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Improving the Study of Multiobjective Optimization of a Stent

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Abstract

This paper is coming with a view to extend and improve the multiobjective optimization of a stent in a fluid structure context studied in the previous works. The stent is assumed to be elastic and is modeled by Euler-Bernoulli equation. To obtain an optimal stent shape, we combine a fluid structure interaction computational method with a ϵ -multiobjective evolutionary algorithm.

Keywords: Multiobjective optimization, Stent, Fluid-Structure, Genetic algorithm, BFGS method

1 Introduction

Stent placement keep artery open but it perturbs more often blood flow. Stent shape in artery can provoke the presence of recirculation zones, blood stagnation zones, thrombosis and embolism. the aim of this work is to find optimal stents shapes in order to reduce blood stagnation and recirculation zones. As it has already been observed [2], [8] a stent associated with a higher value of shear stress is preferred because it lowers the risk of the late restenosis by reducing the presence of blood stagnation. A former paper [3], [9] had shown that a stent associated with a lower value of vorticity is preferred. Qualitative study [3], shows that the variation of vorticity and shear stress depend entirely on the variation of three parameters l , h and w .

In the previous papers [3], [7] the multiobjective optimization of a stent is studied without taking into account the effect of fluid-stent interaction in unsteady blood flow. That hypothesis simplifies the study.

In this paper, the fact that we describe the stent by an elastic structure namely Euler-Bernoulli equation and the fluid by Navier-Stokes equations is the one main contributions of this work. The coupled problem between stent and fluid is taking into account during all the search of optimal stent shape.

An other contribution of this paper is that we combine a quasi-Newton algorithm with a genetic algorithm for multiobjective optimization problem in order to find optimal Pareto front.

We propose the multiobjective optimization approach because we have two competing criteria which are the vorticity and the shear stress on the wall stent.

2 Fluid and stent properties

The symmetric properties of the problem implies us to take in the following work the half fluid domain Ω_t . The boundary of Ω_t is decomposed by : $\partial\Omega_t = \Sigma_{in} \cup \Sigma_{out} \cup \Sigma_{stent} \cup \Sigma_{sym}$ (see Figure 1. below).

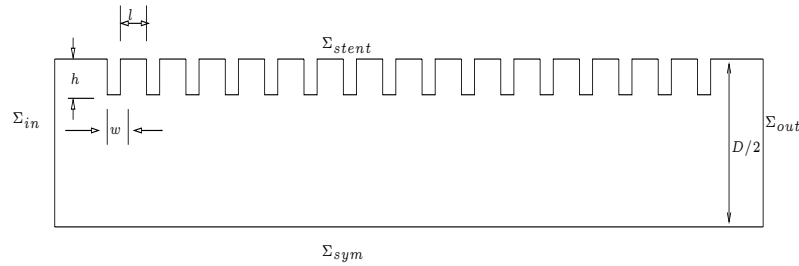


Figure 1: Simplified 2d-geometry of a stent and associated fluid domain Ω_t

The stent is represented by an elastic structure Σ_{stent} which is described by the Euler-Bernouilli equation:

$$\left\{ \begin{array}{l} \rho^S h_S \frac{\partial^2 u}{\partial t^2}(x_1, t) + \frac{E h_S^3}{12(1 - \nu^2)} \frac{\partial^4 u}{\partial x_1^4}(x_1, t) = \gamma(x_1, t), \forall (x_1, t) \in [0, L] \times [0, T] \\ u(0, t) = \frac{\partial u}{\partial x_1}(0, t) = 0, \forall t \in [0, T] \\ u(L, t) = \frac{\partial u}{\partial x_1}(L, t) = 0, \forall t \in [0, T] \\ u(x_1, 0) = u^0(x_1), \forall x_1 \in [0, L] \\ \frac{\partial u}{\partial t}(x_1, 0) = \dot{u}^0(x_1), \forall x_1 \in [0, L] \end{array} \right. \quad (1)$$

Where,

1. $u : [0, T] \times [0, L] \rightarrow \mathbf{R}$ is the transversal displacement of the structure,
2. u^0 is the initial displacement of the structure,
3. \dot{u}^0 is the initial velocity of the structure,
4. E is the Young modulus of the structure,

5. h_S is the structure thickness,
6. ρ^S is the density of the structure,
7. ν is the poisson coefficient,
8. $\gamma(x_1, t) = (-\sigma(v, p) \cdot n) \cdot e_2 \sqrt{1 + (\frac{\partial u}{\partial x_1}(x_1, t))^2}$ the external volume force acting on the structure [6],
9. L is the structure length.

The fluid is viscous, incompressible and Newtonian and is modelled by two dimensional Navier Stokes equations.

Let

$$v : [0, T] \times \Omega_t \rightarrow \mathbf{R}^2$$

the fluid velocity vector and

$$p : [0, T] \times \Omega_t \rightarrow \mathbf{R}$$

the fluid pressure.

We will find the couple (v, p) such that:

$$\left\{ \begin{array}{ll} \rho^F (\frac{\partial v}{\partial t} + (v \cdot \nabla) v) - \mu \Delta v + \nabla p = f & \mathbf{in} \quad [0, T] \times \Omega_t \\ \nabla \cdot v = 0 & \mathbf{in} \quad [0, T] \times \Omega_t \\ v(t = 0) = v_0 & \mathbf{in} \quad \Omega_F \\ -pIn + \mu \nabla v \cdot n = p_{in}In & \mathbf{on} \quad [0, T] \times \Sigma_{in} \\ -pIn + \mu \nabla v \cdot n = 0 & \mathbf{on} \quad [0, T] \times \Sigma_{out} \\ v(x_1, \frac{D}{2} + u(x_1, t)) = (0, \frac{\partial u}{\partial t}(x_1, t)) & \mathbf{on} \quad [0, T] \times \Sigma_{stent} \\ v_2 = 0 & \mathbf{on} \quad [0, T] \times \Sigma_{sym} \\ \frac{\partial v_1}{\partial x_2} = 0 & \mathbf{on} \quad [0, T] \times \Sigma_{sym} \end{array} \right. \quad (2)$$

Where

1. I is the identity matrix,
2. n is the unit outward vector normal to $\partial\Omega_t = \Sigma_{in} \cup \Sigma_{out} \cup \Sigma_{stent} \cup \Sigma_{sym}$,
3. v_0 is the initial condition of the fluid,
4. μ is the viscosity of the fluid,
5. ρ^F is the density of the fluid,
6. f is the volume force of the fluid,
7. p_{in} is the boundary conditions imposed of the pressure,

8. at outflow Σ_{out} , We impose free boundary condition of the pressure,
9. v_1 is the first component of vector v ,
10. v_2 is the second component of vector v ,
11. Σ_{sym} is symmetric axis,
12. on Σ_{sym} the no penetration condition $v \cdot n = v_2 = 0$ and the continuity stress tensor $\sigma \cdot n = \frac{\partial v_1}{\partial x_2} = 0$ are imposed.

We have a fluid structure interaction problem. The deformation of the stent depends on the fluid flow through the vessel and the domain occupied by the fluid Ω_t depends on the displacement of the vessel wall. The computational method to solve this coupled problem is done in [6].

2.1 Multiobjective optimization problem

The multiobjective optimization problem is defined as follow:
to maximize

$$J_1(l, h, w) = \frac{1}{length(\Gamma_\omega)} \sqrt{\int_0^T \int_{\Gamma_\omega} \tau_\omega^2 d\Gamma_\omega dt}$$

and to minimize

$$J_2(l, h, w) = \frac{1}{area(\omega)} \sqrt{\int_0^T \int_\omega |\nabla \times v|^2 d\Omega dt}$$

simultaneously
such that

$$(l, h, w) \in [l_{down}, l_{up}] \times [h_{down}, h_{up}] \times [w_{down}, w_{up}]$$

Where,

1. $\nabla \times v = \frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}$ is the vorticity,
2. $\tau_\omega = \mu \left(\frac{\partial v_2}{\partial x} + \frac{\partial v_1}{\partial y} \right)$ is the shear stress,
3. $l_{down}, l_{up}, h_{down}, h_{up}, w_{down}, w_{up}$ are limits imposed on the design parameters.

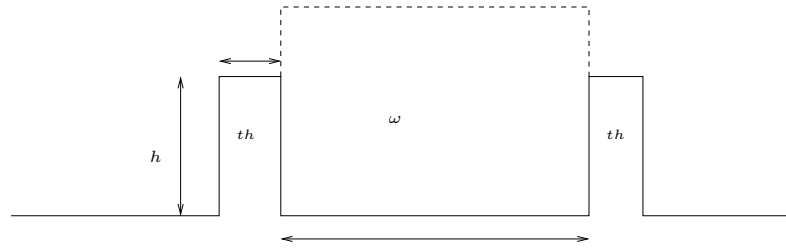


Figure 2: Definition of the integration domains ω and Γ_ω

2.2 Description of the computational method

To find the optimal stent shapes, On the one hand, we solve the fluid-stent interaction problem thanks to the method developed by Mbaye et al [6]. In the later paper authors introduce an approximation of the structure equation by modal decomposition. The coupled problem is translated into an optimization problem. Using ALE formulation, Newmark scheme and a quasi-Newton algorithm namely BFGS method we compute the displacement of the structure, the velocity and the pressure of the fluid, for more details of this method see [6]. On the other hand, we combine the fluid structure interaction computational method with the ϵ -multiobjective evolutionary algorithm developed by K. Deb et al. [4] and freely available at the site <http://www.iitk.ac.in/kangal/soft.htm> in order to maximize J_1 and minimize J_2 simultaneously.

3 Numerical results

3.1 Variational formulation for the fluid equation

Let W and Q the variational spaces:

$$\begin{aligned} W &= \left\{ w \in (H^1(\Omega_t))^2 : w = \hat{w}(\mathcal{A}_t^{-1}), w = 0 \text{ on } \Sigma_{stent} \text{ and } w_2 = 0 \text{ on } \Sigma_{sym} \right\} \\ Q &= \left\{ q \in L^2(\Omega_t) : q = \hat{q}(\mathcal{A}_t^{-1}); \hat{q} \in \hat{Q} \right\}, \end{aligned} \quad (3)$$

where

- w_2 is the second component of w .
- $\mathcal{A}_t : \bar{\Omega}_0 \rightarrow \bar{\Omega}_t$ is a C^1 -diffeomorphism defined as follow:

$$\mathcal{A}_t(\hat{x}_1, \hat{x}_2) = (x_1, x_2) = \begin{cases} x_1 = \hat{x}_1, & \forall (\hat{x}_1, \hat{x}_2) \in \Omega_0 \forall t \in (0, T) \\ x_2 = \frac{H+u(\hat{x}_1, t)}{H} \hat{x}_2, & \forall (\hat{x}_1, \hat{x}_2) \in \Omega_0 \forall t \in (0, T) \end{cases} \quad (4)$$

The transformation \mathcal{A}_t permits us to introduce the ALE formulation and to compute the mesh velocity [6].

- Ω_0 is the reference domain,
- $\hat{Q} = L^2(\Omega_0)$,
- $\hat{w} \in \hat{W} = \{ \hat{w} \in (H^1(\Omega_0))^2 : \hat{w} = 0 \text{ on } \Sigma_{stent} \text{ and } \hat{w} \cdot n = 0 \text{ on } \Sigma_{sym} \}$

Now, we can introduce the variational formulation for the fluid equations for all $t \in [0, T]$, find the velocity $v \in W$ and the pressure $p \in Q$ such that

$$\left\{ \begin{array}{l} \left(\frac{\partial v}{\partial t} |_{\hat{x}}, w \right)_{0, \Omega_t} + c(v, v, w) + a(v, w) + b(w, p) = F(w), \quad \forall w \in W \\ b(v, q) = 0, \quad \forall q \in Q \end{array} \right. \quad (5)$$

Where

$$\begin{aligned} \left(\frac{\partial v}{\partial t} |_{\hat{x}}, w \right)_{0, \Omega_t} &= \frac{d}{dt} \int_{\Omega_t} \rho^F v \cdot w \, dx - \int_{\Omega_t} \rho^F v \cdot w (\nabla \cdot \vartheta) \, dx, \\ c(v, v, w) &= \int_{\Omega_t} \rho^F ((v - \vartheta) \cdot \nabla) v \cdot w \, dx \\ a(v, w) &= \int_{\Omega_t} \mu \nabla v \cdot \nabla w \, dx \\ b(w, p) &= - \int_{\Omega_t} \nabla \cdot w \cdot p \, dx \\ F(w) &= \int_{\Omega_t} f \cdot w \, dx + \int_{\Sigma_1} p_{in} w \cdot n \, d\sigma, \\ \vartheta(x, t) &= \frac{\partial \mathcal{A}_t}{\partial t}(\hat{x}) = \left(0, \frac{\partial u}{\partial t}(x_1, t) \frac{x_2}{H+u(x_1, t)} \right)^T \end{aligned} \quad (6)$$

and $\vartheta(x, t)$ is the mesh velocity.

3.2 Spatial discretization

Let W_h be the finite element approximation spaces for the fluid velocity obtained from W by using the mixed finite element $P^1 + bubble/P^1$, and let Q_h be also the finite element approximation spaces for the fluid pressure obtained from Q by using the finite element P^1 :

$$\begin{aligned} W_h &= \{ w_h \in (C^0(\bar{\Omega}_h))^2 / \forall K \in \mathcal{T}_h, w_h = \hat{w}_h(\mathcal{A}_t^{-1}), \quad w_{h|K} \in P_1 + bubble, \\ &\quad (w_2)_h = 0 \text{ on } \Sigma_{sym}, \quad w_h = 0 \text{ on } \Sigma_{stent} \}, \end{aligned} \quad (7)$$

and

$$Q_h = \{ q_h \in C^0(\bar{\Omega}_h) / \forall K \in \mathcal{T}_h, q_h = \hat{q}_h(\mathcal{A}_t^{-1}), \quad q_{h|K} \in P_1 \}.$$

we have the variational formulation defined as follow: find the velocity $v_h \in W_h$ which satisfies the Dirichlet conditions $v_h(x_1, H + u(x_1, t)) \approx (0, \dot{u}(x_1, t))$ on Σ_{stent} and the pressure $p_h \in Q_h$ such that:

$$\left\{ \begin{aligned} \left(\frac{\partial v_h}{\partial t} |_{\hat{x}}, w_h \right)_{0, \Omega_t} + c(v_h, v_h, w_h) + a(v_h, w_h) + b(w_h, p_h) &= F(w_h), \quad \forall w_h \in W_h \\ b(v_h, q_h) &= 0, \quad \forall q_h \in Q_h \end{aligned} \right. \quad (8)$$

3.3 Time discretization

Let Δt the time step and $t_n = n\Delta t$ the time for n iterations where $n \in \mathbb{N}$. Let the following variational spaces:

$$W_h^{n+1} = \{w_h \in (C^0(\overline{\Omega}_h^{n+1}))^2 / \forall K \in \mathcal{T}_h, w_h = \hat{w}_h(\mathcal{A}_{t_{n+1}}^{-1}), w_h|_K \in P_1 + \text{bubble}, (w_2)_h = 0 \text{ on } \Sigma_{sym}, w_h = 0 \text{ on } \Sigma_{stent}\}, \quad (9)$$

and

$$Q_h^{n+1} = \{q_h \in C^0(\overline{\Omega}_h^{n+1}) / \forall K \in \mathcal{T}_h, q_h = \hat{q}_h(\mathcal{A}_{t_{n+1}}^{-1}), q_h|_K \in P_1\}.$$

The time integration scheme is based on the implicit Euler approximation. Knowing $\Omega_h^n, v_h^n \in W_h^n$ and $p_h^n \in Q_h^n$, find $\Omega_h^{n+1}, (v_h^{n+1}, p_h^{n+1}) \in W_h^{n+1} \times Q_h^{n+1}$ for all $(w, q) \in W_h^{n+1} \times Q_h^{n+1}$ such that:

$$\left\{ \begin{aligned} &\frac{1}{\Delta t} \int_{\Omega_h^{n+1}} \rho^F v_h^{n+1} \cdot w \, dx - \frac{1}{\Delta t} \int_{\Omega_h^n} \rho^F v_h^n \cdot w \, dx - \int_{\Omega_h^{n+1}} \rho^F v_h^{n+1} \cdot w (\nabla \cdot \vartheta^{n+1}) \, dx \\ &\quad + \int_{\Omega_h^{n+1}} \rho^F [(v_h^n - \vartheta^{n+1}) \cdot \nabla] v_h^{n+1} \cdot w \, dx + \int_{\Omega_h^{n+1}} \mu \nabla v_h^{n+1} \cdot \nabla w \, dx \\ & - \int_{\Omega_h^{n+1}} \nabla \cdot w p_h^{n+1} \, dx = \int_{\Omega_h^{n+1}} f^{n+1} \cdot w \, dx - \int_{\Sigma_{in}} p_{in}(\cdot, t_{n+1}) w \cdot n \, d\sigma, \quad \forall w \in W_h^{n+1} \\ &\quad - \int_{\Omega_h^{n+1}} \nabla \cdot v_h^{n+1} q \, dx = 0, \quad \forall q \in Q_h^{n+1} \\ & v_h^{n+1}(x_1, H + u(x_1, t_{n+1}), t_{n+1}) = (0, \dot{u}^{n+1})^T, \text{ on } \Sigma_{stent}^{n+1}, \quad 0 < x_1 < L \end{aligned} \right. \quad (10)$$

3.4 Time discretization of the structure equation

To discretize the structure equation, we use the modal decomposition and the Newmark scheme (for more details see [6].)

3.5 Boundary conditions

The boundary conditions imposed to the pressure [6]:

$$p_{in}(x_1, x_2, t) = \begin{cases} 10^3(1 - \cos(\frac{\pi t}{0.0025})), & (x_1, x_2) \in \Sigma_1, 0 \leq t \leq 0.005 \\ 0, & (x_1, x_2) \in \Sigma_1, 0.005 \leq t \leq T \end{cases} \quad (11)$$

3.6 Parameters value related to fluid

The fluid viscosity is $\mu = 0.035 \frac{g}{cm \cdot s}$, the density of the fluid is $\rho^F = 1 \frac{g}{cm^3}$, the volume forces $f = (0, 0)$ and the width of artery is $D = 0.4cm$ and the length of the artery is between $[1.88, 3.872]$. Time step is $\Delta t = 0.5ms$, and $v_0 = 0$.

3.7 Parameters value related to stent

The density of the structure is $\rho^S = 4.51 \frac{g}{cm^3}$, its Young modulus is $E = 110.10^{10} \frac{g}{cm \cdot s^2}$, its poisson coefficient is $\nu = 0.33$ and its thickness is $h_S = 0.01cm$.

3.8 Parameters value related to stent

The three stent design parameters $(l, w, h) \in [0.1, 0.2] \times [0.005, 0.017] \times [0.01, 0.04]$ (*incm*).

3.9 Parameters value related to genetic algorithm

We use the following parameters for the genetic algorithm: a population of 60 individuals, number of generations 50, the probability of mutation $p_m = 0.33$, the probability of crossover $p_c = 0.9$ and the seed $p_s = 0.123$.

4 Numerical simulations

l	w	h	J_1	J_2
0.103230	0.007831	0.027290	0.088440	12.106800
0.100609	0.014214	0.012138	0.094012	20.976600
0.174287	0.014452	0.037666	0.020916	1.439360
0.199600	0.013776	0.038084	0.004759	0.355302
0.102404	0.013559	0.012438	0.115140	24.660000
0.101965	0.016628	0.038707	0.042090	3.543920
0.118421	0.010580	0.037884	0.072433	6.532220

Table 1: Set of optimal solutions at $t = 2.5 ms$.

l	w	h	J_1	J_2
0.101258	0.006591	0.038870	0.120832	14.209000
0.101258	0.012545	0.039458	0.085801	8.909340
0.110229	0.008405	0.014031	0.196400	43.475300
0.110549	0.006614	0.015282	0.147732	28.601400
0.118381	0.013138	0.037243	0.063835	6.764010
0.103829	0.006033	0.014950	0.173769	33.411800
0.174370	0.014539	0.039114	0.043150	4.329730
0.196559	0.005972	0.036434	0.026643	2.541000
0.110201	0.012948	0.018666	0.128662	20.732600

Table 2: Set of optimal solutions at $t = 5 ms$.

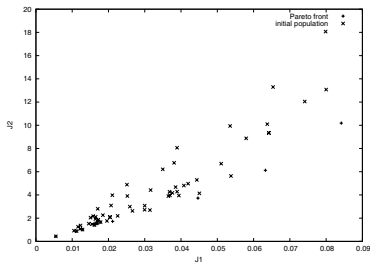


Figure 3: Population initial and Pareto front at $t = 2.5 ms$.

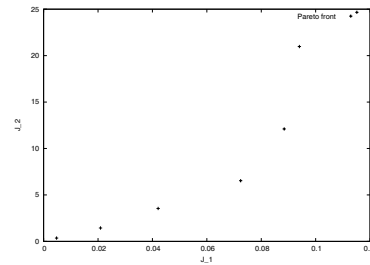


Figure 4: Set of optimal solutions on Pareto front at $t = 2.5 ms$.

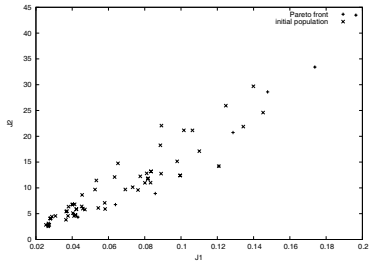


Figure 5: Population initial and Pareto front at $t = 5 ms$.

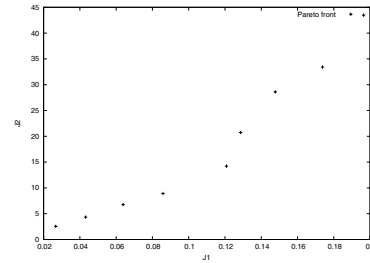


Figure 6: Set of optimal solutions on Pareto front at $t = 5 ms$.

According to optimal parameters $v(l, w, h)$, we represent some optima stents (see the following Figures):

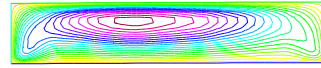
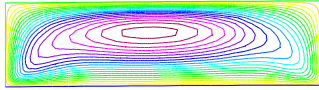


Figure 7: Streamlines between two struts at $t = 2.5$ ms.

Figure 8: Streamlines between two struts at $t = 2.5$ ms.

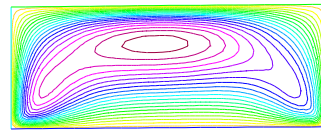
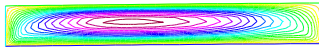


Figure 9: Streamlines between two struts at $t = 5$ ms.

Figure 10: Streamlines between two struts at $t = 5$ ms.

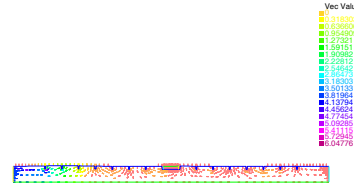
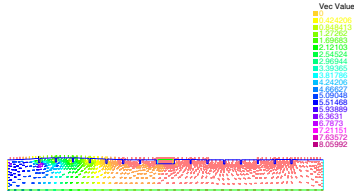


Figure 11: Velocity streamlines at $t = 2.5 \text{ ms}$.

Figure 12: Velocity streamlines at $t = 2.5 \text{ ms}$.

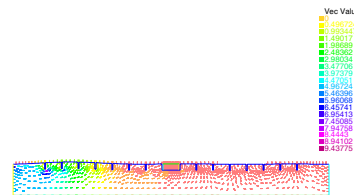
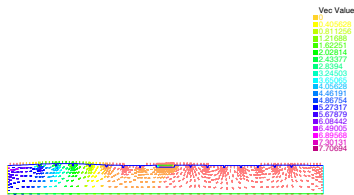


Figure 13: Velocity streamlines at $t = 5 \text{ ms}$.

Figure 14: Velocity streamlines at $t = 5 \text{ ms}$.

4.1 Results and discussion

We observe again a significant advance of Pareto front and a good distribution of the optimal solutions. The difference between the two approaches, without or with taking into account the elasticity of the stent, is situated at the level of the optimal parameters found. In this work, we note that the parameter l takes at present more dispersed values than the previous case where the wall artery is supposed rigid whereas h keeps its opposed character by comparison with two criterias. If h goes to h_{up} then we observe that J_1 and J_2 decrease simultaneously, whereas if h goes to h_{down} then J_1 and J_2 increase simultaneously.

To sum up, the fact that we take into account the interaction fluid-stent confirm the previous tendency but bring a more strong sensibility on a par with the distance between struts than two criterias.

5 Conclusion

A combination of multiobjective optimization based on genetic algorithm and a computational method of fluid-stent interaction problem is used to find a set of optimal solutions. The fact that we take into account the fluid-stent interaction in the multiobjective optimization problem permits us to obtain good distribution of the solutions on the Pareto front. Once the solutions are obtained, the designer may be able to choose a final design with further considerations.

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