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VIBRATION ANALYSIS OF A SLENDER ROTATING SHAFT.

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Abstract

In this paper the vibration analysis of cantilever rotating shaft is considered. In this model the rotary inertia and the gyroscopic effects as well as the coupling effects have been incorporated. Natural frequencies of a rotating shaft have been calculated at whirling speed. A program is elaborated for theoretical calculation of critical speed of rotating shaft. To verify the present model the critical speed of shaft system are compared with those available in the literature. The calculations of effect of pressure on the vibration of a cantilever rotating shaft is carried out in this paper.

INTRODUCTION

Rotating shafts are used for power transmission in many modern machines. Slender shaft or bar plays a crucial part in mechanical industry. For example, the broaching cutters and lead screw of machine tools are typical slender parts. But it is well known that turning operation of slender bar is much more difficult than that of ordinary shafts due to its low stiffness. For designing the slender shaft has to consider length to diameter ratio is high and low stiffness.

Accurate prediction of dynamics of rotating shafts is necessary for a successful design. Free vibrations analysis is one of the important steps in rotor-dynamics. In this free vibrations of an in-extensional cantilever supported shaft with density and inertia are considered. Rotary inertia and gyroscopic effects are included, but shear deformation is neglected. To analyze the free vibrations of the shaft, the finite element method is used.

This method is applied to the static as well as dynamic analysis on shaft, which demonstrates the results. A result is derived which describes the nonlinear free vibrations of the rotating shaft in two transverse planes. It is found that in this case, both forward and backward nonlinear natural frequencies are being excited. The transverse vibrations of a slender beam which is free to rotate. For free vibration (modal analysis) analysis of slender shaft we use the application tool of FEM with various boundary condition and loading conditions.

Grybos[1] considered the effect of shear deformation and rotary inertia of a rotor on its critical speeds. Jei and Leh[3] investigated the whirl speeds and mode shapes of a uniform asymmetrical Rayleigh shaft with asymmetrical rigid disks and isotropic bearings. Sturla and Argento[5] studied the free and forced response of a viscoelastic spinning Rayleigh shaft. Melanson and Zu[6] studied the free vibrations and stability of internally damped rotating shafts with general boundary conditions. El-Mahdy and Gadelrab[10] studied the freeVibrations of unidirectional fiber reinforcement rotating composite rotor. Raffa and Vatta[11] derived the equations of motion for an asymmetric Timoshenko shaft with unequal principal moments of inertia. The critical speeds and mode shapes of a spinning Rayleigh beam with six general boundary conditions are investigated analytically by Sheu and Yang [12]. Gubran and Gupta [13] studied the effect of stacking sequence and coupling mechanisms on the natural frequencies of composite shafts.To simplify the analysis, researchers often try to use the linear analysis.

In this paper, the equations of motion of a continuous cantilever rotating shaft with inertia are derived. Rotary inertia and gyroscopic effects are included but shear deformation is neglected.

NOTATIONS

A - Shaft Cross sectional area,

E - Modulus of elasticity,

I - Moment of inertia,

L - Length of Shaft,

Lo - Length of Shaft subjected to pressure,

M - Bending Moment,

P - External Force,

Po - Pressure at free end of Shaft,

r - Radius of Shaft,

Y - Lateral displacement of Shaft.

ρ - Density Of Shaft,

Ω - Whirling speed of Shaft,

ω - Angular frequency of Shaft.

η - Lateral displacement of shaft at axial coordinate ξ.

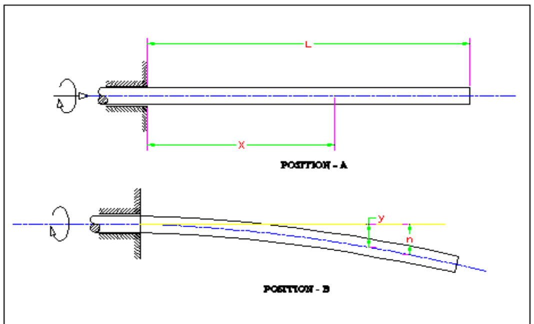

FIG. 1

BUCKLING OF A NON ROTATING SHAFT

It will be assumed that the shaft is a 'slender member' and that its deflection equation is that for an Euler beam

for which

$$M = EI\frac{d^2y}{dx^2} \quad . \quad . \quad . \quad . \quad (1)$$

where,

M is the applied bending moment at section x and

d2y/dx2 is the (approximate) curvature of the deflected beam.

External pressure will cause the shaft to buckle statically if the bending moment at section x from the external pressure exceeds the restraining elastic moment M in the shaft. It may not be immediately clear whether this can happen or not. For instance, consider two comparable situations (Figs 2a and b). In Fig. 2a a cantilever shaft is subjected to uniform negative pressure (i.e. a vacuum). In Fig. 2b a simply supported is subjected to the same negative pressure over its area inside the supporting walls. The cantilever is always stable and will never buckle however high the hydrostatic tension. The simply supported shaft is potentially unstable and, if p is the applied tension per unit area, it will buckle when,

$$pA = \frac{\pi^2 EI}{l^2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If now both shafts are subjected to a uniform positive pressure, neither is unstable, however high the pressure. This behavior may be explained by the following heuristic argument.

From the theory of hydrostatics, if a fluid is in hydrostatic equilibrium, then the pressure forces acting over any closed surface in the fluid will be in static equilibrium with\the body forces acting on the fluid within the volume. The resultant moment of these forces about any point must therefore be zero. If the real shaft can be replaced by a fluid shaft which would be in hydrostatic equilibrium, then there can be no bending moment acting on the deflected shaft and, hence, there will be no possibility of buckling

If, however, the external force system acting on the shaft is not one for which a fluid shaft would be in hydrostatic equilibrium, then there will in general be a resultantbending moment about any section of the deflected shaft, and if this acts to increase the deflection further, buckling is a possibility. Consider the cantilever shaft (Fig. 2a). If a uniform pressure is applied, it is clear that a fluid shaft will always be in equilibrium. Even when the shaft is deflected, it may still be replaced by a fluid shaft which will remain in hydrostatic equilibrium. Hence, the external forces do not cause a bending moment in the deflected shaft and there is never a buckling problem. However, in the case of the simply supported shaft of Fig. **2b,** a fluid shaft would not be in equilibrium. Consider the length of 'fluid shaft' between the supports, as shown shaded in Fig. **3a.** It would be sucked in or squeezed out through the supporting walls.

To maintain equilibrium, additional stresses must be 'applied to the cut ends of the shaft at each end. When the applied stress is a tension stress, tension forces pA must be applied to each end of the fluid shaft as shown. If the shaft then deflects under this force system, there will beno resultant bending moment at any section of the shaft because it is in a state of hydrostatic equilibrium. However, since the external axial tension forces pA are not applied to the shaft, a fluid shaft would not be in hydrostatic equilibrium.

The difference from a hydrostatic force system is an axial compressive force pA (Fig. **3b),** which does exert a resultant bending moment about any section of the deflected shaft, and leads to buckling when pA reaches the critical value given by equation (2).

These conclusions confirm the results given by Peterson **(3)** for the buckling of a shaft of constant crosssectional area subjected to an axial load in the presence of uniform hydrostatic pressure. However, the above argument is useful because it applies also to the case when the external pressure is no longer uniform. Consider the cantilever of Fig. 1, subjected to a varying pressure field p(x).

A fluid shaft would not now be in hydrostatic equilibrium. An additional axial body force A(dp/dx) **dx** must be applied to every section of length dx, acting to the right, to maintain hydrostatic equilibrium. The actual force system differs from that required to maintain hydrostatic equilibrium by A(dp/dx) dx acting to the left. The bending moment at section x of the real shaft when it is deflected is therefore that produced by forces A(dp/dx) dx acting to the left on every section of length dx (Fig. **4).** Integrating from section x to the end of the shaft, the resultant bending moment at x from the external

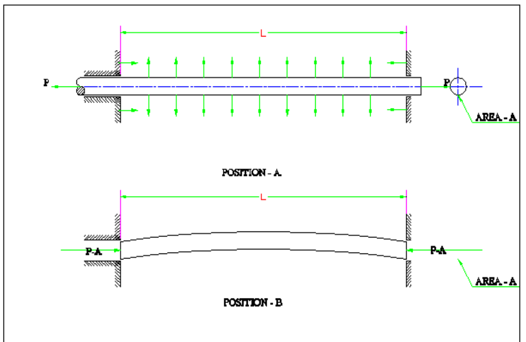

FIG. 3

$$M = \int_x^l A \left(\frac{dp}{dx}\right)_{x=\xi} (\eta - y)\, d\xi \quad . \quad . \quad (3)$$

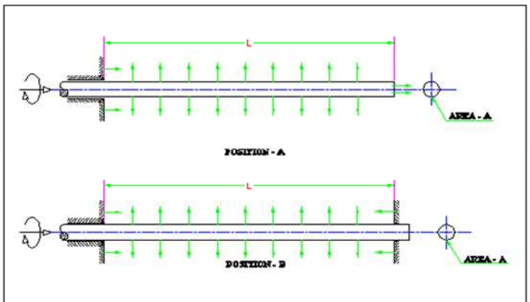

FIG. 2

$$M = \int_{x}^{l} A \left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)_{x=\xi} (\eta - y)\, \mathrm{d}\xi \quad . \quad . \quad (3)$$

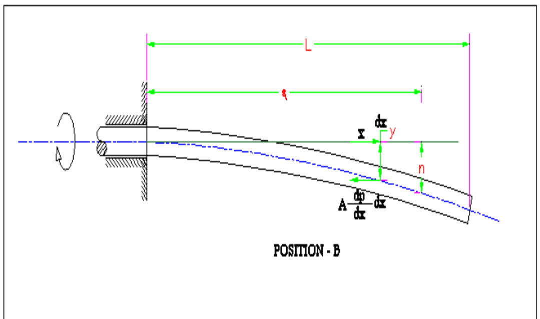

FIG. 4

Where y is the deflection at section x and η is the deflection at section ξ , and this may cause buckling if it exceeds the elastic restoring moment in the shaft. It will be clear from this discussion that the body forces

A(dp/dx) dx shown in Fig. 4 do not exist in the real situation, in which there are no body forces. The forces shown in Fig. 4 are therefore effective body forces which give the same bending moment in the deflected shaft as the actual external surface forces give. In Appendix 1 this equivalence is verified mathematically. If EZ(x) is the bending stiffness of the shaft about a diameter at section x, the differential equation for the

shaft's deflection is, from equation (I),

$$EI\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \int_{x}^{l} A \left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)_{x=\xi} (\eta - y)\, \mathrm{d}\xi \quad . \quad . \quad (4)$$

The exact solution of this equation is the exact solution of the static buckling problem. Unfortunately, a general mathematical solution of equation (4) has not been found, and the only exact solution known to the author is for the case when the cross-sectional area and bending stiffness of the shaft are constant and when the pressure varies linearly from zero at the wall to p, (say) at the free end.

In this case

$$p(x) = p_0\frac{x}{l} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and equation (4) becomes

$$EI\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = A\frac{p_0}{l}\int_{x}^{l} (\eta - y)\, \mathrm{d}\xi \quad . \quad . \quad (6)$$

which is the equation for the buckling of a flagpole under its own weight **(2).** It is shown in **(2)** that, after differentiating equation **(6)** and changing the variable, Bessel's equation is obtained and an exact numerical solution can be found. By comparison with the flagpole case, the lowest buckling pressure can be shown to be given by

$$Ap_0 = 7{\cdot}84\frac{EI}{l^2} \quad . \quad . \quad . \quad . \quad (7)$$

This may be compared with the result for the buckling of a cantilever shaft subjected to a single end load poA
which is

$$Ap_0 = \frac{\pi^2}{4}\frac{EI}{l^2} = 2{\cdot}47\frac{EI}{l^2} \quad . \quad . \quad . \quad (8)$$

It is clear that the distribution of pressure along the shaft affects buckling by a large amount. The assumption made in **(I)** that equation (8) applies to the extruder problem may not therefore be numerically accurate, depending on the pressure distribution inside the extruder.

ENERGY INTEGRALS

Although an exact solution cannot be found for the general case when the shaft is rotating and the applied pressure and cross-sectional area vary along its length, an accurate approximate solution can be obtained by an energy method, as described in **(2)** or, in the form used here, in **(4).** This involves calculating the energy of the shaft in terms of an assumed deflection curve which is unknown but which can be approximated without serious loss of accuracy. Suppose first that the non-rotating shaft is undergoing a transverse vibration about its un deflected axis of symmetry.

The difference between the maximum strain energy of the shaft U (when its deflection is a maximum) and the maximum kinetic energy T (when its deflection is zero) must be the work done on the shaft W by the external pressure forces as the shaft deforms from zero
deflection to maximum deflection during the vibration. Let the shaft vibrate at angular frequency **w** so that

$$y = y(x) \sin \omega t \quad . \quad . \quad . \quad . \quad (9)$$

and, hence

$$\dot{y} = y(x)\omega \cos \omega t \quad . \quad . \quad . \quad (10)$$

The maximum kinetic energy of the shaft is then

$$T = \frac{1}{2}\int_0^l \rho A\dot{y}_{\max}^2 \, dx = \frac{1}{2}\omega^2 \int_0^l \rho A y^2(x)\, dx \quad (11)$$

where p is the density of the shaft, which may be a function of x if necessary. The strain energy of the fully
deflected shaft is, from (4),

$$U = \frac{1}{2}\int_0^l EI\left(\frac{d^2y}{dx^2}\right)^2 dx \quad . \quad . \quad . \quad (12)$$

where, unless otherwise indicated, y means y(x), the maximum deflection of the shaft. Finally, the work done by the external pressure forces can be calculated from first principles (see Appendix 1) or determined by extending the argument of the previous section.

The effective body force A(dp/dx) dx shown in Fig. 4 may be considered to do work, because, as the shaft deflects, its point of application moves a small distance in the direction of the force. From **(4),** this axial displacement of the section at x is

$$\Delta = \frac{1}{2}\int_0^x \left(\frac{dy}{dx}\right)^2_{x=\xi} d\xi \quad . \quad . \quad . \quad (13)$$

and so the work done by the axial force A(dp/dx) dx is

$$dW = \frac{1}{2}A\frac{dp}{dx}\left[\int_0^x \left(\frac{dy}{dx}\right)^2_{x=\xi} d\xi\right] dx \quad . \quad (14)$$

which may be integrated to give

$$W = \int_0^l \frac{1}{2}A\frac{dp}{dx}\left[\int_0^x \left(\frac{dy}{dx}\right)^2_{x=\xi} d\xi\right] dx \quad . \quad (15)$$

Hence, since by the law of conservation of energy,

$$T + W = U \quad . \quad . \quad . \quad . \quad (16)$$

therefore

$$\frac{1}{2}\omega^2\int_0^l \rho A y^2\, dx + \frac{1}{2}\int_0^l A\frac{dp}{dx}\left[\int_0^x \left(\frac{dy}{dx}\right)^2_{x=\xi} d\xi\right] dx = \frac{1}{2}\int_0^l EI\left(\frac{d^2y}{dx^2}\right)^2 dx \quad (17)$$

and the natural frequency of free lateral vibration of the shaft is given by

$$\omega^2 = \left[\frac{\int_0^l EI\left(\frac{d^2y}{dx^2}\right)^2 dx - \int_0^l A\frac{dp}{dx}\left[\int_0^x \left(\frac{dy}{dx}\right)^2_{x=\xi} d\xi\right] d\xi}{\int_0^l \rho A y^2\, dx}\right] \quad . \quad . \quad . \quad (18)$$

An alternative form of this equation may be obtained by noting that the right-hand side of equation (15) may be integrated by parts using the result that,

$$\int u\, dv = uv - \int v\, du \quad . \quad . \quad . \quad (19)$$

where

$$u = \tfrac{1}{2}A \int_0^x \left(\frac{dy}{dx}\right)^2_{x=\xi} d\xi \quad . \quad . \quad . \quad (20)$$

and

$$dv = \frac{dp}{dx}\,dx \quad . \quad . \quad . \quad . \quad (21)$$

to give

whereAo and po are the area and pressure at the free end of the cantilever. Using this

$$W = \frac{1}{2}\int_0^l (A_0p_0 - Ap)\left(\frac{dy}{dx}\right)^2 dx - \frac{1}{2}\int_0^l p\frac{dA}{dx}\left[\int_0^x \left(\frac{dy}{dx}\right)^2_{x=\xi} d\xi\right] dx \quad (22)$$

alternative (but completely equivalent) expression, equation **(1** 8) becomes

$$\omega^2 = \left[\frac{\int_0^l EI\left(\frac{d^2y}{dx^2}\right)^2 dx - \int_0^l (A_0p_0 - Ap)\left(\frac{dy}{dx}\right)^2 dx + \int_0^l p\frac{dA}{dx}\left[\int_0^x \left(\frac{dy}{dx}\right)^2_{x=\xi} d\xi\right] dx}{\int_0^l \rho A y^2\, dx}\right] \quad . \quad . \quad . \quad (23)$$

Equations (18) and (23) are equivalent exact expressions for the natural frequency of vibration of a cantilever elastic shaft subjected to external pressure. If the correct deflection curve y(x) were substituted into equations (18) and (23), the same exact result for the natural frequency would be obtained.

WHIRLING CALCULATIONS

So far only non-rotating shafts have been considered. However, from the theory of whirling, a rotating shaft is likely to run unsteadily when its speed of angular rotation Q is close to the natural frequency of free lateral vibrations

ω. It is assumed that gyroscopic effects are negligible for a shaft of small diameter-to-length ratio. If the shaft

is unrestrained (except by its own stiffness) synchronous whirl at shaft speed will occur when Q =**ω.** Consider

how the (lowest) whirling speed Q is affected by the external pressure field.

Shafts of constant cross-sectional area

In this case dA/dx = 0, so that equation (23) gives

$$\Omega^2 = \left[\frac{EI\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx - A\int_0^l (p_0 - p)\left(\frac{dy}{dx}\right)^2 dx}{\rho A\int_0^l y^2\, dx}\right] \quad (24)$$

where A = A. is the constant area of the shaft. If the correct deflection curve **y (x)** were substituted into equation (24), then the exact value of the critical speed would be obtained. However, since the deflection y(x) is not known exactly (even for the buckling flagpole problem it is a complicated power series) an approximation for y(x) must be used. The flagpole problem is solved approximately in **(2)** and **(4)** by using the relation

$$y = \delta\left(1-\cos\frac{\pi x}{2l}\right) \quad . \quad . \quad . \quad (25)$$

and, by comparison with the known exact solution, it is shown that good accuracy is obtained with this simple relation, which is therefore also adopted here. (A further comment on the accuracy of this approximation is given in Appendix (2)

Substituting equation (25) into equation (24) and integrating gives,

$$\Omega^2 = \frac{EI}{\rho A}\frac{(\pi/2l)^4}{(3-8/\pi)}\left[1-\frac{8Al}{\pi^2 EI}\int_0^l (p_0-p)\sin^2\frac{\pi x}{2l}\,dx\right] \quad . \quad . \quad . \quad (26)$$

which relates the whirling speed Q to (po-p) for a shaft of constant area A. po is the pressure at the free end of the shaft, **p** is the pressure at section x, EI is the bending stiffness, p is the density and 1 is the length of the shaft.

Case 1. Zero pressure gradients

The integral in the right-hand side of equation (26) is then zero, giving,

$$\Omega = \frac{3{\cdot}67}{l^2}\sqrt{\frac{EI}{\rho A}} = \Omega_0\ (\text{say}) \quad . \quad . \quad (27)$$

which compares with the known exact result for this case of

$$\Omega = \frac{3{\cdot}53}{l^2}\sqrt{\frac{EI}{\rho A}} \quad . \quad . \quad . \quad . \quad (28)$$

The approximate critical speed is thus about 4 per cent too high.

Case2. No rotation, constant pressure gradient

This is the flagpole problem. Equation (26) predicts that static buckling will occur when,

$$A\int_0^l (p_0-p)\sin^2\frac{\pi x}{2l}\,dx = \frac{\pi^2 EI}{8l} \quad . \quad (29)$$

Because then the whirling speed is zero.

For constant pressure gradient,

$$p_0-p = p_0\left(1-\frac{x}{l}\right) \quad . \quad . \quad . \quad (30)$$

for which equation (29) gives

$$p_0 A = 8{\cdot}30 \frac{EI}{l^2} \quad . \quad . \quad . \quad . \quad (31)$$

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CONCLUSION

The purpose of these calculations has been to determine theoretically the effect of external pressure on the vibration of a cantilever elastic shaft of variable area. An exact energy equation (18) has been obtained for the natural frequency of transverse (bending) vibrations in the presence of an external pressure field. It is clear from this equation that it is the distribution of pressure along the shaft, rather than the pressure at the free end, which affects the shaft stability. Equation (23) is a completely equivalent alternative form of equation (18). Whirling is assumed to occur when the rotational speed of the shaft coincides with the natural frequency of free lateral vibrations of the non-rotating shaft. Approximate values of the whirling speed can be obtained by substituting an approximate assumed deflection curve into either of the energy equations (18) or (23), and it has been shown that numerical values within about 5 per cent of the exact values may be expected for the lowest whirling speed. Since the problem was raised by a paper on the design of plastics extruders, it is of interest to record that some larger extruding machines apparently operate at speeds near or above their lowest whirling speed. Although the restraint of the barrel, the torque transmitted by the screw and viscous forces in the molten plastic make the extruder problem very much more complicated than the simplified theoretical problem considered here, the significance of this conclusion appears to merit further investigation. Shaft and barrel wear and fatigue of the hardened surfaces of the screw flights are becoming increasingly serious problems with large extruders, and a satisfactory explanation of their mechanical behavior is urgently needed.

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