MULTIPRODUCT MONOPOLY, COMMODITY BUNDLING, AND CORRELATION OF VALUES*

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I. Introduction

Through what selling strategy can a multiproduct monopolist maximize his profits when his knowledge about individual consumers' preferences is limited? One possibility, extensively studied in the context of a single-good monopoly, is to use quantity-dependent pricing as a means of discriminating among customers with differing tastes (see, for example, Oi [1971] and Maskin and Riley [1984]).

An alternative technique for price discrimination, first suggested by Stigler [1968] and analyzed further by Adams and Yellen [1976], is for the monopolist to package two or more products in bundles rather than selling them separately.[1] Through a series of examples Adams and Yellen illustrate that bundling can serve as a useful price discrimination device, even when all consumers' willingnesses to pay for each of the goods individually are unaffected by whether they are also consuming the other product. A typical example is illustrated in Figure I (adapted from Figure IV in Adams and Yellen), where there are two goods, three consumers (A,B,C) who consume at most one unit of each good (with reservation values for each good that are independent of whether the other good is consumed), and zero costs of production. There, a bundle offered at a price of 100 fully extracts all potential surplus, which would be impossible pricing the goods independently. Unfortunately, though, these authors do not provide any general characterization of the circumstances in which bundling is actually a multiproduct monopolist's optimal strategy. Their examples, however (such as Figure I), create the impression that the profitable use of bundling

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1. Examples are film distributors' bundling of assorted movies for sale to exhibitors [Stigler, 1968] and IBM's bundling of maintenance and programming services with computers [Scherer, 1980, p. 319].

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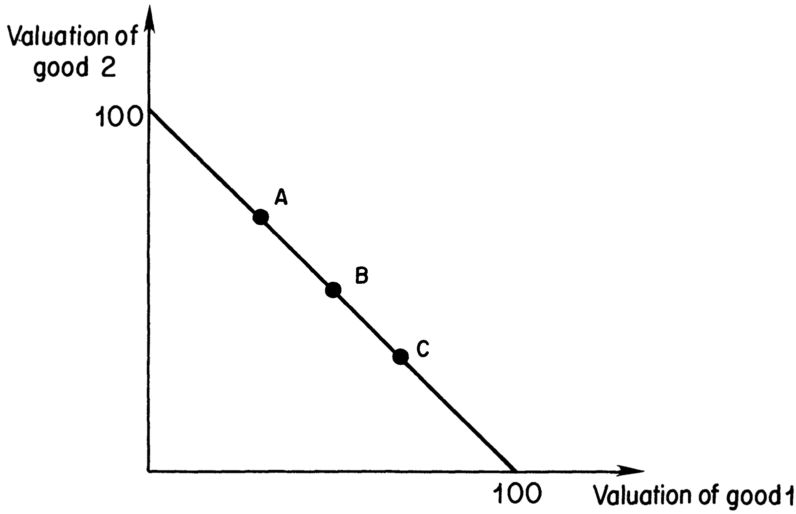

FIGURE I

is somehow tied to the presence of a negative correlation between the reservation values of the goods across the population of consumers.[2]

In this note we investigate the conditions under which bundling is an optimal strategy in the Adams and Yellen [1976] model. Our analysis distinguishes between cases where the monopolist can and cannot monitor the purchases of consumers. For each of these cases we provide and interpret sufficient conditions for bundling to dominate unbundled sales. One implication of these conditions is that bundling is always an optimal strategy whenever reservation values for the various goods are independently distributed in the population of consumers (regardless of the monopolist's ability to monitor purchases). In addition, when purchases can be monitored, bundling dominates unbundled sales for virtually all (in a sense made precise below) joint distributions of reservation values.

The analysis here is related to two other recent papers. Schmalensee [1984] considers the optimality of bundling in the Adams and Yellen model for the special case of a joint normal distribution of reservation values. Using both analytical and numerical technqiues, Schmalensee's results illustrate, among

2. For an expression of this view in the literature, see Schmalensee [1982], p. 71.

other things, that bundling can be optimal even when the correlation between reservation values in the population is nonnegative. Spence [1980], in an elegant paper, considers the multiproduct quantity-dependent pricing problem. Though much of Spence's focus is on computational aspects of the problem, in an appendix he provides an example with an independent distribution of preferences for two goods in which the multiproduct seller's optimal strategy differs from the outcome that would arise with two single-product monopolists engaged in quantity-dependent pricing. Given the differences between Spence's example and our model, his findings provide an interesting complement to the results here.[3]

In Section II we briefly describe the model. In Section III we present our results, distinguishing between cases where monitoring is and is not possible. Finally, in Section IV we briefly discuss the implications of our results for models of multiproduct oligopoly.

II. The Model

The model is essentially that of Adams and Yellen [1976]. A multiproduct monopolist sells two products, goods 1 and 2, which are produced at constant marginal costs of c_1 and c_2, respectively.[4] Each consumer desires at most one unit of each good, demands each independently of his consumption of the other, and is characterized by his reservation values for each of the two goods, $(v_1,v_2) \geq 0$. Reservation values for each of the two goods are jointly distributed in the population according to the distribution function $F(v_1,v_2)$. In contrast to Adams and Yellen, we assume that this distribution possesses no atoms, and represent its density function by $f(v_1,v_2)$. Let $g_i(v_i|v_j)$ and $h_i(v_i)$ denote the conditional and marginal densities derived from $f(\cdot,\cdot)$; $G_i(v_i|v_j)$ and $H_i(v_i)$ denote the conditional and marginal distribution functions (for $i = 1,2$). Also, in order to avoid trivial outcomes, we assume that for each good i there exists a

3. In particular, in Spence's example consumer preferences have the form $\theta_x u(x) + \theta_y u(y) - P$, where x and y are the quantities of the two goods purchased, P is the total payment, θ_x and θ_y are iid on $\{\underline{\theta}, \bar{\theta}\}$, and $u(\cdot)$ is increasing, differentiable, and concave. Thus, the monopolist in Spence's example possesses the ability to completely separate the four consumer types (quantities can be varied continuously). Here, on the other hand, the monopolist faces a continuum of consumer types, but has a much cruder ability to segment consumers (since he can offer only three different bundles for sale: one unit of x, one unit of y, and a bundle of one unit of each). We discuss the relationship between our results and Spence's example further in footnotes 10 and 16.

4. Neither the assumption of two goods nor that of constant marginal costs is actually critical for the results to follow. The assumptions are made to ease the exposition and facilitate comparison with the existing literature.

positive measure of consumers who have $v_i > c_i$. Last, resale by consumers is assumed to be impossible.

The monopolist can choose one of three pricing policies. First, he could simply price each commodity separately. Second, he could offer the goods for sale only as a bundle, with a single bundle price. Third, he could offer to sell either separately or bundled, with a price for the bundle that is different from the sum of the single-good prices. Following Adams and Yellen, we shall refer to the two kinds of bundling as pure bundling and mixed bundling. We can, however, immediately rule out pure bundling as a (uniquely) optimal strategy, because mixed bundling is always (weakly) better: mixed bundling with a bundle price P_B and single-good prices $P_1 = P_B - c_2$ and $P_2 = P_B - c_1$ always yield profits at least as high as pure bundling with price P_B, and typically does better [Adams and Yellen, p. 483].[5] Thus, we now turn to a comparison of the profits obtainable from mixed bundling and unbundled sales.

III. Results

In considering possible mixed bundling strategies, it is important to distinguish between cases where the monopolist can and cannot monitor purchases. In the latter case he is effectively constrained to offer a bundle price P_B which is no larger than the sum of individual goods prices, $P_1 + P_2$, for otherwise consumers would never buy the bundle. On the other hand, if the monopolist can monitor purchases, then he faces no such constraint since he can always prevent consumers from purchasing both good 1 and good 2 separately.[6]

In what follows, we employ one additional assumption on the distribution of preferences: that $g_i(P_i|s)$ is continuous in P_i at P_i^* for all s, where P_i^* is the optimal nonbundling price for good i.[7]

5. In principle, the monopolist could also use a randomized strategy, making the allocation of the goods to a buyer stochastic. We shall ignore this possibility in the text, but see footnote 7 for a further discussion of this point.

6. The literature on single-good quantity-dependent pricing (e.g., Maskin and Riley [1984]), for example, implicitly assumes that purchases can be monitored since the monopolist does not worry about consumers purchasing multiple subquantities (though this problem disappears when quantity discounts exist just as it disappears when bundle discounts are used here).

7. It can be shown that a simple pricing policy of this sort is the optimal single-good selling strategy as long as the single-good profit function is concave in price. In particular, in that case no randomized selling procedure can improve profits. McAfee and McMillan [1988] also derive a sufficient condition for the optimal multiproduct sales policy to be nonstochastic. Their condition actually implies the condition (1) that we derive below, which ensures that bundling dominates unbundled sales.

Obviously, one sufficient condition for this to be true is that $f(v_1,v_2)$ is continuous, but we prefer to state the assumption in this nonprimitive way in order to emphasize that the set of cases in which the results apply is really much larger than that.

We begin by establishing a general sufficient condition (valid regardless of whether purchases can be monitored) for bundling to dominate unbundled sales.

PROPOSITION 1. Let (P_1^*,P_2^*) be the optimal nonbundling prices. Mixed bundling dominates unbundled sales if[8]

$$\int_0^{P_1^*} \{[1 - G_2(P_2^* \mid s)] - g_2(P_2^* \mid s)(P_2^* - c_2)\} h_1(s)ds + (P_1^* - c_1)[1 - G_2(P_2^* \mid P_1^*)]h_1(P_1^*) > 0. \tag{1}$$

Proof of Proposition 1. The proof proceeds by arguing that a local improvement can be made regardless of whether purchases can be monitored if condition (1) holds.

Suppose that the inequality in (1) is satisfied. First, introduce a bundle whose price is $P_B = P_1^* + P_2^*$ (and leave the single-good offers unchanged). Clearly, profits are unchanged since the bundle is irrelevant due to its pricing. Now consider a small increase in the price of good 2 to $\hat{P}_2 = P_2^* + \epsilon$, where $\epsilon > 0$. Note that the ability to monitor purchases is irrelevant here since $P_B < P_1^* + \hat{P}_2$. The resulting pattern of purchases, which is depicted in Figure II, is given by

Good 1 only:

(i) $v_1 - P_1^* \geq 0$

(ii) $v_2 - P_2^* \leq 0$

Good 2 only:

(i) $v_2 - P_2^* - \epsilon \geq 0$

(ii) $v_1 - P_1^* + \epsilon \leq 0$

Bundle:

(i) $v_1 + v_2 - P_1^* - P_2^* \geq 0$

(ii) $v_2 - P_2^* \geq 0$

(iii) $v_1 - P_1^* + \epsilon \geq 0.$

8. Schmalensee [1984] utilizes an analogous condition in his analysis of the special case of a Gaussian distribution of reservation values.

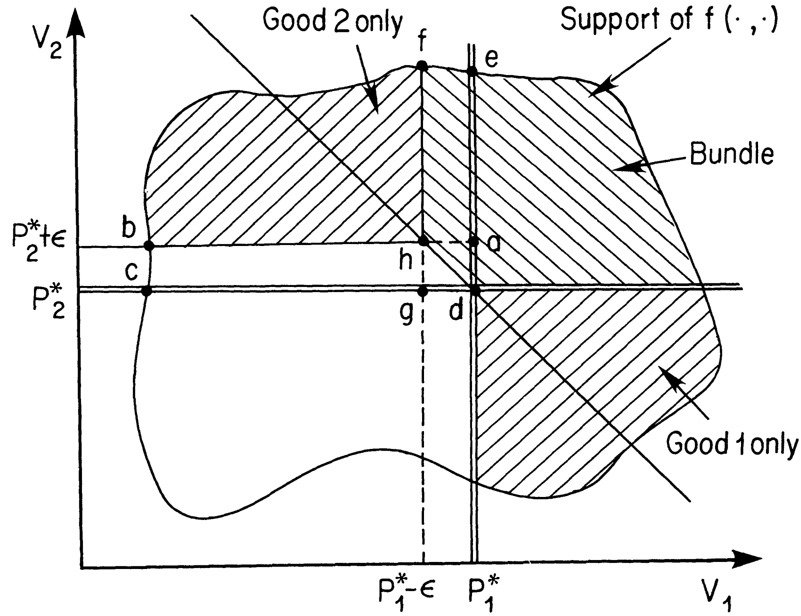

FIGURE II

Since the actions of and profits from consumers with $v_1 > P_1^*$ are unaffected by this change in P_2, we need only focus on the change in profitability from sales to those with $v_1 < P_1^*$. Profits from this group as a function of ϵ are given by

$$\zeta(\epsilon) \equiv (P_2^* + \epsilon - c_2) \int_{P_2^*+\epsilon}^{\infty} \left\{ \int_0^{P_1^*-\epsilon} f(s,q)ds \right\} dq$$
$$+ (P_1^* + P_2^* - c_1 - c_2) \int_{P_1^*-\epsilon}^{P_1^*} \left\{ \int_{P_2^*+P_1^*-s}^{\infty} f(s,q)dq \right\} ds.$$

Differentiating this expression with respect to ϵ and taking the limit as $\epsilon \longrightarrow 0$ yields the expression in condition (1) implying that $\xi'(0) > 0$ so that a strict improvement in profits would be possible through bundling.

Q.E.D.

Condition (1) is derived by starting at an initial position with the optimal independent goods pricing prices (P_1^*,P_2^*) and a bundle price equal to the sum of these prices, and then marginally increasing the price of good 2. In principle, one could also consider marginally raising the price of good 1 or marginally lowering the price of the bundle from the initial position. It is not difficult to show, however, that either of these changes gives rise to a marginal change in profits that is *identical* to that in the expression in condition (1) (this follows from the first-order conditions for P_1^* and P_2^*), and so there is no reason to consider these conditions independently.

One case of special interest is that of independently distributed reservation values. Condition (1) of Proposition 1 leads immediately to the following result:

COROLLARY 1. If v_1 and v_2 are independently distributed, then bundling dominates unbundled sales.

Proof of Corollary 1. For the case of independently distributed reservation values, condition (1) reduces to (note that $h_i(P) = g_i(P|s)$ for all (P,s) and $i = 1,2$):

$$(2) \quad H_1(P_1^*)\{[1 - H_2(P_2^*)] - h_2(P_2^*)(P_2^* - c_2)\} + (P_1^* - c_1^*)h_1(P_1^*)[1 - H_2(P_2^*)] > 0.$$

But, if P_2^* is the optimal unbundled price for good 2, then the first term in (2) is equal to zero, so that (1) reduces to

$$(3) \quad (P_1^* - c_1)h_1(P_1^*)[1 - H_2(P_2^*)] > 0.$$

Now, by the assumptions of no atoms and existence of a positive measure of valuations above cost, $(P_1^* - c_1)[1 - H_2(P_2^*)] > 0$. Also, under our continuity assumption it must be that $h_1(P_1^*) > 0$ (again, from the nonbundling first-order condition). Thus, condition (3) holds, and a local gain from bundling is possible.

Q.E.D.

It is worthwhile to consider the independence case graphically. In Figure II one can see three first-order effects of locally raising P_2.[9] First, there is a direct price effect from raising revenues received from consumers in the set $\{(v_1,v_2)|v_1 \le P_1^*, v_2 \ge P_2^*\}$. Second, sales of good 2 fall by the measure of area (*abcd*). Third, sales of good 1 increase by the measure of area (*defg*) due to consumers switching from purchasing only good 2 to purchasing the bundle. The sum of the first two of these effects is exactly the local gain if the monopolist was able to slightly raise the price of good 2 only to consumers with valuations less than P_1^*. With independence, however, this local gain must be zero at the optimal price P_2^*

9. In the case of independent distributions the set $\{(v_1,v_2) \mid f(v_1,v_2) > 0\}$ would actually be rectangular.

because the monopolist desires the same good 2 (unbundled) price regardless of a consumer's level of v_1 (this follows from the fact that the distribution of reservation values for good 2 is independent of the value of v_1). Thus, the net effect of moving to mixed bundling when reservation values are independently distributed is positive.

It is also instructive to consider graphically (Figure III) the effect of marginally lowering P_B by ϵ (recall that this change in profits is numerically identical to that of raising P_2). To first-order, the monopolist loses ϵ from consumers in area (abe), but gets consumers in area ($aefg$) to switch from buying only good 1 to also buying the bundle, and consumers in area ($bcde$) to buy the bundle instead of good 2 only; the first two of the effects, however, add up to zero due to the initially optimal level of P_2^*. What is interesting to note about this fact is that if we imagine a monopolist who can quote a price for each good i depending on a consumer's level of v_j, the monopolist would have to lower P_1 by ϵ for all $v_2 > P_2^*$ *and* P_2 by ϵ for all $v_1 > P_1^*$ in order to generate these same demand shifts, but at a loss of 2 ϵ from consumers in area ($abeh$). Thus, lowering P_B is

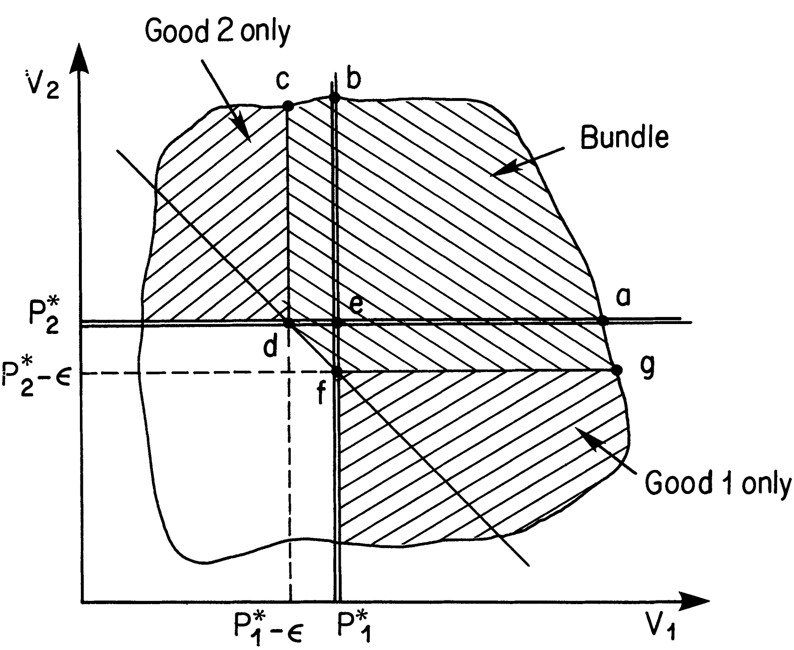

FIGURE III

worthwhile here even though lowering prices in an independent goods pricing regime would not be.[10]

Proposition 1, of course, implies that bundling is generally optimal in a much broader range of cases than just independence. The second term on the left side of inequality (1) corresponds to the rectangular area (*efgd*) in Figure II: this is the extra profit from consumers who buy both goods with bundling when without bundling they would buy only one good. This effect will always tend to generate gains from bundling. The first term in inequality (1), though, may be either positive or negative (in the independence case it is zero). We can, however, provide an intuitive sufficient condition for this term to be nonnegative: imagine again that the monopolist can observe v_j but not v_i, and let $P_i^*(v_j)$ be the monopolist's optimal price for good i conditional on knowing that a consumer's valuation for good j is v_j.[11] Then, if $P_2^*(v_1)$ is decreasing in v_1, the first term inequality (1) must be positive (since we know that $\int_0^\infty \{[1 - G_2(P_2^* \mid s)] - g_2(P_2^* \mid s)\,(P_2^* - c_2)\}\, h_1(s)ds = 0$) thereby ensuring that bundling dominates unbundled sales (regardless of whether purchases can be monitored).[12]

It might be thought that this condition—that those with low values of v_j should be charged a higher price for good i—is directly

10. The effects noted here and in the previous paragraph have interesting parallels to those in Spence's [1980] example. There he shows that if the bundles for types $(\underline{\theta},\underline{\theta})$, $(\underline{\theta},\bar{\theta})$, $(\bar{\theta},\underline{\theta})$, and $(\bar{\theta},\bar{\theta})$ are (x,x), (y,z), (z,y), and (e,e), that $z = e =$ the first-best level for type $\bar{\theta}$, while $x < x^* < y$, where x^* is the level purchased by type $\underline{\theta}$ in the independent goods (quantity-dependent) pricing scheme. The motivation for setting $y > x^*$ there is that a small reduction in the charge to type $(\bar{\theta},\bar{\theta})$ allows an increase in profits from *both* types $(\underline{\theta},\bar{\theta})$ and $(\bar{\theta},\underline{\theta})$ through an increase in their consumption that is equal to what would arise in a single-good pricing problem with just groups $(\underline{\theta},\bar{\theta})$ and $(\bar{\theta},\bar{\theta})$; an effect that closely parallels the effect here of lowering P_B. Likewise, the driving force behind having $x < x^*$ is that increasing the charges to types $(\bar{\theta},\underline{\theta})$ and $(\underline{\theta},\bar{\theta})$ and lowering x from the single-good quantity dependent pricing levels has no first-order effect, but relaxes the constraints between type $(\bar{\theta},\bar{\theta})$ and types $(\underline{\theta},\bar{\theta})$ and $(\bar{\theta},\underline{\theta})$, thereby allowing profits to be raised by increasing the latter groups' consumption levels; an effect analogous to what occurs when we raise P_2 marginally here.

11. This interpretation of condition (4) and what follows assumes that the problem of picking an optimal price for good i given a value of v_j is concave.

12. On the other hand, if the inequality in condition (1) is not satisfied, and if profits are concave in (P_1,P_2,P_B) on the set $\{(P_1,P_2,P_B) \mid P_1 + P_2 \geq P_B\}$, then single-good pricing is optimal when purchases cannot be monitored. To see this, note that the gradient of profits at the single-good optimal prices (with $P_B = P_1^* + P_2^*$), $\nabla\pi^*$, satisfies $\nabla\pi^* \cdot \Delta P \leq 0$ for all ΔP such that $\Delta P_1 + \Delta P_2 < \Delta P_B$. Unfortunately, however, we have not found any general condition under which profits are concave in this manner. In fact, the profit function in this problem appears to have inherent nonconcavities. The usual sorts of monotonicity assumptions on $f(\cdot,\cdot)$, for example, do not ensure concavity (e.g., note that the value of the shift of marginal consumers on segment *fh* in Figure II from single-good to bundled sales may increase or decrease as P_2 increases depending upon whether $(P_B - c_1 - c_2)$ is larger or smaller than $(P_1 - c_1)$).

tied to the presence of a negative correlation of reservation values. This is not completely accurate, however. Defining $\epsilon_i(P_i|v_j) = P_i \{g_i(P_i|v_j)/[1 - G_i(P_i|v_j)]\}$ to be the demand elasticity of good i conditional on valuation v_j for good j, it is easy to see that[13]

$$(4)\quad \text{sign}\, \frac{dP_i^*(v_j)}{dv_j} = \text{sign}\left\{-\frac{d\epsilon_i(P_i^*|v_j)}{dv_j}\right\}$$

$$= \text{sign}\left\{\frac{d}{dv_j}\left[\frac{1 - G_i(P_i^*|v_j)}{g_i(P_i^*|v_j)}\right]\right\}$$

$$= \text{sign}\left\{-[1 - G_i(P_i^*|v_j)]\frac{dg_i(P_i^*|v_j)}{dv_j} - g_i(P_i^*|v_j)\frac{dG_i(P_i^*|v_j)}{dv_j}\right\}.$$

Now, it can be shown that $dG_i(P_i|v_j/dv_j > 0$ (respectively, < 0) for all P_i implies a strictly negative (respectively, positive) correlation between v_1 and v_2.[14] But, the presence of the expression $dg_i(P_i|v_j)/dv_j$ in (4) (which cannot be signed a priori), indicates that the sign of the first term in condition (1) cannot be tied solely to the level of correlation between reservation values.

Our final result demonstrates that if purchases can be monitored, then mixed bundling will dominate unbundled sales for virtually any joint distribution of reservation values.

PROPOSITION 2. Let (P_1^*, P_2^*) be the optimal nonbundling prices. Suppose that the monopolist can monitor sales. Then bundling dominates unbundled sales if

$$(5)\quad \int_0^{P_1^*} \{[1 - G_2(P_2^*|s)] - g_2(P_2^*|s)(P_2^* - c_2)\}h_1(s)ds + (P_1^* - c_1)[1 - G_2(P_2^*|P_1^*)]h_1(P_1^*) \neq 0.$$

Proof of Proposition 2. If (5) is violated because the expression is positive, then Proposition 1 applies. Suppose then that the expression in (5) is negative. Again introduce a bundle with price $P_B = P_1^* + P_2^*$. Now, however, lower P_2 slightly to $P_2^* - \epsilon$, where $\epsilon >$

13. For a slightly different interpretation of the sufficient condition for bundling to be profitable, note that the expression $\{[1 - G_i(\cdot)]/g_i(\cdot)\}$ commonly arises in adverse-selection problems (see, for example, Myerson [1981], Maskin and Riley [1984], McAfee and McMillan [1987]). Its expectation is the expected difference between the first-order and second-order statistics, which is exactly the amount of rent the buyer must be left with if he is not to understate his valuation. If this informational rent decreases in the valuation of the other good, then the optimality of bundling is ensured.

14. The covariance between v_1 and v_2 can be written as $\int\{\int[v_2 - E(v_2)] g_2(v_2|v_1)dv_2\}[v_1 - E(v_1)]h_1(v_1)dv_1$. If, for example, $G_2(\cdot|v_1)$ is decreasing in v_1, then the expression in curly brackets is increasing in v_1 (see Milgrom [1981]), and thus the entire expression is positive.

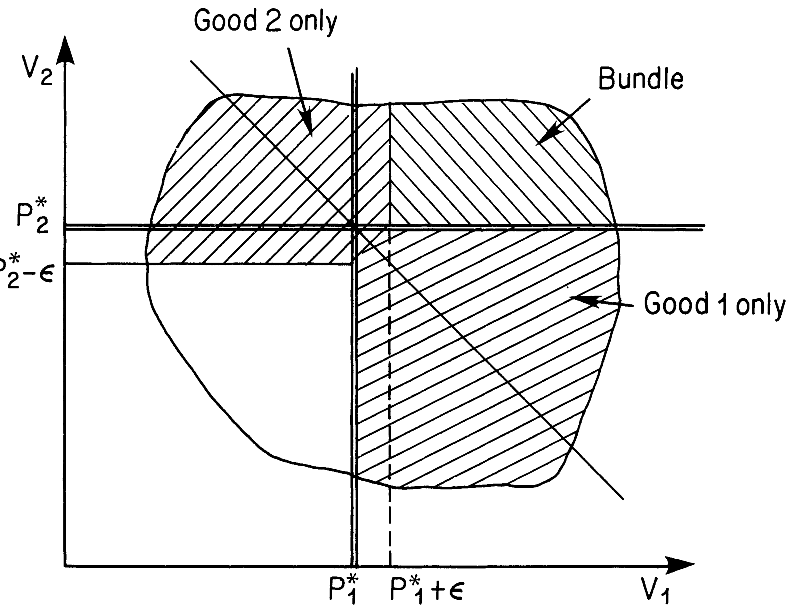

FIGURE IV

0. The resulting distribution of sales is depicted in Figure IV. Analysis of the limit of the derivative of profits with respect to ϵ as $\epsilon \longrightarrow 0$ then indicates that a gain in profits is available.[15]

Q.E.D.

It should be clear from condition (5) that independent goods pricing will virtually never be an optimal sales strategy here when purchases can be monitored.[16] By providing an additional means for the monopolist to segment groups of customers, bundling essentially always raises profits in this case.

IV. OLIGOPOLY

It is reasonably straightforward to apply the results above to the case of multiproduct oligopoly. For example, consider a duopoly

15. We omit the mathematical details; the point should be obvious from Figure IV and is analogous to the argument in the proof of Proposition 1.

16. Spence also considers the use of bundling for nonindependent distributions of preferences in his example. Interestingly, in contrast to our result here, he finds that a range of (positive) levels of correlation exists such that independent pricing is optimal. This finding arises in Spence's model because of a discontinuity in the derivative of profits with respect to a change in the charge to type $(\underline{\theta}, \bar{\theta})$ at the independent goods (quantity dependent) pricing outcome (recall footnote 10). Here, no corresponding discontinuity exists.

comprised of firm A and firm B,[17] each of which produces a version of products 1 and 2. Consumer valuations for the goods are represented by four reservation values: $(v_{1A},v_{1B},v_{2A},v_{2B})$. The firms engage in simultaneous price choices (which may involve independent goods pricing, pure bundling, or mixed bundling).

In this setting, given the pricing choice of its rival, each firm acts as a monopolist relative to the demand structure induced by its rival's prices. When will independent pricing arise as an equilibrium? Suppose that firm B is pricing its products independently at prices (P_{1B},P_{2B}). Then we can define each consumer's "pseudo-reservation value" for firm A's two products as

$$\hat{v}_1 = v_{1A} - \max\{0,v_{1B} - P_{1B}\}$$
$$\hat{v}_2 = v_{2A} - \max\{0,v_{2B} - P_{2B}\}.$$

If we let $\hat{f}(\hat{v}_1,\hat{v}_2)$ be the distribution of these induced reservation values, it is not difficult to show that independent pricing can only be a Nash equilibrium if condition (1) (or (5) if monitoring of purchases is possible) fails to hold for the pseudo-reservation values induced at the independent pricing Nash equilibrium prices. An interesting implication of this fact is that independent pricing cannot be a Nash equilibrium if the distribution of the reservation values for product 1 is independent of the distribution for product 2 since in that case the induced pseudo-reservation values are themselves independent (so Corollary 1 applies).[18]

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REFERENCES

Adams, W. J., and J. L. Yellen, "Commodity Bundling and the Burden of Monopoly," this *Journal,* XC (1976), 475–98.
Maskin, E., and J. Riley, "Monopoly with Incomplete Information," *Rand Journal of Economics,* XV (1984), 171–96.
McAfee, R. P., and J. McMillan, "Commodity Bundling by a Monopolist," mimeo, University of Western Ontario, December 1985.
——, and ——, "Auctions and Bidding," *Journal of Economic Literature,* XXV (1987), 699–738.
——, and ——, "Multidimensional Incentive Compatibility and Mechanism Design," *Journal of Economic Theory,* XLVI (1988), 335–354.

17. The points made here easily extend to any finite number of firms.
18. The reservation values for product 1 are said to be independent of those for product 2 if the density of $(v_{1A},v_{1B},v_{2A},v_{2B})$, $\phi(\ )$, can be written as $\phi(v_{1A},v_{1B},v_{2A},v_{2B}) = g(v_{1A},v_{1B}) \cdot h(v_{2A},v_{2B})$.

Milgrom, P., "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics,* XII (1981), 380–91.
Myerson, R. B., "Optimal Auction Design," *Mathematics of Operations Research,* VI (1981), 58–73.
Oi, W. Y., "A Disneyland Dilemma: Two-Part Tariffs for a Mickey Mouse Monopoly," this *Journal,* LXXXV (1971), 77–90.
Scherer, F. M., *Industrial Market Structure and Economic Performance*(Chicago, IL: Rand McNally, 1980).
Schmalensee, R., "Commodity Bundling by a Single-Product Monopolist," *Journal of Law and Economics,* XXV (1982), 67–71.
——, "Gaussian Demand and Commodity Bundling," *Journal of Business,* LVII (1984), S211–30.
Spence, A. M., "Multi-Product Quantity-Dependent Prices and Profitability Constraints," *Review of Economic Studies,* XLVII (1980), 821–42.
Stigler, G. J., "A Note on Block Booking," in G. J. Stigler, ed., *The Organization of Industry* (Homewood, IL: Richard D. Irwin, 1968).
Whinston, M. D., "Product Bundling and Correlation of Values," mimeo, Harvard University, August 1986.