

## Generalized Super Efficiency Model for Ranking Efficient Decision Making Units in Data Envelopment Analysis

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**Abstract:** In evaluating decision making units (DMU's) by using Data Envelopment Analysis (DEA) technique, we encounter the situation in which more than one unit takes efficiency score of one. In such a case, some criteria should be considered to rank the DMU's. Some efficient techniques such as AP, MAJ, etc may be used in this way. For some sets of data, with special structure in models that above mentioned, may be infeasible and unstable. In this paper, a new model is developed that all the existing drawbacks of previously applied models remove. Some numerical examples are put forward.

**Key words:** DEA, Efficiency, Ranking.

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### INTRODUCTION

Data Envelopment Analysis (DEA) is a non-parametric method for evaluating decision making units (DMU). It is introduced by Charnes *et al.*, (1978), in assessment of an educational center in the USA and then extended by Banker *et al.*, (1984). Mathematical programming was used by them to meet the goal.

In evaluating the relative efficiency of each decision making unit by DEA, the scores between zero and one are obtained. In this way, usually more than one unit may be efficient in the DEA models and the scores will be 1. It should be considered that a number of efficient units in the Variable Return to Scale (VRS) models, are not less than the Constant Return to Scale (CRS) models as well. So, the researchers have proposed some methods to differentiate between the efficient units. It is called "Ranking efficient units in DEA". There are so many ranking methods that each of them has special facilities and properties for ranking efficient units.

Charnes *et al.*, (1958a), have counted a number of times in which, an efficient DMU plays the role of a benchmarking unit for others, and the norm will be used to rank them. Because of difficulties in finding the reference set of a DMU, the model is not an appropriate one. Charnes *et al.*, (1985b), have proposed another method in order to find out a benchmark DMU. The rate of outputs has been changed and evaluated the variations of efficiency scores by them. However, they didn't realize, how they fulfil it.

Sexton *et al.*, (1986), have suggested the "Cross Efficiency method". In this method, the weights, which are obtained by solving each of n-linear problems, are used. They have evaluated the efficiency of each DMU, for several times and then the data would be put in a matrix. Each row of this matrix contains the cross efficiency score of DMUs. The average of each rows is computed and the results are stored as a ranking measure. It seems that, it should be a valid method, but some difficulties would be faced. The most important problem comes in to existence, when the DEA models have alternative solutions.

Finally, it should be noted that, there are some techniques and strategies in DEA, which are imposed on the ranking. For example, Thompson *et al.*, (1992), have used the safe regions. In the technique, a number of efficient DMUs may be reduced. But it the method is not a suitable one because of difficulties in finding the appropriate weights. Adler *et al.*, (2005), have suggested another method to make a difference between DMUs. In the model they have decreased a number of inputs and outputs as well, by using component analysis. It causes, the number of efficient DMUs to be decreased. But, in general, the model may not be used for a perfect ranking.

Anderson and Petersen (AP model) (1993), have ranked extreme efficient units by omitting them from Possibility Production Set (PPS), and then Mehrabian *et al.*, (MAG) (1999), have modified the APE model. In some circumstances, the mentioned models may be infeasible and specially the AP model may be unstable because of extreme sensitivity to small variations in data, where some DMUs have relatively small values for

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some of the inputs. Saati *et al.*, (1999), have modified MAJ model and solved its infeasibility and Jahanshahloo *et al* (2006), have changed the type of data normalization in order to receive a much better result. In order to remove the difficulties from AP and MAJ models, some mathematicians have used specific norms. For instance, Jahanshahloo *et al.*, (2004), have practiced L1 norm for ranking efficient units. Amirteimoori *et al.*, (2001), have experienced L2 norm to find the gap between evaluated efficient units and the new PPS. Gradient line and ellipsoid norms have been used by Jahanshahloo *et al.*, (2004), in order to rank efficient units. Tone, (2001) and (2002), has used SBM model in this way. Shanling Li *et al.*, (2006), have developed the works of Tone, (2001) and (2002), for its infeasibility by using a kind of SBM model. To review ranking methods see also Adler *et al.*, (2002).

**Ranking Models:**

In this part, some ranking models in Data Envelopment Analysis (DEA) are summarized. Consider  $n$ , DMUs with  $m$  inputs and  $s$  outputs. The input and output vectors of DMU $_j$  ( $j = 1, \dots, n$ ) are  $X_j = (x_{1j}, \dots, x_{mj})^t$ ,  $Y_j = (y_{1j}, \dots, y_{sj})^t$  where  $X_j \geq 0, X_j \neq 0, Y_j \geq 0, Y_j \neq 0$ .

By using the constant return to scale, convexity and possibility postulates, the non-empty production possibility set (PPS) is defined as follows:

$$T_c = \left\{ (X, Y) : X \geq \sum_{j=1}^n \lambda_j X_j, Y \leq \sum_{j=1}^n \lambda_j Y_j, \lambda_j \geq 0, j = 1, \dots, n \right\}$$

In order to rank the efficient units, the evaluated DMUo from  $T_c$  are omitted by Anderson and Peterson (2005).

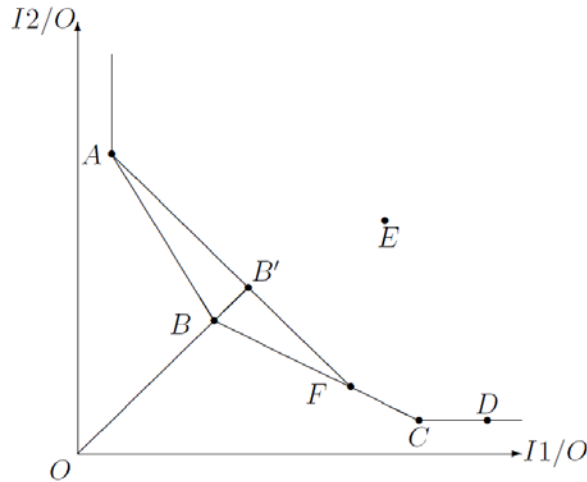
Their proposed model in *CCR/ε* model is as follows:

$$\begin{aligned} & \max \sum_{r=1}^s u_r y_{ro} \\ & S.t \sum_{i=1}^m v_i x_{io} = 1 \\ & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, nj \neq o \\ & u_r \geq \varepsilon, \quad r = 1, \dots, s \\ & v_i \geq \varepsilon, \quad i = 1, \dots, m \end{aligned} \tag{1}$$

and its dual is:

$$\begin{aligned} & \min \theta - \varepsilon \left[ \sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right] \\ & S.t \sum_{j=1, j \neq o}^n \lambda_j x_{ij} + s_i^- = \theta x_{io}, \quad i = 1, \dots, m \\ & \sum_{j=1, j \neq o}^n \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1, \dots, s \\ & \lambda_j \geq 0, \quad j = 1, \dots, n, j \neq o \\ & s_i^- \geq 0, \quad i = 1, \dots, m \\ & s_r^+ \geq 0, \quad r = 1, \dots, s \end{aligned} \tag{2}$$

For efficient units  $\theta^* \geq 1$  and for inefficient units  $0 < \theta^* < 1$ . . For more details see figure 1:

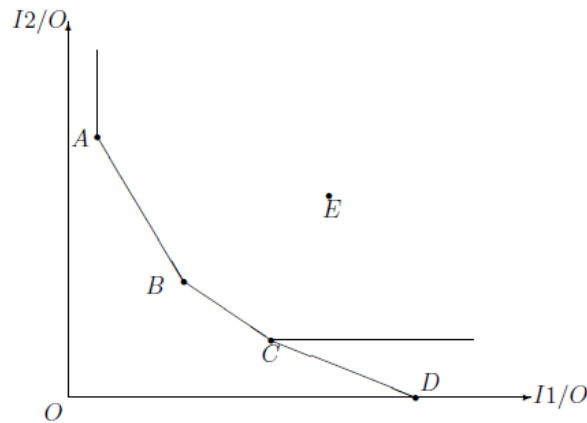


**Fig. 1:** AP model

Unit B is an extreme efficient one. In order to rank B by using AP model, at first, it should be omitted from  $T_c$ . The new frontier is  $AFCD$  and its efficiency score is:  $\theta_B^* = \frac{OB'}{OB}$ . It is obvious that  $\theta_B^* > 1$ .

Omitting non extreme efficient unit F would not change the frontier, so its score remains 1. That means, AP model may not be used for ranking non extreme efficient units. It is the problem of all DEA ranking models. AP model has two important problems:

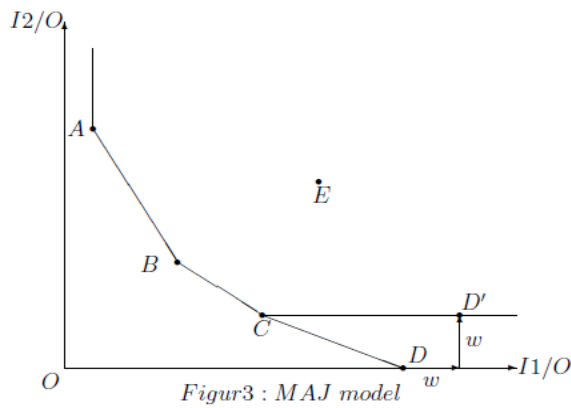
- (1) AP model may be infeasible for special data in input oriented case. In figure 2, For example, unit D has the amount of zero in its first input but none of the other units has zero in the same situation. Hence, AP model is infeasible in this case. It should be considered that the model is always feasible in output oriented case.



**Fig. 2:** AP model is infeasible

- (2) AP model is unstable for some *DMUs* which, one of the data elements is closed to zero.

To remove the problems that come in to account in AP model, Mehrabian *et al.*, (2006) have offered MAJ model. First of all, they the evaluated unit from production possibly set are extricated, then all inputs by  $w$  are increased. Their proposed model is depicted in figure 3:



**Fig. 3:** MAJ model

MAJ model may be written as follows:  $\min 1+w$

$$\min 1 + w$$

$$\begin{aligned}
 S.t \quad & \sum_{j=1, j \neq o}^n \lambda_j x_{ij} \leq x_{io}, + w \quad i = 1, \dots, m \\
 & \sum_{j=1, j \neq o}^n \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \dots, s \\
 & \lambda_j \geq 0, \quad j = 1, \dots, n, j \neq o
 \end{aligned} \tag{3}$$

It can be concluded that the model is not always feasible for any sets of data.

**Theorem 1:**

The necessary and sufficient conditions for feasibility of MAJ model is: in evaluating of  $DMU_o$ , or  $y_{ro} = 0$ ,  $r = 1, \dots, s$  or there exists  $DMU_j, j \neq o$  such that  $y_{rj} \neq 0$ . Proof. see Mehrabian *et al.*, (1999).

**Proposed Model for Ranking Efficient Units:**

In order to extricate the infeasibility of AP and MAJ model, it should be considered different steps for increasing inputs and decreasing outputs, instead of using equal steps. The suggested model for ranking extreme efficient units is as follows:

$$\begin{aligned}
 \min \quad & \theta = \frac{1}{m+s} \left[ \frac{\sum_{i=1}^m \alpha_i w_i}{\sum_{i=1}^m \alpha_i} + \frac{\sum_{r=1}^s \beta_r z_r}{\sum_{r=1}^s \beta_r} \right] + 1 \\
 S.t \quad & \sum_{j=1, j \neq o}^n \lambda_j x_{ij} \leq x_{io}, + w_i \quad i = 1, \dots, m \\
 & \sum_{j=1, j \neq o}^n \lambda_j y_{rj} \geq y_{ro} - z_r, \quad r = 1, \dots, s \\
 & \lambda_j \geq 0, \quad j = 1, \dots, n, j \neq o \\
 & w_i \geq 0, \quad i = 1, \dots, m \\
 & z_r \geq 0, \quad r = 1, \dots, s
 \end{aligned} \tag{4}$$

where  $\alpha_i$  and  $\beta_r$  are the weights that manager states, or they should be found from the data set by their relative importance. Therefore, the model 4 may be operated by managers in share, because of considering managers' options as well. Supposing the entity of objective function which is minimization, the amounts of  $\alpha_i$  and  $\beta_r$  are important. In this status, the little  $\alpha_i$  be used, the more increasing  $w_i$  be obtained. And also we can illustrate this fact for  $\beta_r$ . So the amount of  $\alpha_i$  and  $\beta_r$  influence upon the optimal objective value of model 4 and at last, it will impose on optimal solutions.

If the manager had no suggestion for the weights, the following criteria would be handled:

$$\alpha_i = \begin{cases} \frac{R_i^-}{x_{io}} & \text{if } x_{io} > 0, \\ 0 & \text{otherwise.} \end{cases}$$

and

$$\beta_r = \begin{cases} \frac{y_{ro}}{R_r^+} & \text{if } y_{ro} > 0, \\ 0 & \text{otherwise.} \end{cases}$$

where  $R_i^- = \max_j \{x_{ij}\}$  and  $R_r^+ = \min_j \{y_{rj}\}$  for every  $i$  and  $r$ . By using above formulations, the weights will be "relative importance" of inputs and outputs.

If there is no relative importance between inputs and outputs, then all weights can be considered with the amount of 1 in the model.

Finally, it should be paid attention that all data in model 4 are normalized. For normalizing, the following definitions are needed which would be used for ranking:

$$\bar{x}_{ij} = \frac{x_{ij}}{\max_j \{x_{ij}\}} \text{ and } \bar{y}_{ij} = \frac{y_{rj}}{\max_j \{y_{rj}\}} .$$

It should be underlined here that model 4 ranks only extreme

efficient units. Obviously, the optimal objective value of model 4 is greater than 1 for these units and it will be 1 for the others.

**Theorem 2:**

Model 4 is always feasible and its optimal objective value is bounded.

*Proof:*

To proof feasibility define:  $\lambda_j = \begin{cases} 1 & j = 1, \dots, n, j \neq o, \text{ hence } w_i = \sum_{j=1, j \neq o}^n x_{ij} \quad i = 1, \dots, s \\ 0 & j = o \end{cases}$

$\sum_{j=1, j \neq o}^n y_{rj} - y_{ro} = 0$  then  $z_r = 0$  for all  $r$ . If  $\sum_{j=1, j \neq o}^n y_{rj} - y_{ro} > 0$  then  $z_r = \sum_{j=1, j \neq o}^n y_{rj} - y_{ro}, r=1, \dots, s$ . If  $\sum_{j=1, j \neq o}^n y_{rj} - y_{ro} < 0$  then  $z_r = y_{ro} - \sum_{j=1, j \neq o}^n y_{rj}, r=1, \dots, s$  If is obvious that these solutions are feasible for model 4 and the proof of feasibility is com To proof the second part, using normalized data,  $0 \leq w_i \leq 1$  and because of non-negative:

$$0 \leq \sum_{i=1}^m \alpha_i w_i \leq \sum_{i=1}^m \alpha_i \leq m \sum_{i=1}^m \alpha_i. \text{ Clearly } \sum_{i=1}^m \alpha_i > 0 \text{ then } \frac{\sum_{i=1}^m \alpha_i w_i}{\sum_{i=1}^m \alpha_i} \leq m. \text{ Similarly } \frac{\sum_{r=1}^s \beta_r z_r}{\sum_{r=1}^s \beta_r} \leq s.$$

$$\frac{1}{m+s} \left[ \frac{\sum_{i=1}^m \alpha_i w_i}{\sum_{i=1}^m \alpha_i} + \frac{\sum_{r=1}^s \beta_r z_r}{\sum_{r=1}^s \beta_r} \right] \leq 1, \text{ and the proof is completed.}$$

**Theorem 3:**

If  $R^* = (\lambda^*, W^*, Z^*)$  be an optimal solution of the recent model, then all of the following constraints are active on  $R^*$

$$\begin{aligned} a): & \sum_{j=1, j \neq o}^n \lambda_j x_{ij} \leq x_{io} + w_i \quad i = 1, \dots, m \\ b): & \sum_{j=1, j \neq o}^n \lambda_j y_{ij} \geq y_{ro} - z_r, \quad r = 1, \dots, s \end{aligned} \tag{5}$$

ie, there is no slack in optimality.

Proof. By contradiction, suppose that one of the constraint of (a) (e.g. the first constraint) is not active on  $R^*$ .

Also:  $\sum_{j=1, j \neq o}^n \lambda_j x_{ij} < x_{io} + w_i$ . Then define  $s_1^{-*} = (x_{io} + w_i^*) - \sum_{j=1, j \neq o}^n \lambda_j^* x_{1j}$  Therefore

$$w_1'^* = w_1^* - s_1^{-*} < w_1^* \text{ then } a_1 w_1'^* + \sum_{i=1}^m \alpha_i w_i^* + \sum_{r=1}^s \beta_r z_r^* < \sum_{i=1}^m \alpha_i w_i^* + \sum_{i=1}^s \beta_i z_i^* \text{ and it is violating with}$$

optimality of  $(\lambda^*, W^*, Z^*)$ . Therefore all constraints in (a) and (b) are binding for every optimal solutions.

In model4 if  $w_i = w, i = 1, \dots, m$  and  $z_r = 0, r = 1, \dots, s$ , then the MAJ model will be concluded. Similarly, if  $w_i = w, i = 1, \dots, m$  and  $z_r = w, r = 1, \dots, s$ , then the modified MAJ model [14] will be the consequent. It is as follows:

Min  $1 + w$

$$\begin{aligned} S.t & \sum_{j=1, j \neq o}^n \lambda_j x_{ij} \leq x_{io} + w \quad i = 1, \dots, m \\ & \sum_{j=1, j \neq o}^n \lambda_j y_{ij} \geq y_{ro} - w \quad r = 1, \dots, s \\ & \lambda_j \geq 0, \quad j = 1, \dots, n, j \neq o \end{aligned} \tag{6}$$

**4 Example:**

The mentioned approach is going to be applied by two data sets. In the first example, a data set is constructed to show problems of traditional ranking models. AP and MAJ model (see table 1). The second example consists of 15 Fortune’s top US cities in 1996 (see table 3), (2004).

**4.1 Example1:**

Consider 28 (DMUs) with 3 inputs and 3 outputs. These data are summarized in table 1:

The results of ranking are shown in table 2. In this example, all weights are considered with the amount of 1. In the following table S.AP means the score of AP model and R.AP means the rank of AP model etc.

Consider the first efficient unit. It has only a zero input in its data, therefore, it would be infeasible in AP model. Unit 2 has a small input in second component, hence AP model is unstable for this DMU and its efficiency score is not a real one. Unit 1 is non-zero in its first output and all the other units are zero in the

component, so unit 1 is infeasible in MAJ model. Obviously, there is no problem for the new model and it is always feasible and stable for special data such as the example.

**4.2 Example2: Ranking of 15 US cities:**

Here, high-end housing price (1000 US dollars) lower-end housing monthly rental price (US dollars) and number of violent crimes, as three DEA inputs and median household income (US dollar), number of people with bachelor’s degree (million) and number of doctors (thou-sand) as three DEA outputs in evaluating 15 US cities. These data are summarized in table 3:

**Table 1:** Data

DMUs	Input <sub>1</sub>	Input <sub>2</sub>	Input <sub>3</sub>	Output <sub>1</sub>	Output <sub>2</sub>	Output <sub>3</sub>
1	0	1397736	616961	6785798	1594957	1088699
2	371.95	100	385453	0	545140	835745
3	268.23	685584	341941	0	406947	473600
4	202.02	452713	117429	0	135939	336165
5	197.93	471650	112634	0	204909	317709
6	178.96	423124	189743	0	190178	605037
7	148.04	367012	97004	0	86514	239760
8	189.93	408311	111904	0	1411954	353896
9	23.33	245542	91861	0	135327	239360
10	116.91	305316	91710	0	78357	208188
11	129.62	295812	92409	0	114365	298112
12	106.26	198703	53499	0	67154	233733
13	89.70	210891	95642	0	78992	118553
14	109.26	282209	84202	0	149186	243361
15	85.50	184992	49357	0	116974	234875
16	72.17	222327	73907	0	117854	118924
17	76.18	161159	47977	0	67857	158250
18	73.21	144163	43312	0	114883	101231
19	86.72	190043	55326	0	173099	130423
20	69.09	158439	66640	0	74126	123968
21	77.69	135046	46198	0	65229	262876
22	97.42	206926	66120	0	128279	242773
23	54.96	79563	43192	0	37245	184055
24	67.00	144092	43350	0	86859	194416
25	46.30	100431	31428	0	55989	127586
26	65.12	96873	28112	0	37088	224855
27	20.09	50717	54650	0	11816	24442
28	69.81	117790	30976	0	31726	169051

**Table 2:** The results of ranking

Eff Units	S.AP	R.AP	S.MAJ	R.MAJ	S.new model	R.new model
DMU 1	Infeasible.	-	Infeasible.	-	1.09016	1
DMU 2	4.3289E+3	1	1.30964	2	1.01720	3
DMU 8	4.03281	2	1.65877	1	1.03773	2
DMU 9	1.06436	4	1.00692	4	1.00073	5
DMU 26	1.40566	3	1.01848	3	1.00183	4

**Table 3:** Data of 15 US cities.

DMUs	Input <sub>1</sub>	Input <sub>2</sub>	Input <sub>3</sub>	Output <sub>1</sub>	Output <sub>2</sub>	Output <sub>3</sub>
1	586	581	1193.06	46928	0.6514	9.878
2	475	558	1131.64	42879	0.5529	5.301
3	201	600	3468	43576	1.35	18.2
4	299	609	1340.55	45673	0.729	7.209
5	318	613	634.7	40990	0.319	4.94
6	265	558	657.5	39079	0.515	8.5
7	467	580	882.4	38455	0.3184	4.48
8	583	625	3286.7	54291	1.7158	15.41
9	347	535	917.04	34514	0.4512	8.784
10	296	650	3714.3	41984	1.2195	8.82
11	600	740	2963.1	43249	0.9205	7.805
12	575	775	3240.75	43291	0.5825	10.05
13	351	888	2197.12	46444	1.04	18.208
14	283	727	778.35	41841	0.321	4.665
15	431	695	1245.75	40221	0.2365	3.575

**Table 4:** Relative importance of inputs and outputs.

EFF units	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\beta_1$	$\beta_2$	$\beta_3$
DMU1	1.0238	1.5283	2.9068	1.3596	2.7043	2.7630
DMU3	2.9875	1.4800	1	1.2625	5.7082	5.0909
DMU4	2.0066	1.4581	2.5869	1.3223	3.0824	2.0165
DMU5	1.8867	1.4486	5.4639	1.1876	1.3488	1.3818
DMU6	2.2641	1.5913	5.2745	1.1322	2.1775	2.3776
DMU8	1.0291	1.4208	1.0551	1.5730	7.2549	4.3104
DMU13	1.7094	1	1.5784	1.3456	4.3974	5.0931

**Table 5:** The results of ranking.

Eff Units	S.AP	R.AP	S.MAJ	R.MAJ	S.new model	R.new model
DMU 1	1.0866	5	1.0499	5	1.0055	5
DMU 3	2.0173	1	1.3724	1	1.0920	1
DMU 4	1.0447	7	1.0278	6	1.0026	7
DMU 5	1.0866	6	1.0148	7	1.0039	6
DMU 6	1.5155	2	1.1023	3	1.0371	2
DMU 8	1.3075	3	1.2539	2	1.0299	3
DMU 13	1.1030	4	1.0606	4	1.0101	4

By using the mentioned criteria, the weights  $\alpha_i$  and  $\beta_r$  for each efficient DMU are figured out in the following table:

The results of ranking concluded from 15 US cities are shown in table 4. In the following table S.AP means score of AP model and R.AP means the rank of AP model etc.

**Conclusion:**

In this paper a new model is proposed in order to rank the extreme efficient units which are always restricted and feasible. One of the advantages of the offered model is its cooperative feature for the managers as well as considering their ideas in evaluating the units. Further-more, the model has no slack in optimality, and most probably, it doesn't take place in AP and MAJ models.

In the first example, unit 1 is infeasible in AP and MAJ models but it reached the highest score in the new model. The AP model for unit 2 is not suitable. As a conclusion, the new model has removed the problems of AP and MAJ models.

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