Research Journal of Applied Sciences, Engineering and Technology 4(23): 5034-5039, 2012
ISSN: 2040-7467

Submitted: March 03, 2012 Accepted: March 24, 2012 Published: December 01, 2012

The New Mathematical Models for Inventory Management under Uncertain Market

A. Mirzazadeh
Department of Industrial Engineering, Islamic Azad University, Karaj Branch, Karaj, Iran

Abstract: This paper presents the new mathematical model for determining the optimal ordering policy for industrial and commercial companies. In the previous research, the numerous inventory models under inflationary conditions have been developed. In these models, the demand rate, usually, has been considered constant and well known, time-varying, stock dependent or price-dependent. But, the demand rate, usually, is uncertain in the real world. Therefore, in this study, the new inflationary inventory models under stochastic demand conditions have been developed. The inventory system is in the state of multi-items with budget constraint. The numerical examples have also been given to illustrate and validate the theoretical results.

Keywords: Budget constraint, inflation, inventory systems, stochastic market demand

INTRODUCTION

One of the most important parts of Supply Chain Management (SCM) is inventory system management which is inherently in non-deterministic situation. The many departments of organization such as warehouse, marketing, sale, purchasing, financial, planning, production, maintenance and etc. are relevance to the inventory problem. The problem of inventory systems under inflationary conditions has received attention in recent years. Since 1975 a series of related papers appeared that considered the effects of inflation on the inventory system. Before the 1990s, the earlier efforts have been considered simple situations. Buzacott (1975) made an Economic Order Quantity (EOQ) model with inflation subject to different types of pricing policies. Misra (1979) developed a discounted-cost model and included internal (company) and external (general economy) inflation rates for various costs associated with an inventory system.

Inventoried goods can be broadly classified into four meta-categories. First, obsolescence which refers to items those lose their value through time due to rapid changes of technology or the introduction of a new product by a competitor. Second, deterioration items refer to the damage, spoilage, dryness, vaporization, etc. of the products. Products such as vegetables, fish, medicine, blood, gasoline and radioactive chemicals have a finite shelf life, and start to deteriorate once they are produced. Third, amelioration refers to items whose value or utility or quantity increase with time. Fourth, items with no obsolescence, no deterioration and no amelioration.

If the rate of obsolescence, deterioration or amelioration is not sufficiently low, its impact on modeling of such an inventory system cannot be ignored. There are a few papers for obsolescing and ameliorating items. Moon *et al*. (2005) considered the ameliorating/deteriorating items on an inventory model with a time-varying demand pattern. Another research for ameliorating items has been done by Sana (2010).

The no obsolescing, deteriorating and ameliorating items have been considered in some researches on the inflationary inventory system. Sarker and Pan (1994) surveyed the effects of inflation and the time value of money on order quantity with finite replenishment rate. Some efforts were extended the previous works to consider more complex and realistic assumption, such as Uthayakumar and Geetha (2009), Maity (2010), Vrat and Padmanabhan (1990), Datta and Pal (1991), Hariga (1995), Hariga and Ben-Daya (1996) and Chung (2003).

The deteriorating inventory systems have been studied considerably in the recent years. For example, Chung and Tsai (2001) presented an inventory model for deteriorating items with the demand of linear trend considering the time-value of money. Wee and Law (2001) derived a deteriorating inventory model under inflationary conditions when the demand rate is a linear decreasing function of the selling price. Chen and Lin (2002) discussed an inventory model for deteriorating items with a normally distributed shelf life, continuous time-varying demand, and shortages under an inflationary and time discounting environment. Yang (2004, 2006) discussed the two-warehouse inventory problem for deteriorating items with a constant demand rate and shortages. Chang (2004) established a deteriorating EOQ model when the supplier offers a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity.

Maiti *et al*. (2006) proposed an inventory model with stock-dependent demand rate and two storage facilities under inflation and time value of money. Lo *et al*. (2007) developed an integrated production-inventory model with assumptions of varying rate of deterioration, partial backordering, inflation, imperfect production processes and multiple deliveries. A Two storage inventory problem with dynamic demand and interval valued lead-time over a finite time horizon under inflation and time-value of money considered by Dey *et al*. (2008). Other efforts on inflationary inventory systems for deteriorating items have been made by Hsieh and Dye (2010), Su *et al*. (1996), Chen (1998), Wee and Law (1999), Sarker *et al*. (2000), Yang *et al*. (2001, 2010), Liao and chen (2003), Balkhi (2004a, 2004b), Hou and Lin (2004), Hou (2006), Jaggi *et al*. (2006), Chern *et al*. (2008), Sarkar and Moon (2011) and Khanra *et al*. (2011a, b).

The mentioned papers have considered a constant and well-known inflation rate over the time horizon. Yet, inflation enters the inventory picture only because it may have an impact on the future inventory costs, and the future rate of inflation is inherently uncertain and unstable. But, there are a few works in the inflationary inventory researches under stochastic conditions, especially with multiple stochastic parameters. Mirzazadeh and Sarfaraz (1997) presented multiple-items inventory system with a budget constraint and the uniform distribution function for the external inflation rate and Horowitz (2000) discussed a simple EOQ model with a Normal distribution for the inflation rate and the firm's cost of capital. He showed the importance of taking into account the inflation rate and time discounting, especially when the former is relatively high or when there is considerable uncertainty as to either the inflation rate or the marginal cost of capital. Mirzazadeh (2007) compared the average annual cost and the discounted cost methods in the inventory system's modeling with considering stochastic inflation. The results show that there is a negligible difference between two procedures for wide range values of the parameters. Furthermore, Mirzazadeh (2008) in another work, proposed an inventory model under time-varying inflationary conditions for deteriorating items. Mirzazadeh (2009) developed A Partial Backlogging Mathematical Model under Variable Inflation and Demand. In another study, Mirzazadeh (2012) prepared an optimal production model for an inventory control system where the time horizon i.e., period of business, is random in nature.

In the literature, the demand rate usually has been considered constant and well known, time-varying, stock dependent or price-dependent Furthermore, in some practical situations, the demand rate may be uncertain. Therefore, in this paper the non-deterministic inventory model with inflation under stochastic demand has been developed. Also, the numerical example has been provided to illustrate and validate the theoretical results.

METHODOLOGY

The basic assumptions and notations: The following assumptions have been considered in the developed model:

- The inventory system costs are known at the beginning of the time horizon and will be increasing through the inflation rates.
- The demand rates are stochastic.
- A multiple items inventory system has been considered.
- The available budget is constrained and will be increased through the inflation rate.
- Replenishment is instantaneous, i.e., the replenishment rate is infinite.

The following notations are used in the model:

n	:	The number of items
Q_i	:	The order quantity for i-th item
D_i	:	The annual demand rate for i-th item
S_i	:	The ordering cost for i-th item at the beginning of the time horizon
C_i	:	The purchasing cost per unit for i-th item at time zero
h_{ki}	:	The internal (for k = 1) and the external (for k = 2) holding cost per unit per unit time for i-th item at time zero
f_k	:	The Internal Inflation Rate (IIR) for k = 1 and the External Inflation Rate (EIR) for k = 2
R_k	:	The discount rate net of inflation: $R_k = r-f_k$ where r is the discount rate
B	:	The maximum available budget at the beginning of the time horizon
E [$EUAC_i$]	:	The total expected annual costs for i-th item, where: i = 1, 2,..., n

The models development: The many studies of the inventory management systems have been reported. Analysis of the inventory systems in the literature is carried out using two procedures. First method, determine the optimal values of the control variables by minimizing the average annual cost and the alternative (and in theory more correct) procedure determine the optimal ordering policy by minimizing the discounted value of all future costs. As stated previously, Mirzazadeh (2007) showed by detailed computations that there is a negligible difference between two procedures for wide range values of the parameters. The average annual cost method will be used in this paper.

The total annual inventory systems costs include purchasing, ordering and carrying costs. The annual purchasing cost for the i-th item is equal to:

$$[1+ (1- Q_i/D_i)f_2/2]C_iD_i \qquad (1)$$

also, the annual ordering cost for the i-th item is as follows:

$$[1+ (1- Q_i/D_i)f_1/2](S_iD_i / Q_i) \tag{2}$$

Finally, the annual carrying cost for the i-th item after calculating will be:

$$\sum_{k=1}^{2}\left[1+\left(1-Q_i/D_i\right)f_k/2\right]\left(h_{ki}rQ_i/2\right) \tag{3}$$

Therefore, the total annual inventory system costs for the i-th item are:

$$\begin{aligned} EUAC_i &= \left[1+\left(1-Q_i/D_i\right)f_1/2\right]\left(S_iD_i/Q_i\right)+ \\ &+\sum_{k=1}^{2}\left[1+\left(1-Q_i/D_i\right)f_k 2\right]\left(h_{ki}rQ_i/2\right) \\ &+\left[1+\left(1-Q_i/D_i\right)f_1/2\right]C_iD_i \end{aligned} \tag{4}$$

Budget constraint: The available budget at time zero is B, which will be increasing by EIR. Also, the unit price, C_i, increases by EIR. Therefore, the budget constraint at the time t is:

$$\sum_{i=1}^{n} Q_iC_ie^{f_2t} \le Be^{f_2t} \tag{5}$$

By simplifying Eq. (5) we have:

$$\sum_{i=1}^{n} Q_iC_i \le B \tag{6}$$

The demand rates in Eq. (1)-(4) are stochastic. The expected value method can be used in this direction. Therefore, the optimal ordering quantities may be calculated using this model:

$$\begin{aligned} & MinZ = E\left[EUAC_i\right] \\ ` \ & s.t.: \\ & \sum_{i=1}^{n} Q_iC_i \le B \end{aligned} \tag{7}$$

The experimental results reveal that three probability density functions (pdf) are suitable for the demand rate in the inventory systems:

- Uniform
- Normal
- Exponential

These cases will be explained as follows.

Case (I): Stochastic demand rate with uniform distribution function: Let demand rate has a Uniform pdf as follows:

$$f\left(D_i\right) = \frac{1}{d_{2i}-d_{1i}} I\left[d_{1i}, d_{2i}\right] \tag{8}$$

Objective is to minimization of the total expected annual costs:

$$MinZ_1 = \sum_{i=1}^{n} E\left[EUAC_i\right] \tag{9}$$

where:

$$E\left[EUAC_i\right] = \int_{d_{1i}}^{d_{2i}} EUAC_{if}\,(D_i)dD_i \tag{10}$$

By substituting Eq. (4) in (10) and then substituting in Eq. (9) and simplifying the terms we have:

$$MinZ_1 = \sum_{i=1}^{n}\left\{\begin{aligned} &\frac{-rQ_i^2\left\lfloor f_1h_{1i}+f_2h_{2i}\right\rfloor Ln(d_{2i}/d1i)}{4(d_{2i}-d_{1i})} \\ &+\frac{Q_ih_{1i}r\left(1+f_1/2\right)+h_{2i}r\left(1+f_2/2\right)-f_2C_i}{2} \\ &+\frac{C_i\left(1+f_2/2\right)\left(d_{2i}+d_{1i}\right)-f_1C_i}{2}+\frac{S_i\left(1+f_1/2\right)\left(d_{2i}+d_{1i}\right)}{2Q_i} \end{aligned}\right\} \tag{11}$$

Therefore, the constrained multiple item inventory model can be stated as follow:

$$Min\ \ Z_1\sum_{i=1}^{n} Q_iC_i \le B \tag{12}$$

The problem has been solved with using the Lagrangian method:

$$L(Q_i,\lambda) = Z_1 + \lambda\left(\sum_{i=1}^{n} Q_iC_i - B\right) \tag{13}$$

By taking the first deviation of the above function respect to Q_I and l, set them equal to zero and simplification we have:

$$\left\{\begin{aligned} &Q_i = \sqrt{\frac{S_i\left(1+f_1/2\right)\left(d_{1i}+d_{2i}\right)}{2\left[r\left(h_{1i}+h_{2i}\right)/2\left(f_2/2-\lambda\right)C_i\right]}} \\ &\sum_{i=1}^{n} Q_iC_i = B \end{aligned}\right. \quad \text{For i = 1, ..., n} \tag{14}$$

The Q_i, for i = 1,…, n, will be obtained by calculating the above equations.

Case (II): Stochastic demand rate with normal distribution function: Now assume the demand rate of i-th item has a Normal distribution with the mean of m_i and the standard deviation of σ_i. The Eq. (4), may be rewriting as follows with considering $rf_1 \approx rf_2 \approx 0$:

$$EUAC_i = \left[\frac{S_i\left(1+ f_1 / 2\right)}{Q_i} + C_i(1+ f_2 / 2)\right] D_i - \frac{f_1 S_i - rQ_i(h_{1i} + h_{2i}) + Q_1 C_i f_2}{2} \tag{15}$$

Therefore, the total expected annual costs are:

$$MinZ_2 = \sum_{i=1}^{n} E\left[EUAC_i\right] \tag{16}$$

So, the mathematical model can be explained as follows:

$$\begin{aligned} MinZ_2 &= \sum_{i=1}^{n} \left\{\left[\frac{S_i(1+ f_1 / 2)}{Q_i} + C_i(1+ f_2 / 2)\right] \mu_i \frac{f_1 S_i - rQ_i(h_{1i} + h_{2i}) + Q_i C_i f_2}{2}\right\} \\ s.t.&: \\ &\sum_{i=1}^{n} Q_i C_i \le B \end{aligned} \tag{17}$$

Similarly case (i), the Lagrange-multiplier can be used. Therefore, the optimal ordering quantities will be obtained with using these equations:

$$\begin{cases} Q_i = \sqrt{\dfrac{S_i(1+ f_1 / 2)\mu_i}{(r\left(h_{1i} + h_{2i}\right) / 2 - (f_2 / 2 - \lambda)C_i}} & for \ \ i = 1,\ldots,n \\ \sum_{i=1}^{n} Q_i C_i = B \end{cases} \tag{18}$$

Case (III): Stochastic demand rate with exponential distribution function: The demand rate of i-th item has the exponential pdf with the mean of $1/q_i$. Similarly, the previous cases, the optimal ordering quantities can be calculated with using:

$$\begin{cases} Q_i = \sqrt{\dfrac{S_i(1+ f_1 / 2)\theta_i}{(r\left(h_{1i} + h_{2i}\right) / 2 - (f_2 / 2 - \lambda)C_i}} & for \ \ i = 1,\ldots,n \\ \sum_{i=1}^{n} Q_i C_i = B \end{cases} \tag{19}$$

The numerical example: The following numerical example is provided to illustrate the theoretical results.

Let

n = 2
S_1 = \$100 per order
S_2 = \$100 per order
C_1 = \$30 per unit
C_2 = \$20 per unit
h_{11} = \$0.5 per unit per year
h_{12} = \$0.25 per unit per year
h_{21} = \$1.5 per unit per year
h_{22} = \$0.75 per unit per year

Table 1: The numerical example

Case	Parameters	Q_1*	Q_2*	λ*
I	d_{11} = 5000 d_{21} = 11000 d_{12} = 4000 d_{22} = 8000	390	415	0.233586
II	μ_1 = 12000 σ_1 = 100 μ_1 = 5000 σ_1 = 50	436	346	0.27216
III	θ_1 = 1/8000 θ_2 = 1/9000	357	465	0.27137

B = \$20000
r = 20% per unit cost per year
f_1 = 8%
f_2 = 12%.

The models have been solved with considering the above mentioned values and the optimal values of Q_1, Q_2 and l have been obtained (Table 1).

Summary: Since 1975 a series of related papers appeared that considered the effects of inflation on the inventory system. In the previous research, the demand rate has been considered Constant and well known, Time-varying, Stock dependent, Price-dependent. Furthermore, the demand rate has a non-deterministic situation in the real world. The significant and unique findings of this research in comparison with the previous work, is developing the new inflationary inventory models with assuming stochastic demand rate. The multiple items have been considered in the system and the replenishment is instantaneous, i.e., the replenishment rate is infinite. The available budget is constrained and will be increased through the inflation rate. The numerical example has been provided to illustrate the theoretical results.

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