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Research Article

An Introduction to Fuzzy Testing of Multialternative Hypotheses for Group of Samples with the Single Parameter: Through the Fuzzy Confidence Interval of Region of Acceptance

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Classical statistics and many data mining methods rely on "statistical significance" as a sole criterion for evaluating alternative hypotheses. It is very useful to find out the significant difference existing between the samples as well as the population or between two samples. But in this paper, the researchers try to apply the concepts of fuzzy group testing of hypothesis problem between multi group of samples of same size or different, through comparing the parameters like mean, standard deviation, and so forth. Hence we can compare multigroups such that they have the significant difference in their mean or standard deviation or other parameters through the fuzzy group testing of multihypotheses. The authors introduced and investigated the concepts very first time through fuzzy analysis that can decide which group(s) or samples can be taken for further investigation and either H_0 is rejected or accepted and hence the next discussion provides the properties of group of samples which may result in the optimized solution for the problem.

1. Introduction

The fuzzy technique can be used to generate solutions to the problems based on "*vague, ambiguous, qualitative, incomplete, or imprecise information*." Fuzzy logic is an extension of fuzzy set theory that has developed since five decades approximately. The problem of dealing with imprecision and uncertainty is a part of the wider human experiences before the introduction of fuzzy theory.

In the traditional approach for hypotheses testing, all of the concepts are assumed to be precise and well-defined. But sometimes we have to take decision in an unrealistic manner, because in realistic problems we may come across fuzzy data and fuzzy hypotheses. Sometimes the decisions on samples will not help us to proceed. Arnold [1] studied on fuzzy hypotheses testing with crisp data. The problem of testing fuzzy hypotheses when the observations were crisp was considered by Taheri and Behboodian [2]. Torabi et al. [3] used Neyman-Pearson Lemma for fuzzy hypotheses by testing with vague data. Chachi et al. [4] also studied on the same problem in the context of fuzzy decision problems. Viertl [5] studied testing fuzzy hypotheses using fuzzy data based on fuzzy test statistic.

We know that the analysis of variance (ANOVA) is a powerful statistical tool for test of significance between two or more samples. It is used in a situation where three or more samples have been considered at a time, towards the testing of hypothesis that all the samples are drawn from the same population; otherwise their parameters have the significant difference. The purpose of this analysis is to test the homogeneity of several means. This analysis found to test the null hypothesis H_0 is that all the means are equal against the alternative hypotheses that some of them are not equal. So the variance ratio

$$F = \frac{\text{variances between the samples}}{\text{variances within the samples}} \quad (1)$$

which is used for the comparison in the following designs:

(1) completely randomized design (CRD, ANOVA for one-way classification);

(2) randomized block design (RBD, ANOVA for two-way classification);

(3) latin square design (LSD, ANOVA for three-way classification).

But these designs suffer by the disadvantage of being inherently less informative than other more sophisticated layouts. This makes the design less efficient and results in less sensitivity in detecting significant effects. For example, if null hypothesis is rejected then it concludes that the treatments differ significantly but it is not clear that which samples could be considered for the further investigation also which are inconsiderable in the future analysis. So we take this for our consideration, and we assign a method which would be very useful to test the null hypothesis $H_0 : \theta = \theta_1 = \theta_2 = \cdots = \theta_n$ versus multialternative hypotheses:

$$\text{Alternative hypothesis} \begin{cases} H_1 : \theta = \theta_1 = \theta_2 \cdots = \theta_{n-1} \neq \theta_n \\ H_2 : \theta = \theta_1 = \theta_2 \cdots \neq \theta_{n-1} \neq \theta_n \\ \cdot \\ \cdot \\ H_\phi : \theta \neq \theta_1 \neq \theta_2 \cdots \neq \theta_{n-1} \neq \theta_n. \end{cases} \quad (2)$$

In the classical theory of statistical inference there is a one-to-one relationship between the parameter values for which the null hypothesis is accepted and the structure of the confidence intervals. Namely, a family of α-level acceptance regions for a statistical test concerning the parameter θ is equivalent to a certain $1 - \alpha$ family of confidence intervals for θ. The results of a statistical test, therefore, can alternatively be stated in terms of the corresponding confidence interval. By developing the concepts of fuzzy confidence intervals and fuzzy statistical tests and using the one-to-one relationship between tests and confidence intervals may be convenient to generalize a correspondence between fuzzy confidence interval and fuzzy statistical test. In such a case, we have, for example, the degree of membership of the null hypothesized fuzzy parameter in the fuzzy confidence interval as the degree of acceptability of the null hypothesis. In the following, we use such a relationship in fuzzy environment to test the hypothesis corresponding to the parameters of the multisamples simultaneously via fuzzy group testing of hypothesis by fixing the level of confidence values for different fuzzy hypotheses. In this test two or more numbers of samples can be taken for our consideration and simultaneously we tried here to check the hypothesis whether all the samples coincide with the population parameter or some of the samples have much deviated from the population.

2. Fuzzy Group Testing of Hypothesis with Confidence Intervals

2.1. The Fuzzy Group Environment

Definition 1 (see [6]). Let X be any nonempty set. A fuzzy subset μ of X is a function $\mu : X \rightarrow [0, 1]$.

Definition 2 (see [7]). A fuzzy set μ on G is called fuzzy subgroup of G if for $x, y \in G$,

(i) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$,

(ii) $\mu(x^{-1}) = \mu(x)$.

Definition 3 (see [8]). Let G be a finite group. In $2^G - \{\phi\}$, a nonempty set $\vartheta \subset 2^G - \{\phi\}$ is called a HX group on G, if ϑ is a group with respect to the algebraic operation defined by $AB = \{ab/a \in A \text{ and } b \in B\}$, in which its unit element is denoted by E.

Definition 4 (see [8]). Let μ be a fuzzy subset defined on G. Let $\vartheta \subset 2^G - \{\phi\}$ be a HX group on G. A fuzzy set λ^μ defined on ϑ is said to be a fuzzy subgroup induced by μ on ϑ or a fuzzy HX subgroup on ϑ, if, for any $A, B \in \vartheta$,

(i) $\lambda^\mu(AB) \geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}$,

(ii) $\lambda^\mu(A^{-1}) = \lambda^\mu(A)$,

where $\lambda^\mu(A) = \max\{\mu(x)/\text{for all } x \in A \subseteq G\}$.

Assumptions. Let $(\Omega, A(\vartheta_i), \mathbb{P})$ be a probability space (where Ω is a set of all possible outcomes of an experiment), $\vartheta_i \subseteq 2^G - \phi$ (collection of subsets of G such that $G_1, G_2, \ldots \subseteq \vartheta_i$ or ϑ_i is collection of G_i), and $\mathbb{P}$ is a set of probability measure. $A(\vartheta_i)$ is the collection of λ_i where $\lambda_i : G_i \rightarrow \mathcal{F}(\mathbb{R})$ is a fuzzy valued function and G_i is a random sample having distribution $f_{\underline{\theta_i}}$ with parameter $\underline{\theta_i} = (\theta_{i1}, \theta_{i2}, \theta_{i3}, \ldots, \theta_{in})$, for all $i = 1, 2, \ldots, \overline{k}$.

The fuzzy valued function $\lambda_i : G_i \rightarrow \mathcal{F}(\mathbb{R})$ is a fuzzy random variable if $X^l_{i(h)} : G_i \rightarrow \mathcal{F}(\mathbb{R})$ and $X^u_{i(h)} : G_i \rightarrow \mathcal{F}(\mathbb{R})$ are two real valued random variables for all $h \in [0, 1]$ where $\forall x \in G_i; \lambda_{ih}(x) = [X^l_{ih}, X^u_{ih}]$.

Fuzzy random variables λ_1 and λ_2 are identical if X^l_{ih}, Y^l_{ih} and X^u_{ih}, Y^u_{ih} are identical for all $h \in [0, 1]$. If $\theta = \theta_{ij}$ for all $i = 1, 2, \ldots$ for some j, then θ_{ij} is known as identity element of the group θ_i. If all θ_i are elements of the HX group ϑ, then the comparison may vary between the different collection of θ_i and the identity element of ϑ be $E = \theta_k$ which consisting the identity element "e" of G. Also it is very clear that $\lambda_{ij}(e) \geq \lambda_{ij}(x)$ for all i. The element "e" is called reaching height element (RHT) of each group of samples.

Definition 5. Here let us consider G_1, and G_2 be the two samples contain equal number of elements, considered for group testing of hypothesis. We say that it is a fuzzy random sample of size "n" from $f_{\underline{\theta_i}}$, if λ_{ij}'s are independent and identically distributed fuzzy random variables from $f_{\underline{\theta_i}}$ where $j = 1, 2, \ldots n$. It is very clear that from the definition of independent and identically distributed (i.i.d) fuzzy random variable (FRV), for fuzzy random sample, $\lambda_{i1}, \lambda_{i2}, \lambda_{i3}, \ldots, \lambda_{in} \xrightarrow{\text{i.i.d}} f_{\underline{\theta_i}}$ for $i = 1, 2$, we can see that $\lambda^l_{i1}, \lambda^l_{i2}, \lambda^l_{i3}, \ldots, \lambda^l_{in} \xrightarrow{\text{i.i.d}} f^l_{\underline{\theta_i}}$ and $\lambda^u_{i1}, \lambda^u_{i2}, \lambda^u_{i3}, \ldots, \lambda^u_{in} \xrightarrow{\text{i.i.d}} f^u_{\underline{\theta_i}}$ for all $h \in [0, 1]$.

2.2. Group Testing of Hypotheses: Classical Approach. Consider the problem of testing the null hypothesis $H_0 : \theta = \theta_1 = \theta_2$ versus multialternative hypotheses:

$$\text{Alternative hypothesis} \begin{cases} H_1 : \theta = \theta_1 \neq \theta_2, \\ H_2 : \theta = \theta_2 \neq \theta_1, \\ H_\phi : \theta \neq \theta_1 \neq \theta_2. \end{cases} \tag{3}$$

For each $\theta = \theta_1 = \theta_2$, let A $(\theta = \theta_1 = \theta_2)$ denote the acceptance region of a level α-group testing of hypothesis $H_0 : \theta = \theta_1 = \theta_2$ and $\alpha = \alpha_K$, where $K = \Omega$ or 1 or 2 or j.

If $S_i(X) = \{\theta_i : X_i \in A(\theta_i)\}$ then

$$\theta_i \in S_i(X) \Longleftrightarrow X_i \in A(\theta_i), \tag{4}$$

and hence

$$P_{\theta_i}\{\theta_i \in S_i(X)\} \geq 1 - \alpha, \quad \forall \theta_i. \tag{5}$$

We can define the area of level of significance for different hypotheses as follows:

$$Ar_i = \begin{cases} Ar_\Omega = A_{S_1(X)}(\theta_i) \cap A_{S_2(X)}(\theta_i), \\ Ar_1 = A_{S_1(X)}(\theta_i), \\ Ar_2 = A_{S_2(X)}(\theta_i), \\ Ar_3 = R - \left[A_{S_1(X)}(\theta_i) \cup A_{S_2(X)}(\theta_i)\right]. \end{cases} \tag{6}$$

The indicator function $I_{S_\zeta(X)}(\theta)$ is defined as

$$\begin{gathered} \theta \in S_\zeta(X) \text{ then } I_{S_\zeta(X)}(\theta) = 1, \\ \theta \notin S_\zeta(X) \text{ then } 1 - I_{S_\zeta(X)}(\theta) = 1. \end{gathered} \tag{7}$$

So, a confidence set can be viewed as a statement about testing hypothesis H_i, which exhibits the values for which the hypothesis is completely accepted if

$$\varphi(X) = \begin{cases} \dfrac{I_{S_\Omega(X)}(X)}{\text{Accept } H_0}, \\ \dfrac{I_{S_1(X)}(X)}{\text{Accept } H_1}, \\ \dfrac{I_{S_2(X)}(X)}{\text{Accept } H_2}, \\ \dfrac{I_{S_\phi(X)}(X)}{\text{Accept } H_\phi}. \end{cases} \tag{8}$$

The above said value of the test is a rule stating that the null hypothesis cannot be rejected if the interval contains the hypothesized value and can be rejected if the other intervals completely contain the hypothesized value of the given problem. So here directly we can have the chance for making the conclusion about the multihypotheses using the hypothesized value and the confidence interval of respective hypothesis.

Example 6. Let $G_1 = \{X_{11}, X_{12}, X_{13}, \ldots, X_{1n}\}$ and $G_2 = \{X_{21}, X_{22}, X_{23}, \ldots, X_{2n}\}$ be $\xrightarrow{\text{i.i.d}}$ from normal distribution $N(\theta, 1)$ with the unknown means θ_1 and θ_2, respectively. A confidence interval at confidence level $1 - \alpha = 1 - \alpha_K$, where $K = \Omega$ or 1 or 2 or ϕ for θ_i of the form

$$S_K(X_i) = \begin{cases} \left[\overline{X}_1 - \left(\frac{1}{\sqrt{n}}Z_1 - \frac{\alpha}{2}\right), \overline{X}_1 + \left(\frac{1}{\sqrt{n}}Z_1 - \frac{\alpha}{2}\right)\right] \\ \quad \cap \left[\overline{X}_2 - \left(\frac{1}{\sqrt{n}}Z_2 - \frac{\alpha}{2}\right), \overline{X}_2 + \left(\frac{1}{\sqrt{n}}Z_2 - \frac{\alpha}{2}\right)\right] \\ \left[\overline{X}_2 + \left(\frac{1}{\sqrt{n}}Z_1 - \frac{\alpha}{2}\right), \overline{X}_1 + \left(\frac{1}{\sqrt{n}}Z_1 - \frac{\alpha}{2}\right)\right] \\ \left[\overline{X}_2 - \left(\frac{1}{\sqrt{n}}Z_2 - \frac{\alpha}{2}\right), \overline{X}_1 - \left(\frac{1}{\sqrt{n}}Z_2 - \frac{\alpha}{2}\right)\right] \\ R - \left[\left[\overline{X}_1 - \left(\frac{1}{\sqrt{n}}Z_1 - \frac{\alpha}{2}\right), \overline{X}_1 + \left(\frac{1}{\sqrt{n}}Z_1 - \frac{\alpha}{2}\right)\right]\right. \\ \quad \left.\cup \left[\overline{X}_2 - \left(\frac{1}{\sqrt{n}}Z_2 - \frac{\alpha}{2}\right), \overline{X}_2 + \left(\frac{1}{\sqrt{n}}Z_2 - \frac{\alpha}{2}\right)\right]\right]. \end{cases} \tag{9}$$

This can be derived easily, where Z_α is the α-quartile of standard normal distribution.

Assume the two random samples with sizes $n = 21$, $\overline{X}_1 = 0.8$, and $\overline{X}_2 = 0.75$ are observed and we want to test $H_0(0.6) : \theta = 0.6$ versus the other alternative hypotheses H_1, H_2, and H_φ at level $\alpha = 0.05$. Then we can have

$$S_K(X_i) = \begin{cases} (0.39729, 1.20270) \cap (0.34729, 1.15270) \\ (1.15270, 1.20270) \\ (0.34729, 0.39729) \\ (-\infty, 0.34729) \cup (1.20270, \infty), \end{cases} \tag{10}$$

where $Z_\alpha = 1.96$ and $K = \Omega, 1, 2$, and ϕ.

Hence the graphical representations of $S_K(X_i)$'s are given in Figure 1.

Consider the approximate test statistic

$$\begin{gathered} Z_c = \frac{2\overline{X} - \left(\overline{X}_1 + \overline{X}_2\right)}{10^{2-1}\sqrt{4\left(S^2/N\right) + \left(\left(s_1^2 + s_2^2\right)/\left(n_1 + n_2\right)\right)}}, \\ \text{if 2 samples are given,} \\ Z_c = \frac{3\overline{X} - \left(\overline{X}_1 + \overline{X}_2 + \overline{X}_3\right)}{10^{3-1}\sqrt{9\left(S^2/N\right) + \left(\left(s_1^2 + s_2^2 + s_3^2\right)/\left(n_1 + n_2 + n_3\right)\right)}}, \\ \text{if 3 samples are given,} \end{gathered} \tag{11}$$

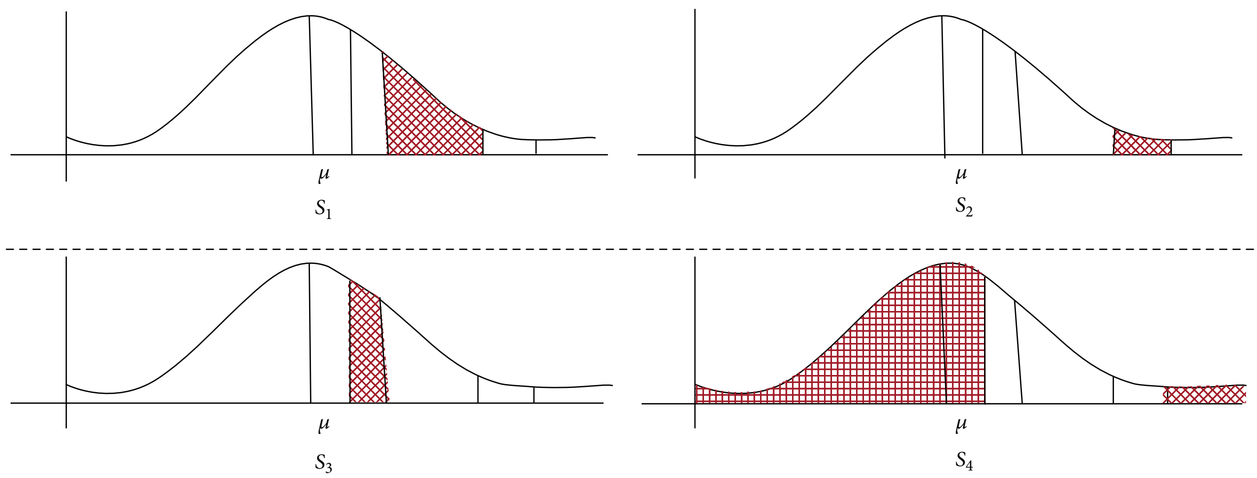


Figure 1: Graphical representations of H_0 and the alternative hypotheses.

where $\overline{X}$ = population mean (parameter), $\overline{X}_1, \overline{X}_2, \overline{X}_3$ = sample means, S^2 = population variance, s_1^2, s_2^2, s_3^2 = variances of samples, and n_1, n_2, n_3 = sample sizes.

Hence we have

$$\varphi(X) = \begin{cases} \dfrac{1}{\text{Accept } H_0}, \dfrac{0}{\text{Reject } H_1}, \dfrac{0}{\text{Reject } H_2}, \dfrac{0}{\text{Reject } H_\phi} \\ \dfrac{0}{\text{Reject } H_0}, \dfrac{1}{\text{Accept } H_1}, \dfrac{0}{\text{Reject } H_2}, \dfrac{0}{\text{Reject } H_\phi} \\ \dfrac{0}{\text{Reject } H_0}, \dfrac{0}{\text{Reject } H_1}, \dfrac{1}{\text{Accept } H_2}, \dfrac{0}{\text{Reject } H_\phi} \\ \dfrac{0}{\text{Reject } H_0}, \dfrac{0}{\text{Reject } H_1}, \dfrac{0}{\text{Reject } H_2}, \dfrac{1}{\text{Accept } H_\phi}. \end{cases} \tag{12}$$

In this case, based on the observed value of $\overline{X}_1$ and $\overline{X}_2$, we accept the null hypotheses at level of $\alpha = 0.05$ but the rejection of null hypothesis does not mean H_1 as accepted.

2.3. The Proposal in Fuzzy Environment. In this section, we investigate a procedure to provide a fuzzy test for group testing of a fuzzy parameter for fuzzy data which is based on fuzzy confidence intervals. In order to derive the degrees of acceptability of the null and alternative hypotheses, we introduce the following method in this test. It is very clear that the data available are observations of a normal fuzzy random sample with the unknown fuzzy mean θ and known variance σ^2.

That is, $X_{11}, X_{12}, \ldots, X_{1n} \xrightarrow{\text{i.i.d}} N(\theta, \sigma^2)$ and $X_{21}, X_{22}, \ldots, X_{2n} \xrightarrow{\text{i.i.d}} N(\theta, \sigma^2)$. We restrict our attention only for two sided alternative hypotheses.

(1) First we transform the original problem

$$H_0(\overline{\theta}) : \overline{\theta} = \overline{\theta}_1 = \overline{\theta}_2 \text{ versus}$$

$$\text{Alternative hypotheses} \begin{cases} H_1(\overline{\theta}) : \overline{\theta} = \overline{\theta}_1 \neq \overline{\theta}_2 \\ H_2(\overline{\theta}) : \overline{\theta} = \overline{\theta}_2 \neq \overline{\theta}_1 \\ H_\phi(\overline{\theta}) : \overline{\theta} \neq \overline{\theta}_1 \neq \overline{\theta}_2 \end{cases} \tag{13}$$

into a set of crisp group testing problems concerning h-levels of the fuzzy parameter. For each h level, based on the samples,

$$\begin{aligned} X_{1h}^l &= \left(X_{11h}^l, X_{12h}^l, \ldots, X_{1nh}^l\right), \\ X_{2h}^l &= \left(X_{21h}^l, X_{22h}^l, \ldots, X_{2nh}^l\right), \\ X_{1h}^u &= \left(X_{11h}^u, X_{12h}^u, \ldots, X_{1nh}^u\right), \\ X_{2h}^u &= \left(X_{21h}^u, X_{22h}^u, \ldots, X_{2nh}^u\right). \end{aligned} \tag{14}$$

The following classical group testing problems are solved at level α:

$$H_{0h}^l : \overline{\theta}_h^l = \overline{\theta}_{1h}^l = \overline{\theta}_{2h}^l \text{ versus } \begin{cases} H_{1h}^l : \overline{\theta}_h^l = \overline{\theta}_{1h}^l \neq \overline{\theta}_{2h}^l \\ H_{2h}^l : \overline{\theta}_h^l = \overline{\theta}_{2h}^l \neq \overline{\theta}_{1h}^l \\ H_{\phi h}^l : \overline{\theta}_h^l \neq \overline{\theta}_{1h}^l \neq \overline{\theta}_{2h}^l \end{cases}$$

$$H_{0h}^u : \overline{\theta}_h^u = \overline{\theta}_{1h}^u = \overline{\theta}_{2h}^u \text{ versus } \begin{cases} H_{1h}^u : \overline{\theta}_h^u = \overline{\theta}_{1h}^u \neq \overline{\theta}_{2h}^u \\ H_{2h}^u : \overline{\theta}_h^u = \overline{\theta}_{2h}^u \neq \overline{\theta}_{1h}^u \\ H_{\phi h}^u : \overline{\theta}_h^u \neq \overline{\theta}_{1h}^u \neq \overline{\theta}_{2h}^u. \end{cases} \tag{15}$$

We obtain the $1 - \alpha$ confidence intervals for the crisp parameters θ_h^l, θ_h^u for each $h \in [0, 1]$, denoted by $[L_1(X_{1h}^l), L_2(X_{1h}^l)]$, $[L_1(X_{2h}^l), L_2(X_{2h}^l)]$ and

$[U_1(X^l_{1h}), U_2(X^l_{1h})]$, $[U_1(X^l_{2h}), U_2(X^l_{2h})]$, respectively. The test functions are defined as follows:

$$\varphi\left(X^l_{1h}\right) = \left\{ \frac{I_{[L_1(X^l_{1h}),L_2(X^l_{1h})]}(X)}{\text{Accept } H_0}, \frac{I_{1-[L_1(X^l_{1h}),L_2(X^l_{1h})]}(X)}{\text{Reject } H_0}, \right.$$

$$\varphi\left(X^u_{1h}\right) = \left\{ \frac{I_{[U_1(X^u_{1h}),U_2(X^u_{1h})]}(X)}{\text{Accept } H_0}, \frac{I_{1-[U_1(X^u_{1h}),U_2(X^u_{1h})]}(X)}{\text{Reject } H_0} \right.$$

$$\longrightarrow 1$$

$$\varphi\left(X^l_{2h}\right) = \left\{ \frac{I_{[L_1(X^l_{2h}),L_2(X^l_{2h})]}(X)}{\text{Accept } H_0}, \frac{I_{1-[L_1(X^l_{2h}),L_2(X^l_{2h})]}(X)}{\text{Reject } H_0} \right.$$

$$\varphi\left(X^u_{1h}\right) = \left\{ \frac{I_{[U_1(X^u_{2h}),U_2(X^u_{2h})]}(X)}{\text{Accept } H_0}, \frac{I_{1-[U_1(X^u_{2h}),U_2(X^u_{2h})]}(X)}{\text{Reject } H_0} \right.$$

$$\longrightarrow 2. \tag{16}$$

Let us consider two brands of sizes 21 to be compared with the population such that their average life differs or coincides. Consider the average life time of the population to be around 20,000 working life time with the variance 4,00,000 represented by a triangular fuzzy number $\overline{\theta}_0 = (20000, 500, 500)_T$. We consider the physical problem of testing the hypothesis $H_0 : \theta_0 = \theta_{10} = \theta_{20}$ at level $\alpha = 0.05$, using fuzzy group testing of hypotheses. We have

$$\begin{aligned} X_1 &= (18850, 165, 180)_T, \\ X_2 &= (19760, 154, 172)_T, \\ X_{1h} &= \left[X^l_{1h}, X^u_{1h}\right] = [18685 + 165h, 19030 - 180h], \\ X_{2h} &= \left[X^l_{2h}, X^u_{2h}\right] = [19606 + 154h, 19932 - 172h], \\ \overline{\theta}_{0h} &= \left[\theta^l_{0h}, \theta^u_{0h}\right] = [19500 + 500h, 20500 - 500h]. \end{aligned} \tag{17}$$

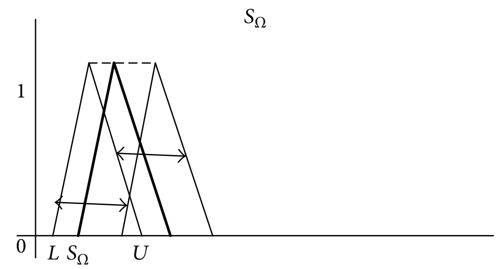
Hence, the two sided 0.95 confidence intervals for the θ^l_{1h}, θ^u_{1h} and θ^l_{2h}, θ^u_{2h} are

(i) $\theta^l_{1h} = [18543.58 + 165h, 18826.42 + 165h]$ and $\theta^u_{1h} = [18888.58 - 180h, 19171.42 - 180h]$;

(ii) $\theta^l_{2h} = [19464.58 + 154h, 19747.42 + 154h]$ and $\theta^u_{2h} = [19790.58 - 172h, 20073.42 - 172h]$.

Similarly, we can also calculate the membership values of the variables X_1 and X_2. Then the membership value of fuzzy confidence interval C_T is defined as $C_T(\theta_0)^l = W^l/(W^l + S^l)$, $C_T(\theta_0)^u = W^u/(W^u + S^u)$.

Here the values of W^l, W^u, S^l, and S^u are calculated by using the formula given in Appendix A.



$L = (a^l_0, b^l_0, c^l_0)_T$
$U = (a^u_0, b^u_0, c^u_0)_T$
$S_\Omega = [(a_1, b_1, c_1)_T$ and $(a_2, b_2, c_2)_T]$

Figure 2: Graphical representation of confidence interval for H_0 (graph for S_Ω).

The degree of membership of $\overline{\theta_0} = (20000, 500, 500)_T$ in two sided confidence interval is

$$\begin{aligned} C_T(\theta_0)^l &= \frac{W^l}{W^l + S^l}, \\ &\text{where } W^l = W^l_{1h} + W^l_{2h},\ S^l = S^l_{1h} + S^l_{2h}, \\ C_T(\theta_0)^u &= \frac{W^u}{W^u + S^u}, \\ &\text{where } W^u = W^u_{1h} + W^u_{2h},\ S^u = S^u_{1h} + S^u_{2h}. \end{aligned} \tag{18}$$

Therefore we have the confidence interval for accepting H_0 as $[C_T(\theta_0)^l, C_T(\theta_0)^u]$ (Figure 2).

Let $C_T(\theta_0) = W/(W+S)$, where $W = W^l_{1h} + W^u_{1h} + W^l_{2h} + W^u_{2h}$ and $S = S^l_{1h} + S^u_{1h} + S^l_{2h} + S^u_{2h}$.

(1) If $C_T(\theta_0) = 0.325$ (say) lies inside $[C_T(\theta_0)^l, C_T(\theta_0)^u]$, then briefly the function of acceptance of H_0 is

$$S_\Omega(X) = \left\{ \frac{0.325}{\text{Accept } H_0}, \frac{0.675}{\text{Reject } H_0} \right. . \tag{19}$$

(2) Hence we can calculate the membership values of θ_{10} and θ_{20} by using $C_T(\theta_{10}) = (W_1)/(W_1 + S_1)$; $C_T(\theta_{20}) = (W_2)/(W_2 + S_2)$ where $W_1 = W^l_1 + W^u_1$; $S_1 = S^l_1 + S^u_1$ and $W_2 = W^l_2 + W^u_2$; $S_2 = S^l_2 + S^u_2$.

Consider $C_T(\theta_{10}) = 0.156$ and $C_T(\theta_{20}) = 0.195$ and the rejection of H_0 may result as

$$S_K(X) = \left\{ \begin{array}{l} \dfrac{0.675}{\text{Reject } H_0}, \dfrac{0.156}{\text{Accept } H_1}, \dfrac{0}{\text{Reject } H_2}, \dfrac{0}{\text{Reject } H_\phi} \\ \dfrac{0.675}{\text{Reject } H_0}, \dfrac{0.675 - 0.156}{\text{Reject } H_1}, \dfrac{0.195}{\text{Accept } H_2}, \dfrac{0}{\text{Reject } H_\phi} \\ \dfrac{0.675}{\text{Reject } H_0}, \dfrac{0.675 - 0.156}{\text{Reject } H_1}, \dfrac{0.675 - 0.195}{\text{Reject } H_2}, \\ \dfrac{0.675 - (0.156 + 0.195)}{\text{Reject } H_\phi}. \end{array} \right. \tag{20}$$

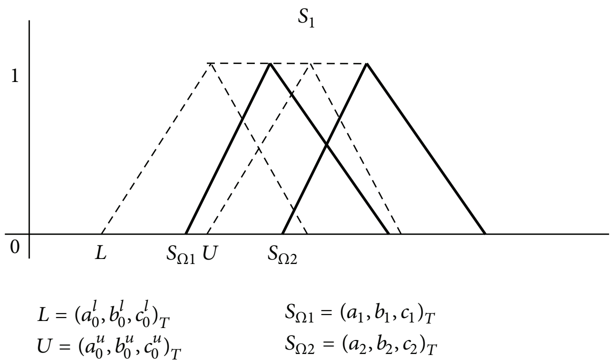


Figure 3: Graphical representation of confidence interval for H_1 (graph for S_1).

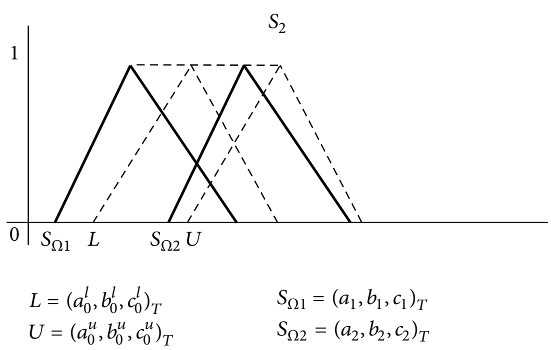


Figure 4: Graphical representation of confidence interval for H_2 (graph for S_2).

(3) If $C_T(\overline{\theta}_{10})$ lies within the interval of $C_T(\overline{\theta}_0)$ and $C_T(\overline{\theta}_{20})$ which lies partially outside of $C_T(\overline{\theta}_0)$, then the first group can be accepted and the second can be rejected with the help of fuzzy membership value of $C_T(\overline{\theta}_0)$. Hence H_1 is accepted (Figure 3).

(4) If $C_T(\overline{\theta}_{10})$ lies outside the interval of $C_T(\overline{\theta}_0)$ and $C_T(\overline{\theta}_{20})$ lies inside $C_T(\overline{\theta}_0)$, then the first sample can be rejected and the second may be accepted. Hence H_2 is partially accepted (Figure 4).

(5) If $C_T(\overline{\theta}_{10})$ and $C_T(\overline{\theta}_{20})$ both partially lie inside, then it is very clear that both the samples have significant difference by comparing the population parameter. Hence H_φ can be chosen (Figure 5).

3. Conclusion

This analysis can be extended for multi groups (more than two) and we can check the chosen groups are coinciding with the population parameter or within the groups. So in a single attempt, we can decide the suitable samples for our further investigation according to the necessity, instead of rejection of null hypothesis. This is an important advantage of this method, so we simultaneously can make the decision that would be better to improve the property analyze of the variables in future. Classical approach of a statistical problem

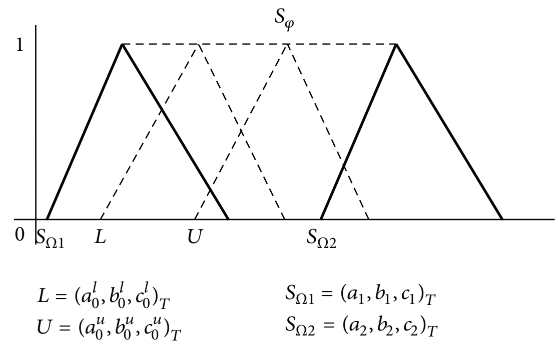


Figure 5: Graphical representation for accepting H_φ (rejection of all samples) (graph for S_φ).

will not be very precise sometimes and it will not help us to make better decisions about the properties of the variables. Many of the methods like this were already introduced by mathematicians to solve problems using concepts of fuzzy. So we try to find one of the solutions for the testing of hypotheses problem through fuzzy group testing of hypotheses which may help us in this aspect and also it may be very useful to make the broad decisions upon the groups of samples.

So through this discussion, we may get the following conclusions.

(1) This methodology can be extended to compare more than two groups; meanwhile multialternative hypotheses can be discussed and further property analysis of the variables can also be very effective for future discussion.

(2) This can be extended in future for collection of multi-subgroups of a special group called HX group. For example, number of groups (systems) can be developed by choosing different collection of elements of a single group which can be analyzed by using fuzzy group testing method to identify the better performing system(s) to complete the task in the successful manner as well as to consume time. This may be very useful to induce the performance level to the expected height. These are the expected advantages of this research work in future.

Appendices

A. Estimation of Primary Statistics

Consider

$$K^l_{\theta_1:\alpha} = \left\{h : \theta^l_{1h} \in \left[\overline{X}^l_{1h} - \frac{\sigma}{\sqrt{n_1}} z_c - \frac{\alpha}{2}, \overline{X}^l_{1h} + \frac{\sigma}{\sqrt{n_1}} z_c - \frac{\alpha}{2}\right]\right\}, \quad \text{(A.1)}$$

$$C^l_{1;\theta_1;\alpha} = \left\{h : \theta^l_{1h} < \overline{X}^l_{1h} - \frac{\sigma}{\sqrt{n_1}} z_c - \frac{\alpha}{2}\right\}, \quad \text{(A.2)}$$

Table 1: The rough values of X_{ij}^{l} and X_{ij}^{u} (app.) of two groups are explained briefly by the following table through triangular fuzzy numbers.

S. number	X_1			X_1^l	X_1^u	X_2			X_2^l	X_2^u
1	18769	160	170	18609	18939	19900	180	188	19720	20088
2	19654	165	174	19489	19828	18750	170	190	18580	18940
3	18800	168	170	18632	18970	20100	165	159	19935	20259
4	19000	175	182	18825	19182	18655	166	170	18489	18825
5	19502	160	166	19342	19668	19780	164	158	19616	19938
6	19600	156	162	19444	19762	20003	156	166	19847	20169
7	18875	184	177	18691	19052	18365	180	190	18185	18555
8	18365	165	160	18200	18525	19400	171	175	19229	19575
9	18900	160	160	18740	19060	18770	179	168	18591	18938
10	19102	160	154	18942	19256	18880	168	169	18712	19049
11	19800	182	173	19618	19973	19663	182	175	19481	19838
12	19258	163	170	19095	19428	19950	159	166	19791	20116
13	19346	155	175	19191	19521	20900	172	177	20728	21077
14	18755	154	149	18601	18904	18700	158	165	18542	18865
15	19444	135	132	19309	19576	18600	182	190	18418	18790
16	20010	169	166	19841	20176	18955	160	150	18795	19105
18	19874	150	160	19724	20034	19100	162	159	18938	19259
19	18910	165	170	18745	19080	19235	174	168	19061	19403
20	18850	135	142	18715	18992	18850	153	156	18697	19006
21	19390	158	165	19232	19555	19390	161	166	19229	19556

$$C_{2;\theta_1;\alpha}^{l}=\left\{h:\theta_{1h}^{l}>\overline{X}_{1h}^{l}+\frac{\sigma}{\sqrt{n_1}}z_c-\frac{\alpha}{2}\right\}, \quad (A.3)$$

$$K_{\theta_2:\alpha}^{l}=\left\{h:\theta_{2h}^{l}\in\left[\overline{X}_{2h}^{l}-\frac{\sigma}{\sqrt{n_2}}z_c-\frac{\alpha}{2},\overline{X}_{2h}^{l}+\frac{\sigma}{\sqrt{n_2}}z_c-\frac{\alpha}{2}\right]\right\}, \quad (A.4)$$

$$C_{1;\theta_2;\alpha}^{l}=\left\{h:\theta_{2h}^{l}<\overline{X}_{2h}^{l}-\frac{\sigma}{\sqrt{n_2}}z_c-\frac{\alpha}{2}\right\}, \quad (A.5)$$

$$C_{2;\theta_2;\alpha}^{l}=\left\{h:\theta_{2h}^{l}>\overline{X}_{2h}^{l}+\frac{\sigma}{\sqrt{n_2}}z_c-\frac{\alpha}{2}\right\}, \quad (A.6)$$

$$K_{\theta_1:\alpha}^{u}=\left\{h:\theta_{1h}^{u}\in\left[\overline{X}_{1h}^{u}-\frac{\sigma}{\sqrt{n_1}}z_c-\frac{\alpha}{2},\overline{X}_{1h}^{u}+\frac{\sigma}{\sqrt{n_1}}z_c-\frac{\alpha}{2}\right]\right\}, \quad (A.7)$$

$$C_{1;\theta_1;\alpha}^{u}=\left\{h:\theta_{1h}^{u}<\overline{X}_{1h}^{u}-\frac{\sigma}{\sqrt{n_1}}z_c-\frac{\alpha}{2}\right\}, \quad (A.8)$$

$$C_{2;\theta_1;\alpha}^{u}=\left\{h:\theta_{1h}^{u}>\overline{X}_{1h}^{u}+\frac{\sigma}{\sqrt{n_1}}z_c-\frac{\alpha}{2}\right\}, \quad (A.9)$$

$$K_{\theta_2:\alpha}^{u}=\left\{h:\theta_{2h}^{u}\in\left[\overline{X}_{2h}^{u}-\frac{\sigma}{\sqrt{n_2}}z_c-\frac{\alpha}{2},\overline{X}_{2h}^{u}+\frac{\sigma}{\sqrt{n_2}}z_c-\frac{\alpha}{2}\right]\right\}, \quad (A.10)$$

$$C_{1;\theta_2;\alpha}^{u}=\left\{h:\theta_{2h}^{u}<\overline{X}_{2h}^{u}-\frac{\sigma}{\sqrt{n_2}}z_c-\frac{\alpha}{2}\right\}, \quad (A.11)$$

$$C_{2;\theta_2;\alpha}^{u}=\left\{h:\theta_{2h}^{u}>\overline{X}_{2h}^{u}+\frac{\sigma}{\sqrt{n_2}}z_c-\frac{\alpha}{2}\right\}, \quad (A.12)$$

$$S_1^{l}=\int_{C_{1;\theta_1;\alpha}^{l}}\left[\overline{X}_{1h}^{l}-\frac{\sigma}{\sqrt{n_1}}z_c-\frac{\alpha}{2}-\theta_{1h}^{l}\right]dh+\int_{C_{2;\theta_1;\alpha}^{l}}\theta_{1h}^{l}-\left[\overline{X}_{1h}^{l}+\frac{\sigma}{\sqrt{n_1}}z_c-\frac{\alpha}{2}\right]dh, \quad (A.13)$$

$$S_2^{l}=\int_{C_{1;\theta_2;\alpha}^{l}}\left[\overline{X}_{2h}^{l}-\frac{\sigma}{\sqrt{n_2}}z_c-\frac{\alpha}{2}-\theta_{2h}^{l}\right]dh+\int_{C_{2;\theta_2;\alpha}^{l}}\theta_{2h}^{l}-\left[\overline{X}_{2h}^{l}+\frac{\sigma}{\sqrt{n_2}}z_c-\frac{\alpha}{2}\right]dh, \quad (A.14)$$

$$S_1^{u}=\int_{C_{1;\theta_1;\alpha}^{u}}\left[\overline{X}_{1h}^{u}-\frac{\sigma}{\sqrt{n_1}}z_c-\frac{\alpha}{2}-\theta_{1h}^{u}\right]dh+\int_{C_{2;\theta_1;\alpha}^{u}}\theta_{1h}^{u}-\left[-\overline{X}_{1h}^{u}+\frac{\sigma}{\sqrt{n_1}}z_c\frac{\alpha}{2}\right]dh, \quad (A.15)$$

$$S_2^{u}=\int_{C_{1;\theta_2;\alpha}^{u}}\left[\overline{X}_{2h}^{u}-\frac{\sigma}{\sqrt{n_2}}z_c-\frac{\alpha}{2}-\theta_{2h}^{u}\right]dh+\int_{C_{2;\theta_2;\alpha}^{u}}\theta_{2h}^{u}-\left[\overline{X}_{2h}^{u}+\frac{\sigma}{\sqrt{n_2}}z_c-\frac{\alpha}{2}\right]dh, \quad (A.16)$$

$$W_1^{l}=\int_{K_{\theta_1;\alpha}^{l}}\left[\theta_{1h}^{l}-\overline{X}_{1h}^{l}-\frac{\sigma}{\sqrt{n_1}}z_c-\frac{\alpha}{2}\right]dh+\int_{K_{\theta_1;\alpha}^{l}}\left[\overline{X}_{1h}^{l}+\frac{\sigma}{\sqrt{n_1}}z_c-\frac{\alpha}{2}-\theta_{1h}^{l}\right]dh, \quad (A.17)$$

$$W_2^l = \int_{K^l_{\theta_2;\alpha}} \left[\theta^l_{2h} - \overline{X}^l_{2h} - \frac{\sigma}{\sqrt{n_2}} z_c - \frac{\alpha}{2}\right] dh + \int_{K^l_{\theta_2;\alpha}} \left[\overline{X}^l_{2h} + \frac{\sigma}{\sqrt{n_2}} z_c - \frac{\alpha}{2} - \theta^l_{2h}\right] dh, \quad (A.18)$$

$$W_1^u = \int_{K^u_{\theta_1;\alpha}} \left[\theta^u_{1h} - \overline{X}^u_{1h} - \frac{\sigma}{\sqrt{n_1}} z_c - \frac{\alpha}{2}\right] dh + \int_{K^u_{\theta_1;\alpha}} \left[\overline{X}^u_{1h} + \frac{\sigma}{\sqrt{n_1}} z_c - \frac{\alpha}{2} - \theta^u_{1h}\right] dh, \quad (A.19)$$

$$W_2^u = \int_{K^u_{\theta_2;\alpha}} \left[\theta^u_{2h} - \overline{X}^u_{2h} - \frac{\sigma}{\sqrt{n_2}} z_c - \frac{\alpha}{2}\right] dh + \int_{K^u_{\theta_2;\alpha}} \left[\overline{X}^u_{2h} + \frac{\sigma}{\sqrt{n_2}} z_c - \frac{\alpha}{2} - \theta^u_{2h}\right] dh. \quad (A.20)$$

Note. We considered 2 groups to be compared; hence we need 20 formulae to conclude. Similarly if we consider "n" groups for our test, then we need "$10n$" formulae to test the hypotheses.

B. Tentative Intervals of X_1 and X_2

See Table 1.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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