_2)^T;\ \Omega\bigcap\ker L;\ 0\right]\\
&=\deg\left[\phi\left(x_1,x_2,1\right);\ \Omega\bigcap\ker L;\ 0\right]\\
&=\deg\left[\phi\left(x_1,x_2,0\right);\ \Omega\bigcap\ker L;\ 0\right]\\
&=\operatorname{sign}\left\{\det\begin{bmatrix}\sum_{l=0}^{+\infty}k_1\left(-l\right)\exp\left(x_1^*\right) & 0\\ 0 & -\sum_{l=0}^{+\infty}k_3\left(-l\right)\exp\left(x_2^*\right)\end{bmatrix}\right\}\\
&=\operatorname{sign}\left[-\sum_{l=0}^{+\infty}k_1\left(-l\right)\sum_{l=0}^{+\infty}k_3\left(-l\right)\exp\left(x_2^*\right)\exp\left(x_1^*+x_2^*\right)\right]\\
&=-1\neq 0.
\end{aligned}\tag{27}$$

By now, we have proved that Ω verifies all requirements of Lemma 1; then it follows that $Lz = Nz$ has at least one solution in $\operatorname{Dom} L\cap\overline{\Omega}$; that is to say, (6) has at least one ω-periodic solution in $\operatorname{Dom} L\cap\overline{\Omega}$, say $z^*=\{z^*(k)\}=\{(x_1^*(k),x_2^*(k))^T\}$. Let $\overline{x}_1^*(k)=\exp(x_1^*(k))$, $\overline{x}_2^*(k)=\exp(x_2^*(k))$; then by Lemma 3 we know that $\overline{z}^*=\{\overline{x}^*(k)\}=\{\overline{x}_1^*(k),\overline{x}_2^*(k))^T\}$ is an ω-periodic solution of system (2) with strictly positive components. The proof is complete. □

3. Global Asymptotic Stability

Let the delays be zero; then (2) takes the form

$$\begin{aligned}
x_1\left(k+1\right)&=x_1\left(k\right)\exp\left\{\left[r_1\left(k\right)-\sum_{l=0}^{+\infty}k_1\left(-l\right)x_1\left(k\right)-\sum_{l=0}^{+\infty}\frac{k_2\left(-l\right)x_2\left(k\right)}{1+mx_2\left(k\right)}\right]\right\},\\
x_2\left(k+1\right)&=x_2\left(k\right)\exp\left\{\left[-r_2\left(k\right)-\sum_{l=0}^{+\infty}k_3\left(-l\right)x_2\left(k\right)+\sum_{l=0}^{+\infty}\frac{k_4\left(-l\right)x_1\left(k\right)}{1+mx_1\left(k\right)}\right]\right\}.
\end{aligned}\tag{28}$$

In this section, we will present sufficient conditions for the global asymptotic stability of system (28).

Theorem 5. *Assume that (H1) and (H2) are satisfied and furthermore suppose that there exist positive constants υ,μ_1 and μ_2 such that*

$$\begin{aligned}
\mu_1\left[\sum_{l=0}^{+\infty}k_1\left(-l\right)\right]-\mu_2\left[\frac{1}{\left(1+mx_1^*\right)^2}\right]&>\upsilon,\\
\mu_2\left[\sum_{l=0}^{+\infty}k_3\left(-l\right)\right]-\mu_1\left[\frac{1}{\left(1+mx_2^*\right)^2}\right]&>\upsilon.
\end{aligned}\tag{29}$$

Then the positive ω-periodic solution of system (28) *is globally asymptotically stable.*

Proof . Since the delays in system (2) have no effect on the periodic solution, then system (28) has a positive solution $(x_1^*(k),x_2^*(k))^T$. Now we prove below that it is uniformly asymptotically stable. First, we make the change of variable

$$u_i\left(k\right)=x_i\left(k\right)-x_i^*\left(k\right)\quad\left(i=1,2\right).\tag{30}$$

It follows from (28) that

$$\begin{aligned}
&u_1\left(k+1\right)\\
&=x_1\left(k+1\right)-x_1^*\left(k+1\right)\\
&=x_1\left(k\right)\exp\left\{\left[r_1\left(k\right)-\sum_{l=0}^{+\infty}k_1\left(-l\right)x_1\left(k\right)-\sum_{l=0}^{+\infty}\frac{k_2\left(-l\right)x_2\left(k\right)}{1+mx_2\left(k\right)}\right]\right\}\\
&\quad-x_1^*\left(k\right)\exp\left\{\left[r_1\left(k\right)-\sum_{l=0}^{+\infty}k_1\left(-l\right)x_1^*\left(k\right)-\sum_{l=0}^{+\infty}\frac{k_2\left(-l\right)x_2^*\left(k\right)}{1+mx_2^*\left(k\right)}\right]\right\}\\
&=\left\{x_1\left(k\right)\exp\left[\left(-\sum_{l=0}^{+\infty}k_1\left(-l\right)\right)u_1\left(k\right)-\left(\sum_{l=0}^{+\infty}\frac{k_2\left(-l\right)x_2\left(k\right)}{1+mx_2\left(k\right)}-\sum_{l=0}^{+\infty}\frac{k_2\left(-l\right)x_2^*\left(k\right)}{1+mx_2\left(k\right)}\right)\right]-x_1^*\left(k\right)\right\}\frac{x_1^*\left(k+1\right)}{x_1^*\left(k\right)}\\
&=\left\{\left[1-\sum_{l=0}^{+\infty}k_1\left(-l\right)x_1^*\left(k\right)\right]\frac{u_1\left(k\right)}{x_1^*\left(k\right)}-\frac{1}{\left(1+mx_2^*\right)^2}u_2\left(k\right)+\rho_1\right\}x_1^*\left(k+1\right),
\end{aligned}$$

$$\begin{aligned}
&u_2(k+1) \\
&= x_2(k+1) - x_2^*(k+1) \\
&= x_2(k)\exp\left\{\left[-r_2(k) - \sum_{l=0}^{+\infty} k_3(-l)\, x_2(k) + \sum_{l=0}^{+\infty} \frac{k_4(-l)\, x_1(k)}{1+m x_1(k)}\right]\right\} \\
&\quad - x_2^*(k)\exp\left\{\left[-r_2(k) - \sum_{l=0}^{+\infty} k_3(-l)\, x_2^*(k) + \sum_{l=0}^{+\infty} \frac{k_4(-l)\, x_1^*(k)}{1+m x_1(k)}\right]\right\} \\
&= \left\{x_2(k)\exp\left[\left(-\sum_{l=0}^{+\infty} k_3(-l)\right) u_2(k) - \left(\sum_{l=0}^{+\infty} \frac{k_4(-l)\, x_1(k)}{1+m x_1(k)} - \sum_{l=0}^{+\infty} \frac{k_4(-l)\, x_1^*(k)}{1+m x_1(k)}\right)\right] - x_2^*(k)\right\} \frac{u_2^*(k+1)}{x_2^*(k)} \\
&= \left\{\left[1 - \sum_{l=0}^{+\infty} k_3(-l)\, x_2^*(k)\right] \frac{u_2(k)}{x_2^*(k)} - \frac{1}{(1+m x_1^*)^2} u_1(k) + \rho_2\right\} x_2^*(k+1),
\end{aligned} \tag{31}$$

where $\|\rho_i\|/\|u\|$ $(i = 1, 2)$ converges to zero as $\|u\| \to 0$.

Define a function V by

$$V(N(k)) = \mu_1 \left|\frac{u_1(k)}{x_1^*(k)}\right| + \mu_2 \left|\frac{u_2(k)}{x_2^*(k)}\right|, \tag{32}$$

where μ_1 and μ_2 are all positive constants given by (34) and (35), respectively. Calculating the difference of V along the solution of system (31), we get

$$\begin{aligned}
\Delta V &= \mu_1 \left(\left|\frac{u_1(k+1)}{x_1^*(k+1)} - \frac{u_1(k)}{x_1^*(k)}\right|\right) + \mu_2 \left(\left|\frac{u_2(k+1)}{x_2^*(k+1)} - \frac{u_2(k)}{x_2^*(k)}\right|\right) \\
&\le -\mu_1 \left[\sum_{l=0}^{+\infty} k_1(-l)\right] |u_1(k)| + \mu_1 \left[\frac{1}{(1+m x_2^*)^2}\right] |u_2(k)| \\
&\quad - \mu_2 \left[\sum_{l=0}^{+\infty} k_3(-l)\right] |u_2(k)| + \mu_2 \left[\frac{1}{(1+m x_1^*)^2}\right] |u_1(k)| \\
&\le -\Delta_1 |u_1(k)| - \Delta_2 |u_2(k)|,
\end{aligned} \tag{33}$$

where

$$\Delta_1 = \mu_1 \left[\sum_{l=0}^{+\infty} k_1(-l)\right] - \mu_2 \left[\frac{1}{(1+m x_1^*)^2}\right], \tag{34}$$

$$\Delta_2 = \mu_2 \left[\sum_{l=0}^{+\infty} k_3(-l)\right] - \mu_1 \left[\frac{1}{(1+m x_2^*)^2}\right]. \tag{35}$$

It follows from the condition (29) that there exists a positive constant σ such that if k is sufficiently large and $\|u\| < \epsilon$, then

$$\Delta V \le -\frac{\sigma}{2} \{|u_1(k)| + |u_2(k)|\} < -\sigma\epsilon. \tag{36}$$

In view of Freedman [38], we can see that the trivial solutions of (31) are uniformly asymptotically stable and so is the solution $\{(x^*(k), y^*(k))^T\}$ of (28). Thus we can conclude that the positive periodic solution of (28) is globally asymptotically stable. The proof is complete. □

4. Numerical Example

In this section, we present some numerical results of system (2) to verify the analytical predictions obtained in the previous section. Let us consider the following discrete system:

$$\begin{aligned}
x_1(k+1) &= x_1(k)\exp\left\{\left[0.5 + 0.3\sin k\pi - \sum_{l=0}^{+\infty} k_1(-l)\, x_1(k-l) - \sum_{l=0}^{+\infty} \frac{k_2(-l)\, x_2(k-l)}{1+2x_2(k-l)}\right]\right\}, \\
x_2(k+1) &= x_2(k)\exp\left\{\left[-(0.6 + 0.2\sin k\pi) - \sum_{l=0}^{+\infty} k_3(-l)\, x_2(k-l) + \sum_{l=0}^{+\infty} \frac{k_4(-l)\, x_1(k-l)}{1+2x_1(k)}\right]\right\},
\end{aligned} \tag{37}$$

where $r_1(k) = 0.5 + 0.3\sin k\pi, r_2(k) = 0.6 + 0.2\sin k\pi, k_i(s) = e^{0.5s}$ $(i = 1, 2, 3, 4), m = 2$, and it is easy to see that all the conditions of Theorem 4 are fulfilled. Thus system (37) has at least a positive two-periodic solution (see Figures 1 and 2).

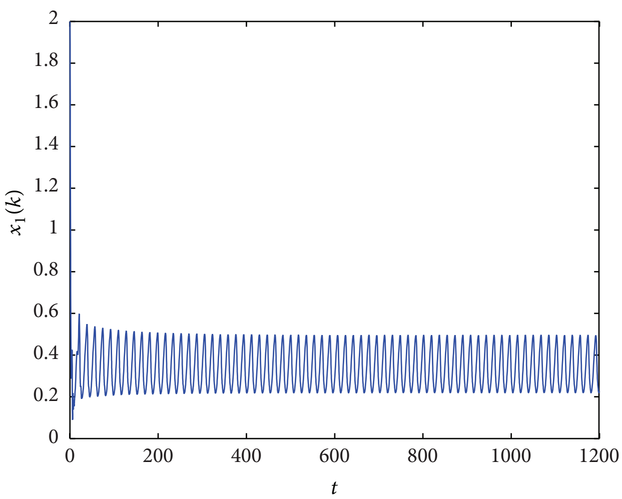

Figure 1: The time series graph of t-x_1 for system (37).

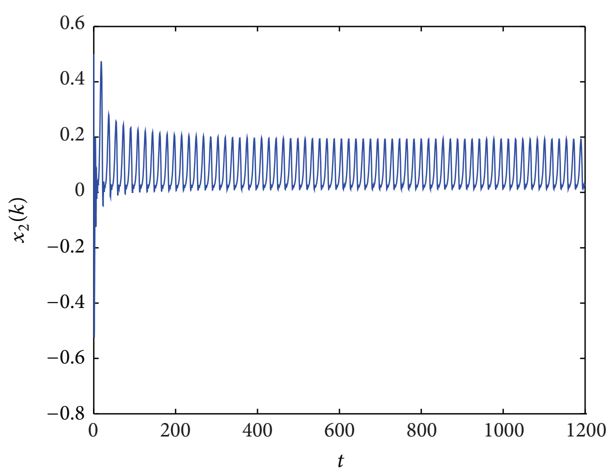

Figure 2: The time series graph of t-x_2 for system (37).

5. Conclusions

In this paper, a discrete predator-prey model with Holling II functional response and delays is investigated. With the aid of Gaines and Mawhin's continuation theorem of coincidence degree theory and the method of Lyapunov function, we establish some sufficient conditions for the existence and global asymptotic stability of positive periodic solutions of the model. Since the time scales can unify the continuous and discrete situations, it is meaningful to investigate the predator-prey model with Holling II functional response and delays on time scales. We leave it for future work.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work was supported by State Science and Technology Support Program (Grant no. 2013BAD15B02), General Scientific Research Projects in Hunan Province, Department of Education, China (Grant no. 14C0542), Graduate Student Research Innovation Project of Hunan Province (Grant no. CX2014B306), and Guangxi Experiment Centre of Science and Technology (Grant no. LGZXKF201112).

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