

SENSITIVITY ANALYSIS OF SOLUTIONS FOR A SYSTEM OF GENERALIZED PARAMETRIC NONLINEAR QUASIVARIATIONAL INEQUALITIES

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A new class of system of generalized parametric nonlinear quasivariational inequalities involving various classes of mappings is introduced and studied. With the properties of maximal monotone mappings, the equivalence between the class of system of generalized parametric nonlinear quasivariational inequalities and a class of fixed point problems is proved and an iterative algorithm with errors is constructed. A few existence and uniqueness results and sensitivity analysis of solutions are also established for the system of generalized nonlinear parametric quasivariational inequalities and some convergence results of iterative sequence generated by the algorithm with errors are proved.

1. Introduction

Recently, variational inequalities constitute an important modelling tool in pure and applied mathematics. In 1996, Zhu and Marcotte [28] introduced and investigated a class of system of variational inequalities in R^n . Afterwards, Nie et al. [21], Verma [22, 23, 24, 25, 26], Wu et al. [27], and others studied the approximation and solvability of a few kinds of various systems of variational inequalities in Hilbert spaces. Moreover, Agarwal et al. [1], Dafermos [3], Dong et al. [4], and Liu et al. [20], and others considered the sensitivity of solutions for several kinds of parametric variational inequalities in Hilbert spaces.

Motivated and inspired by the research work [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28], in this paper, we introduce and study a new class of system of generalized parametric nonlinear quasivariational inequalities involving various classes of mappings in Hilbert spaces. Using some properties of the maximal monotone mapping, we prove the equivalence between the class of system of generalized parametric nonlinear quasivariational inequalities and a class of fixed point problems and also construct an iterative algorithm with errors for solving the system of generalized parametric nonlinear quasivariational inequalities. We also establish a few existence and uniqueness results as well as the sensitivity analysis of solutions for the system of generalized nonlinear parametric quasivariational inequalities, and prove some convergence results of iterative sequence generated by the algorithm with errors. The results presented in this paper extend, improve, and unify some known results in [21, 26, 27] and others.

2. Preliminaries

Let H be a real Hilbert space with an inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$, respectively. Let K be a nonempty closed convex subset of H and let G be an open subset of H in which the parameter λ takes values. Suppose that $M : H \times G \rightarrow 2^H$ is such that for each $\lambda \in G$, $M(\cdot, \lambda) : H \rightarrow 2^H$ is maximal monotone. Let $S, T : H \times G \rightarrow H$ be any mappings, let ρ and β be positive constants, and let f and g be arbitrary elements in H . For each $\lambda \in G$, we consider the following problem: find $x, y \in H$ such that

$$\begin{aligned} 0 &\in \rho(S(y, \lambda) - T(y, \lambda) - f) + x - y + \rho M(x, \lambda), \\ 0 &\in \beta(S(x, \lambda) - T(x, \lambda) - g) + y - x + \beta M(y, \lambda), \end{aligned} \quad (2.1)$$

which is known as the *system of generalized parametric nonlinear quasivariational inequalities*.

If $S(x, \lambda) = x$, $T(x, \lambda) = T(x)$, and $M(x, \lambda) = \partial\varphi(x)$ for any $(x, \lambda) \in H \times G$, where $\partial\varphi$ denotes the subdifferential of a proper, convex, and lower semicontinuous functional $\varphi : H \rightarrow R \cup \{+\infty\}$, then the problem (2.1) reduces to the following problem: determine elements $x, y \in H$ such that

$$\begin{aligned} \langle \rho(S(y) - T(y) - f) + x - y, u - x \rangle &\geq \rho\varphi(x) - \rho\varphi(u), \\ \langle \beta(S(x) - T(x) - g) + y - x, u - y \rangle &\geq \beta\varphi(y) - \beta\varphi(u) \quad \forall u \in H, \end{aligned} \quad (2.2)$$

which is said to be the *system of generalized nonlinear variational inequalities* studied by Nie et al. in [21].

In case $\varphi = \delta_K$, where δ_K denotes the indicator function of the nonempty closed convex subset K of H , then the problem (2.2) reduces to the following problem:

$$\begin{aligned} \langle \rho(S(y) - T(y) - f) + x - y, u - x \rangle &\geq 0, \\ \langle \beta(S(x) - T(x) - g) + y - x, u - y \rangle &\geq 0 \quad \forall u \in K, \end{aligned} \quad (2.3)$$

which was introduced and studied by Wu et al. in [27].

If $f = g = T = 0$, then the problem (2.3) is equivalent to finding $x, y \in K$ such that

$$\begin{aligned} \langle \rho S(y) + x - y, u - x \rangle &\geq 0, \\ \langle \beta S(x) + y - x, u - y \rangle &\geq 0 \quad \forall u \in K, \end{aligned} \quad (2.4)$$

which is called the *system of nonlinear variational inequalities* and has been introduced and studied by Verma [26].

For suitable and appropriate choices of the elements f and g and the mappings S and T , one can obtain various new and previously known systems of variational inequalities as special cases of the system of generalized parametric nonlinear quasivariational inequalities (2.1).

We recall the following concept.

Definition 2.1 [1]. Let $M : H \times G \rightarrow 2^H$ be such that for each $\lambda \in G$, $M(\cdot, \lambda) : H \rightarrow 2^H$ is maximal monotone. Then the implicit resolvent mapping $J_\rho^{M(\cdot, \lambda)}$ associated with $M(\cdot, \lambda)$ is defined by $J_\rho^{M(\cdot, \lambda)}(u) = (I + \rho M(\cdot, \lambda))^{-1}(u)$ for all $u \in H$, where $\rho > 0$ is a constant and I is the identity mapping.

It is known that

$$\|J_\rho^{M(\cdot, \lambda)}(x) - J_\rho^{M(\cdot, \lambda)}(y)\| \leq \|x - y\| \quad \forall x, y \in H. \tag{2.5}$$

Definition 2.2. Let $S : H \times G \rightarrow H$ be a mapping.

(1) S is said to be t -Lipschitz continuous with respect to the first argument if there exists a constant $t > 0$ satisfying

$$\|S(x, \lambda) - S(y, \lambda)\| \leq t\|x - y\| \quad \forall (x, y, \lambda) \in H \times H \times G. \tag{2.6}$$

(2) S is said to be t -strongly monotone with respect to the first argument if there exists a constant $t > 0$ satisfying

$$\langle x - y, S(x, \lambda) - S(y, \lambda) \rangle \geq t\|x - y\|^2 \quad \forall (x, y, \lambda) \in H \times H \times G. \tag{2.7}$$

(3) S is said to be t -generalized pseudocontractive with respect to the first argument if there exists a constant $t > 0$ satisfying

$$\langle x - y, S(x, \lambda) - S(y, \lambda) \rangle \leq t\|x - y\|^2 \quad \forall (x, y, \lambda) \in H \times H \times G. \tag{2.8}$$

(4) S is said to be t -relaxed Lipschitz with respect to the first argument if there exists a constant $t > 0$ satisfying

$$\langle x - y, S(x, \lambda) - S(y, \lambda) \rangle \leq -t\|x - y\|^2 \quad \forall (x, y, \lambda) \in H \times H \times G. \tag{2.9}$$

(5) S is said to be t -relaxed monotone with respect to the first argument if there exists a constant $t > 0$ satisfying

$$\langle x - y, S(x, \lambda) - S(y, \lambda) \rangle \geq -t\|x - y\|^2 \quad \forall (x, y, \lambda) \in H \times H \times G. \tag{2.10}$$

LEMMA 2.3 [5]. Let $\{a_n\}_{n \geq 0}$, $\{b_n\}_{n \geq 0}$, $\{c_n\}_{n \geq 0}$, and $\{t_n\}_{n \geq 0}$ be four sequences of nonnegative numbers satisfying

$$a_{n+1} \leq (1 - t_n)a_n + t_nb_n + c_n \quad \forall n \geq 0, \tag{2.11}$$

where $\{t_n\}_{n \geq 0} \subseteq [0, 1]$, $\sum_{n=0}^\infty t_n = +\infty$, $\lim_{n \rightarrow \infty} b_n = 0$, and $\sum_{n=0}^\infty c_n < \infty$. Then $\lim_{n \rightarrow \infty} a_n = 0$.

3. Main results

The goal of this section is to establish a few existence and uniqueness results as well as sensitivity analysis of solutions for the system of generalized parametric nonlinear quasivariational inequalities (2.1), and prove some convergence results of iterative sequences generated by the algorithm with errors which is based on the following Lemma 3.1.

LEMMA 3.1. Let $S, T : H \times G \rightarrow H$ be mappings, let ρ and β be positive constants, let f and g be arbitrary elements in H , and $\lambda \in G$. Then the following statements are equivalent:

(a) the system of generalized parametric nonlinear quasivariational inequalities (2.1) has a solution $(x, y) \in H \times H$;

(b) there exists $(x, y) \in H \times H$ satisfying

$$\begin{aligned} x &= J_{\rho}^{M(\cdot, \lambda)}(y - \rho(S(y, \lambda) - T(y, \lambda) - f)), \\ y &= J_{\beta}^{M(\cdot, \lambda)}(x - \beta(S(x, \lambda) - T(x, \lambda) - g)); \end{aligned} \tag{3.1}$$

(c) the mapping $F(\cdot, \lambda) : H \rightarrow H$ defined by

$$\begin{aligned} F(u, \lambda) &= J_{\rho}^{M(\cdot, \lambda)} \left[J_{\beta}^{M(\cdot, \lambda)}(u - \beta(S(u, \lambda) - T(u, \lambda) - g)) \right. \\ &\quad \left. - \rho \left[S \left(J_{\beta}^{M(\cdot, \lambda)}(u - \beta(S(u, \lambda) - T(u, \lambda) - g)), \lambda \right) \right. \right. \\ &\quad \left. \left. - T \left(J_{\beta}^{M(\cdot, \lambda)}(u - \beta(S(u, \lambda) - T(u, \lambda) - g)), \lambda \right) - f \right] \right] \quad \forall u \in H, \end{aligned} \tag{3.2}$$

has a fixed point $x \in H$ and $y = J_{\beta}^{M(\cdot, \lambda)}(x - \beta(S(x, \lambda) - T(x, \lambda) - g))$.

Proof. It is easy to see that

$$\begin{aligned} 0 &\in \rho(S(y, \lambda) - T(y, \lambda) - f) + x - y + \rho M(x, \lambda) \\ &\iff y - \rho(S(y, \lambda) - T(y, \lambda) - f) \in [I + \rho M(\cdot, \lambda)](x) \\ &\iff x = J_{\rho}^{M(\cdot, \lambda)}(y - \rho(S(y, \lambda) - T(y, \lambda) - f)). \end{aligned} \tag{3.3}$$

Analogously we can obtain that $y = J_{\beta}^{M(\cdot, \lambda)}(x - \beta(S(x, \lambda) - T(x, \lambda) - g))$. That is, (a) \Leftrightarrow (b). Suppose that (c) holds. Obviously, the mapping F has a fixed point $x \in H$ and $y = J_{\beta}^{M(\cdot, \lambda)}(x - \beta(S(x, \lambda) - T(x, \lambda) - g))$. It follows from (3.2) that $x = F(x, \lambda) = J_{\rho}^{M(\cdot, \lambda)}(y - \rho(S(y, \lambda) - T(y, \lambda) - f))$. That is, (3.1) is satisfied, therefore, (b) holds. Conversely, if (b) holds, then (3.1) and (3.2) yield that

$$\begin{aligned} x &= J_{\rho}^{M(\cdot, \lambda)}(y - \rho(S(y, \lambda) - T(y, \lambda) - f)) \\ &= J_{\rho}^{M(\cdot, \lambda)} \left[J_{\beta}^{M(\cdot, \lambda)}(x - \beta(S(x, \lambda) - T(x, \lambda) - g)) \right. \\ &\quad \left. - \rho \left[S \left(J_{\beta}^{M(\cdot, \lambda)}(x - \beta(S(x, \lambda) - T(x, \lambda) - g)), \lambda \right) \right. \right. \\ &\quad \left. \left. - T \left(J_{\beta}^{M(\cdot, \lambda)}(x - \beta(S(x, \lambda) - T(x, \lambda) - g)), \lambda \right) - f \right] \right] \\ &= F(x, \lambda), \end{aligned} \tag{3.4}$$

that is, (c) holds. This completes the proof. □

Remark 3.2. Lemma 2.1 of Nie et al. in [21], [26, Lemma 1.3] of Verma, and [27, Lemma 2.1] of Wu et al. are special cases of Lemma 3.1 in this paper.

Based on Lemma 3.1 we suggest the following general iterative algorithm with errors for the system of generalized parametric nonlinear quasivariational inequalities (2.1).

Algorithm 3.3. For an arbitrarily chosen initial element $x_0 \in H, \lambda \in G$, compute sequences $\{x_n\}_{n \geq 0}$ and $\{y_n\}_{n \geq 0}$ by the following iterative procedure:

$$\begin{aligned} z_n &= (1 - b_n)x_n + b_nF(x_n, \lambda) + u_n, \\ x_{n+1} &= (1 - a_n)x_n + a_nF(z_n, \lambda) + v_n, \\ y_n &= J_\beta^{M(\cdot, \lambda)}(x_n - \beta(S(x_n, \lambda) - T(x_n, \lambda) - g)) + w_n \quad \forall n \geq 0, \end{aligned} \tag{3.5}$$

where F is defined by (3.2), $\{a_n\}_{n \geq 0}$ and $\{b_n\}_{n \geq 0}$ are any sequences in $[0, 1]$, and $\{u_n\}_{n \geq 0}$, $\{v_n\}_{n \geq 0}$, and $\{w_n\}_{n \geq 0}$ are any sequences satisfying

$$\begin{aligned} \sum_{n=0}^{\infty} a_n &= +\infty, & \lim_{n \rightarrow \infty} \|u_n\| &= 0, \\ \sum_{n=0}^{\infty} \|v_n\| &< +\infty, & \lim_{n \rightarrow \infty} \|w_n\| &= 0. \end{aligned} \tag{3.6}$$

THEOREM 3.4. Assume that $S : H \times G \rightarrow H$ is both s -Lipschitz continuous and a -relaxed monotone with respect to the first argument and $T : H \times G \rightarrow H$ is both t -Lipschitz continuous and b -relaxed Lipschitz with respect to the first argument. Suppose that the sequences $\{x_n\}_{n \geq 0}$ and $\{y_n\}_{n \geq 0}$ generated by Algorithm 3.3 satisfy (3.5) and (3.6). If there exist positive constants ρ and β satisfying

$$\max\{\rho, \beta\} < \frac{2(b - a)}{(s + t)^2}, \tag{3.7}$$

then for any given $f, g \in H, \lambda \in G$, the system of generalized parametric nonlinear quasivariational inequalities (2.1) has a unique solution $(x, y) \in H \times H$ and $\lim_{n \rightarrow \infty} x_n = x$ and $\lim_{n \rightarrow \infty} y_n = y$. Furthermore, if there exists a constant $\eta > 0$ such that

$$\|J_\rho^{M(\cdot, \lambda)}(w) - J_\rho^{M(\cdot, \bar{\lambda})}(w)\| \leq \eta \|\lambda - \bar{\lambda}\| \tag{3.8}$$

for all $(w, \bar{\lambda}, \lambda) \in H \times G \times G$, and S and T are continuous (resp., uniformly continuous or Lipschitz continuous) with respect to the second argument, then the solutions of the system of generalized parametric nonlinear quasivariational inequalities (2.1) are continuous (resp., uniformly continuous or Lipschitz continuous).

Proof. For each given $\lambda \in G$, we want to prove that $F(\cdot, \lambda) : H \rightarrow H$ defined by (3.2) is a contraction mapping. Since S is both s -Lipschitz continuous and a -relaxed monotone

with respect to the first argument, T is both t -Lipschitz continuous and b -relaxed Lipschitz with respect to the first argument and (2.5), we get that

$$\begin{aligned}
& \|F(u, \lambda) - F(v, \lambda)\|^2 \\
&= \left\| J_\rho^{M(\cdot, \lambda)} \left[J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)) \right. \right. \\
&\quad \left. \left. - \rho \left[S \left(J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)), \lambda \right) \right. \right. \right. \\
&\quad \left. \left. \left. - T \left(J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)), \lambda \right) - f \right] \right] \right. \\
&\quad \left. - J_\rho^{M(\cdot, \lambda)} \left[J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)) \right. \right. \\
&\quad \left. \left. - \rho \left[S \left(J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)), \lambda \right) \right. \right. \right. \\
&\quad \left. \left. \left. - T \left(J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)), \lambda \right) - f \right] \right] \right\|^2 \\
&\leq \left\| J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)) \right. \\
&\quad \left. - J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)) \right. \\
&\quad \left. - \rho \left[S \left(J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)), \lambda \right) \right. \right. \\
&\quad \left. \left. - S \left(J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)), \lambda \right) \right] \right. \\
&\quad \left. + \rho \left[T \left(J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)), \lambda \right) \right. \right. \\
&\quad \left. \left. - T \left(J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)), \lambda \right) \right] \right\|^2 \\
&= \left\| J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)) \right. \\
&\quad \left. - J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)) \right\|^2 \\
&\quad - 2\rho \left\langle J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)) \right. \\
&\quad \quad \left. - J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)), \right. \\
&\quad \quad \left. S \left(J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)), \lambda \right) \right. \\
&\quad \quad \left. - S \left(J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)), \lambda \right) \right\rangle \\
&\quad + 2\rho \left\langle J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)) \right. \\
&\quad \quad \left. - J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)), \right. \\
&\quad \quad \left. T \left(J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)), \lambda \right) \right. \\
&\quad \quad \left. - T \left(J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)), \lambda \right) \right\rangle
\end{aligned}$$

$$\begin{aligned}
 & + \rho^2 \left\| S \left(J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)), \lambda \right) \right. \\
 & \quad - S \left(J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)), \lambda \right) \\
 & \quad - T \left(J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)), \lambda \right) \\
 & \quad \left. + T \left(J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)), \lambda \right) \right\|^2 \\
 \leq & \left\| J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)) \right. \\
 & \quad \left. - J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)) \right\|^2 \\
 & + 2\rho a \left\| J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)) \right. \\
 & \quad \left. - J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)) \right\|^2 \\
 & - 2\rho b \left\| J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)) \right. \\
 & \quad \left. - J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)) \right\|^2 \\
 & + \rho^2 \left(\left\| S \left(J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)), \lambda \right) \right. \right. \\
 & \quad \left. - S \left(J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)), \lambda \right) \right\| \\
 & \quad \left. + \left\| T \left(J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)), \lambda \right) \right. \right. \\
 & \quad \left. \left. - T \left(J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)), \lambda \right) \right\|^2 \right) \\
 \leq & (1 - 2(b - a)\rho + (s + t)^2\rho^2) \left\| J_\beta^{M(\cdot, \lambda)} (u - \beta(S(u, \lambda) - T(u, \lambda) - g)) \right. \\
 & \quad \left. - J_\beta^{M(\cdot, \lambda)} (v - \beta(S(v, \lambda) - T(v, \lambda) - g)) \right\|^2 \\
 \leq & (1 - 2(b - a)\rho + (s + t)^2\rho^2) \\
 & \quad \times \|u - v - \beta[S(u, \lambda) - S(v, \lambda) - T(u, \lambda) + T(v, \lambda)]\|^2 \\
 = & (1 - 2(b - a)\rho + (s + t)^2\rho^2) \left[\|u - v\|^2 - 2\beta \langle u - v, S(u, \lambda) - S(v, \lambda) \rangle \right. \\
 & \quad + 2\beta \langle u - v, T(u, \lambda) - T(v, \lambda) \rangle \\
 & \quad \left. + \beta^2 \|S(u, \lambda) - S(v, \lambda) - T(u, \lambda) + T(v, \lambda)\|^2 \right] \\
 \leq & (1 - 2(b - a)\rho + (s + t)^2\rho^2) (1 - 2(b - a)\beta + (s + t)^2\beta^2) \|u - v\|^2
 \end{aligned} \tag{3.9}$$

for all $u, v \in H$. Put

$$\theta = \sqrt{1 - 2(b - a)\rho + (s + t)^2\rho^2} \cdot \sqrt{1 - 2(b - a)\beta + (s + t)^2\beta^2}. \tag{3.10}$$

According to (3.7), we know that $\theta \in (0, 1)$. It follows from (3.9) that

$$\|F(u, \lambda) - F(v, \lambda)\| \leq \theta \|u - v\| \quad \forall u, v \in H. \tag{3.11}$$

That is, $F(\cdot, \lambda)$ is a contraction mapping and hence it has a unique fixed point $x \in H$ for each given $\lambda \in G$. Set $y = J_\beta^{M(\cdot, \lambda)}(x - \beta(S(x, \lambda) - T(x, \lambda) - g))$. It follows from Lemma 3.1 that the system of generalized parametric nonlinear quasivariational inequalities (2.1) has a solution $(x, y) \in H \times H$.

Now we claim that (x, y) is the unique solution of the system of generalized parametric nonlinear quasivariational inequalities (2.1). In fact, if $(u, v) \in H \times H$ is also a solution of the system of generalized parametric nonlinear quasivariational inequalities (2.1), by Lemma 3.1 we know that $u = F(u, \lambda)$ and $v = J_\beta^{M(\cdot, \lambda)}(u - \beta(S(u, \lambda) - T(u, \lambda) - g))$. It follows from the uniqueness of fixed point of F that $u = x$ and hence $v = J_\beta^{M(\cdot, \lambda)}(x - \beta(S(x, \lambda) - T(x, \lambda) - g)) = y$.

Next we assert that the sequences $\{x_n\}_{n \geq 0}$ and $\{y_n\}_{n \geq 0}$ generated by Algorithm 3.3 converge strongly to x and y , respectively. In view of (3.1), (3.5), and (3.11), we conclude that

$$\begin{aligned} \|z_n - x\| &\leq (1 - b_n)\|x_n - x\| + b_n\|F(x_n, \lambda) - F(x, \lambda)\| + \|u_n\| \\ &\leq \|x_n - x\| + \|u_n\|, \\ \|x_{n+1} - x\| &\leq (1 - a_n)\|x_n - x\| + a_n\theta\|z_n - x\| + \|v_n\| \\ &\leq (1 - (1 - \theta)a_n)\|x_n - x\| + a_n\theta\|u_n\| + \|v_n\|, \end{aligned} \tag{3.12}$$

$$\begin{aligned} \|y_n - y\| &= \left\| J_\beta^{M(\cdot, \lambda)}(x_n - \beta(S(x_n, \lambda) - T(x_n, \lambda) - g)) \right. \\ &\quad \left. - J_\beta^{M(\cdot, \lambda)}(x - \beta(S(x, \lambda) - T(x, \lambda) - g)) \right\| + \|w_n\| \\ &\leq \|(x_n - x) - \beta(S(x_n, \lambda) - S(x, \lambda) - T(x_n, \lambda) + T(x, \lambda))\| + \|w_n\| \\ &= \left[\|x_n - x\|^2 - 2\beta\langle x_n - x, S(x_n, \lambda) - S(x, \lambda) \rangle \right. \\ &\quad \left. + 2\beta\langle x_n - x, T(x_n, \lambda) - T(x, \lambda) \rangle \right. \\ &\quad \left. + \beta^2\|S(x_n, \lambda) - S(x, \lambda) - T(x_n, \lambda) + T(x, \lambda)\|^2 \right]^{1/2} + \|w_n\| \\ &\leq \sqrt{1 - 2(b - a)\beta + (s + t)^2\beta^2}\|x_n - x\| + \|w_n\| \end{aligned} \tag{3.13}$$

for all $n \geq 0$, where F and θ are defined by (3.2) and (3.10), respectively. It follows from Lemma 2.3 and (3.6) that $\lim_{n \rightarrow \infty} x_n = x$. Letting $n \rightarrow \infty$ in (3.13), by (3.7) we infer that $\lim_{n \rightarrow \infty} y_n = y$.

Now we analyze the sensitivity of solutions of the generalized parametric nonlinear quasivariational inequalities (2.1). For each given $\lambda \in G$, there exists a unique solution $(x, y) \in H \times H$ denoted by $x(\lambda)$ and $y(\lambda)$ such that (2.1) holds. Hence for each $\lambda, \bar{\lambda} \in G$, we get that

$$\begin{aligned} x(\lambda) &= F(x(\lambda), \lambda), & x(\bar{\lambda}) &= F(x(\bar{\lambda}), \bar{\lambda}), \\ y(\lambda) &= J_\beta^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x, \lambda) - T(x, \lambda) - g)), \\ y(\bar{\lambda}) &= J_\beta^{M(\cdot, \bar{\lambda})}(x(\bar{\lambda}) - \beta(S(x, \bar{\lambda}) - T(x, \bar{\lambda}) - g)), \end{aligned} \tag{3.14}$$

$$\|x(\lambda) - x(\bar{\lambda})\| \leq \|F(x(\lambda), \lambda) - F(x(\lambda), \bar{\lambda})\| + \|F(x(\lambda), \bar{\lambda}) - F(x(\bar{\lambda}), \bar{\lambda})\|, \tag{3.15}$$

$$\begin{aligned} \|y(\lambda) - y(\bar{\lambda})\| &= \left\| J_\beta^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x, \lambda) - T(x, \lambda) - g)) \right. \\ &\quad \left. - J_\beta^{M(\cdot, \bar{\lambda})}(x(\bar{\lambda}) - \beta(S(x, \bar{\lambda}) - T(x, \bar{\lambda}) - g)) \right\|. \end{aligned} \tag{3.16}$$

It follows from (2.5) and (3.8) that

$$\begin{aligned} &\|F(x(\lambda), \lambda) - F(x(\lambda), \bar{\lambda})\| \\ &\leq \left\| J_\rho^{M(\cdot, \lambda)} \left[J_\beta^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)) \right. \right. \\ &\quad \left. \left. - \rho \left[S \left(J_\beta^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)), \lambda \right) \right. \right. \right. \\ &\quad \left. \left. \left. - T \left(J_\beta^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)), \lambda \right) - f \right] \right] \right. \\ &\quad \left. - J_\rho^{M(\cdot, \bar{\lambda})} \left[J_\beta^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)) \right. \right. \\ &\quad \left. \left. - \rho \left[S \left(J_\beta^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)), \lambda \right) \right. \right. \right. \\ &\quad \left. \left. \left. - T \left(J_\beta^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)), \lambda \right) - f \right] \right] \right\| \\ &+ \left\| J_\rho^{M(\cdot, \bar{\lambda})} \left[J_\beta^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)) \right. \right. \\ &\quad \left. \left. - \rho \left[S \left(J_\beta^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)), \lambda \right) \right. \right. \right. \\ &\quad \left. \left. \left. - T \left(J_\beta^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)), \lambda \right) - f \right] \right] \right. \\ &\quad \left. - J_\rho^{M(\cdot, \bar{\lambda})} \left[J_\beta^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \bar{\lambda}) - T(x(\lambda), \bar{\lambda}) - g)) \right. \right. \\ &\quad \left. \left. - \rho \left[S \left(J_\beta^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \bar{\lambda}) - T(x(\lambda), \bar{\lambda}) - g)), \bar{\lambda} \right) \right. \right. \right. \\ &\quad \left. \left. \left. - T \left(J_\beta^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \bar{\lambda}) - T(x(\lambda), \bar{\lambda}) - g)), \bar{\lambda} \right) - f \right] \right] \right\| \\ &\leq \eta \|\lambda - \bar{\lambda}\| + \left\| J_\beta^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)) \right. \\ &\quad \left. - J_\beta^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)) \right\| \end{aligned}$$

$$\begin{aligned}
 & + \left\| J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)) \right. \\
 & \quad \left. - J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \bar{\lambda}) - T(x(\lambda), \bar{\lambda}) - g)) \right\| \\
 & + \rho \left(\left\| S(J_{\beta}^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)), \lambda) \right. \right. \\
 & \quad \left. \left. - S(J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)), \lambda) \right\| \right. \\
 & \quad \left. + \left\| S(J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)), \lambda) \right. \right. \\
 & \quad \left. \left. - S(J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \bar{\lambda}) - T(x(\lambda), \bar{\lambda}) - g)), \lambda) \right\| \right. \\
 & \quad \left. + \left\| S(J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \bar{\lambda}) - T(x(\lambda), \bar{\lambda}) - g)), \lambda) \right. \right. \\
 & \quad \left. \left. - S(J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \bar{\lambda}) - T(x(\lambda), \bar{\lambda}) - g)), \bar{\lambda}) \right\| \right. \\
 & \quad \left. + \left\| T(J_{\beta}^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)), \lambda) \right. \right. \\
 & \quad \left. \left. - T(J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)), \lambda) \right\| \right. \\
 & \quad \left. + \left\| T(J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \lambda) - T(x(\lambda), \lambda) - g)), \lambda) \right. \right. \\
 & \quad \left. \left. - T(J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \bar{\lambda}) - T(x(\lambda), \bar{\lambda}) - g)), \lambda) \right\| \right. \\
 & \quad \left. + \left\| T(J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \bar{\lambda}) - T(x(\lambda), \bar{\lambda}) - g)), \lambda) \right. \right. \\
 & \quad \left. \left. - T(J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \bar{\lambda}) - T(x(\lambda), \bar{\lambda}) - g)), \bar{\lambda}) \right\| \right) \\
 & \leq 2\eta \|\lambda - \bar{\lambda}\| + \beta(\|S(x(\lambda), \lambda) - S(x(\lambda), \bar{\lambda})\| \\
 & \quad + \|T(x(\lambda), \lambda) - T(x(\lambda), \bar{\lambda})\|) \\
 & + \rho \left[\eta s \|\lambda - \bar{\lambda}\| + \beta s (\|S(x(\lambda), \lambda) - S(x(\lambda), \bar{\lambda})\| + \|T(x(\lambda), \lambda) - T(x(\lambda), \bar{\lambda})\|) \right. \\
 & \quad + \|S(z, \lambda) - S(z, \bar{\lambda})\| + \eta t \|\lambda - \bar{\lambda}\| \\
 & \quad + \beta t (\|S(x(\lambda), \lambda) - S(x(\lambda), \bar{\lambda})\| + \|T(x(\lambda), \lambda) - T(x(\lambda), \bar{\lambda})\|) \\
 & \quad \left. + \|T(z, \lambda) - T(z, \bar{\lambda})\| \right] \\
 & = [2 + (s + t)\rho] \eta \|\lambda - \bar{\lambda}\| \\
 & \quad + [1 + (s + t)\rho] \beta (\|S(x(\lambda), \lambda) - S(x(\lambda), \bar{\lambda})\| + \|T(x(\lambda), \lambda) - T(x(\lambda), \bar{\lambda})\|) \\
 & \quad + \rho (\|S(z, \lambda) - S(z, \bar{\lambda})\| + \|T(z, \lambda) - T(z, \bar{\lambda})\|),
 \end{aligned} \tag{3.17}$$

where $z = J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x(\lambda), \bar{\lambda}) - T(x(\lambda), \bar{\lambda}) - g))$. It follows from (3.11) that

$$\|F(x(\lambda), \bar{\lambda}) - F(x(\bar{\lambda}), \bar{\lambda})\| \leq \theta \|x(\lambda) - x(\bar{\lambda})\|. \tag{3.18}$$

Combining (3.15), (3.17), and (3.18), we infer that

$$\begin{aligned} & \|x(\lambda) - x(\bar{\lambda})\| \\ & \leq (1 - \theta)^{-1} \{ [2 + (s + t)\rho] \eta \|\lambda - \bar{\lambda}\| \\ & \quad + [1 + (s + t)\rho] \beta (\|S(x(\lambda), \lambda) - S(x(\lambda), \bar{\lambda})\| \\ & \quad + \|T(x(\lambda), \lambda) - T(x(\lambda), \bar{\lambda})\|) \\ & \quad + \rho (\|S(z, \lambda) - S(z, \bar{\lambda})\| + \|T(z, \lambda) - T(z, \bar{\lambda})\|) \}. \end{aligned} \tag{3.19}$$

From (3.16), we get that

$$\begin{aligned} & \|y(\lambda) - y(\bar{\lambda})\| \\ & \leq \left\| J_{\beta}^{M(\cdot, \lambda)}(x(\lambda) - \beta(S(x, \lambda) - T(x, \lambda) - g)) \right. \\ & \quad \left. - J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x, \lambda) - T(x, \lambda) - g)) \right\| \\ & \quad + \left\| J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\lambda) - \beta(S(x, \lambda) - T(x, \lambda) - g)) \right. \\ & \quad \left. - J_{\beta}^{M(\cdot, \bar{\lambda})}(x(\bar{\lambda}) - \beta(S(x, \bar{\lambda}) - T(x, \bar{\lambda}) - g)) \right\| \\ & \leq \eta \|\lambda - \bar{\lambda}\| + \|x(\lambda) - x(\bar{\lambda})\| \\ & \quad + \beta (\|S(x, \lambda) - S(x, \bar{\lambda})\| + \|T(x, \lambda) - T(x, \bar{\lambda})\|). \end{aligned} \tag{3.20}$$

It follows from (3.19), (3.20), and the continuities of S and T (resp., uniform continuities or Lipschitz continuities) with respect to the second argument that the solutions of the system of generalized parametric nonlinear quasivariational inequalities (2.1) are continuous (resp., uniformly continuous or Lipschitz continuous). This completes the proof. \square

As in the proof of Theorem 3.4, we get the following result.

THEOREM 3.5. *Assume that $S : H \times G \rightarrow H$ is both s -Lipschitz continuous and a -strongly monotone with respect to the first argument and $T : H \times G \rightarrow H$ is both t -Lipschitz continuous and b -generalized pseudocontractive with respect to the first argument. Suppose that the sequences $\{x_n\}_{n \geq 0}$ and $\{y_n\}_{n \geq 0}$ generated by Algorithm 3.3 satisfy (3.5) and (3.6). If there exist positive constants ρ and β satisfying*

$$\max\{\rho, \beta\} < \frac{2(a - b)}{(s + t)^2}, \tag{3.21}$$

then for any given $f, g \in H, \lambda \in G$, the system of generalized parametric nonlinear quasivariational inequalities (2.1) has a unique solution $(x, y) \in H \times H$ and $\lim_{n \rightarrow \infty} x_n = x$ and $\lim_{n \rightarrow \infty} y_n = y$. Furthermore, if there exists a constant $\eta > 0$ satisfying (3.8) and S and T are continuous (resp., uniformly continuous or Lipschitz continuous) with respect to the second argument, then the solutions of the system of generalized parametric nonlinear quasivariational inequalities (2.1) are continuous (resp., uniformly continuous or Lipschitz continuous).

Remark 3.6. Theorem 2.1 in [21] is a special case of Theorem 3.5.

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