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Stability of Switched Linear Discrete-Time Descriptor Systems with Explicit Calculation of a Common Quadratic Lyapunov Sequence

Asier Ibeas, Manuel de la Sen, Ramón Vilanova and J. Herrera

Abstract— **In this paper, the stability of a switched linear regular descriptor system is considered. It will be shown that if a certain simultaneous triangularization condition on the subsystems is fulfilled and all the subsystems are stable then the switched system is stable under arbitrary switching. The result involves different descriptor matrices and extends to the singular case well-known results from the standard one. Furthermore, an explicit construction of a common Lyapunov sequence for a set of discrete-time regular linear descriptor subsystems is performed. The main novelty of the proposed approach is that the common Lyapunov sequence can be easily computed in comparison with previous works which either presented computationally-demanding methods or did not construct the common Lyapunov sequence explicitly.**

I. Introduction

Switched systems have attracted a lot of interest during the last years spoiling an active research activity. Main reason for this relies on their challenging theoretical background as well as their potential applications in real-world control such as the control of robot manipulators, electrical networks and chemical reactors, for citing few of them [1,2]. Thus, many theoretical works have been reported concerning switched systems including the study of their controllability and observability properties and, especially, the stability and stabilization issues, [3-10]. The latter is indeed the main problem when designing a control system. Hence, there are two main questions that can be formulated related to the stability of switched systems; 1) when is a switched system stable under arbitrary switching? 2) Which switching sequences are able to guarantee the stability of a system?

This paper concentrates on the first question since the most interesting feature for control purposes appears when the system is stable under any switching sequence. One of the classical approaches to the analysis of the stability of a switched system under arbitrary switching relies on the use of the Lyapunov theory. In this way, it is widely known [5,6] that if all the subsystems share a common Lyapunov function, then the switched system is asymptotically stable. For linear systems and quadratic Lyapunov functions there are two possibilities to verify when this occurs. The first one is to set up a system of linear matrix inequalities (LMI) whose feasibility implies the existence of such a common Lyapunov function, [11,12]. On the other hand, many algebraic-type conditions have been proposed in the literature to determine when there is a common Lyapunov function [4,13,14]. The main result establishes that if the set of stable linear matrices describing the switched linear system is simultaneously triangularizable, then they share a common Lyapunov function. Therefore, the switched system will be asymptotically stable under any switching law. There are many characterizations to verify when a family of matrices simultaneously triangularizes. Among them, Lie-type criteria have revealed especially useful [4,14,15]. Later, triangularizability criteria were relaxed to a certain class of pair-wise triangularizable set of matrices in [16].

However, both approaches posses the disadvantage that if we are interested in the explicit calculation of the common Lyapunov function, then high-demanding calculations have to be used. As a consequence, little insight in the properties of the common Lyapunov function is gained, [4,13].

This drawback is inherited when the switched system is defined by descriptor systems. Descriptor (also known as *implicit* or *singular*) systems are the most general class of linear systems (in comparison with standard systems) and are capable of describing a wide variety of economic, physical and engineering models, [17-21]. As a consequence, many works have been reported on descriptor systems including the feedback stabilization, Lyapunov stability theory, LMI approach, etc. [17,20-24]. Moreover, the interplay between switched and descriptor systems has also been explored in the literature. Thus, some of the above results concerning standard switched systems have been naturally extended to the singular case during the last years [18,19,26] while the same limitation referred to the calculation of the common Lyapunov function has also been pointed out, [12].

Especially, the recent results in [18,19,26] extend the asymptotic stability property under arbitrary switching of stable switched standard systems satisfying the quite restrictive condition of pair-wise commutativity to the singular case.

The purpose of this paper is twofold. Firstly, the simultaneous triangularizability condition on standard systems, which is a sufficient condition to guarantee the asymptotic stability of the system under arbitrary switching,

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A. Ibeas, R. Vilanova and J. Herrera are with the Departamento de Telecomunicaciones e Ingeniería de Sistemas, Escuela Técnica Superior de Ingeniería, Universitat Autònoma de Barcelona. 08193 Bellaterra. Barcelona. Spain. (corresponding author e-mail: Asier.Ibeas@uab.es).

M. de la Sen is with the Departamento de Electricidad y Electrónica, Facultad de Ciencia y Tecnología, Universidad del País Vasco, P.O. 644, 48080 Bilbao. Vizcaya. Spain.

is extended to the singular case. Furthermore, for the commutativity case a less restrictive condition than that exposed in [18] is stated as sufficient to guarantee stability. Additionally, the result is obtained by using similar techniques to the ones used in the standard case in comparison to the involved algebraic manipulations used in the recent work [26]. Hence, not only are the results extended to the singular case but also obtained by using adequate variations of previous techniques which have not been presented so neat in the literature before.

Secondly, an alternative construction to that included in [4,25] of a common Lyapunov sequence is presented for the case of discrete-time linear switched descriptor systems. To this end, the set of matrices is represented in a special basis where simultaneous triangularizability with the absolute values of all the off-diagonal elements being as small as desired is achieved. This transformation allows the explicit construction of a common Lyapunov sequence in an easy way in comparison with the previous work [4].

The paper is organized as follows. Section II contains the problem formulation. The main result of the paper is presented and proved in Section III while conclusions end the paper.

Notation. $A > (\geq) B$ for matrices A, B means that $A - B$ is a positive (semi)definite matrix.

II. PROBLEM FORMULATION

Consider the class of discrete-time switched linear descriptor systems of the form

$$E_{\sigma(k)} x_{k+1} = A_{\sigma(k)} x_k \quad (1)$$

with $x_k \in \mathbb{R}^n, E_i, A_i \in \mathbb{R}^{n\times n}$ for $i \in \mathcal{I} = \{1,2,...,m\}$ and finite integer *m*, $n \geq 2$ to avoid trivial cases and $\sigma : \mathbb{N} \rightarrow \mathcal{I}$ is the so called switching function. The matrices E_i may be singular, i.e., $d_i = rank\ E_i \leq n$, for $i \in \mathcal{I}$. The following definitions will be important in the sequel.

Definition 1. The switched descriptor system (1) is said regular if there are $z_i \in \mathbb{C}$ such that $\det(z_i E_i - A_i) \neq 0$ for $i \in \mathcal{I}$. ■

Remark 1. Note that $n \geq d_i = rank\ E_i \geq r_i$, $r_i = \deg\det(zE_i - A_i)$ holds for any regular descriptor system (1). When $d_i = r_i$ the descriptor system defined by (E_i, A_i) is said causal. ■

In this paper we will concentrate on extending the results on simultaneous triangularizability based stability to regular switched systems of the class (1). The proposed solution is based on a calculation of an explicit solution to (1). To define the stability and to give such an explicit solution it is needed to previously introduce some results. For the sake of simplicity consider the descriptor system

$$Ex_{k+1} = Ax_k \quad (2)$$

For system (2) the following definition can be stated.

Definition 2. The descriptor system (2) or equivalently, the pair (E, A) is said to be asymptotically stable if the system trajectories satisfy $\lim_{k\rightarrow\infty} \|x_k\| = 0$. ■

The conditions which will be used to set up the existence of a common Lyapunov sequence for (1) are based on the calculation of an explicit solution to (1). To this end, we need to introduce the following matrices:

$$\hat{E}_i = (\lambda_i E_i - A_i)^{-1} E_i, \qquad \hat{A}_i = (\lambda_i E_i - A_i)^{-1} A_i \quad (4)$$

for arbitrary real numbers λ_i such that $(\lambda_i E_i - A_i)^{-1}$ exists for $i \in \mathcal{I}$. Furthermore, the symbol $(*)^D$ will denote the Drazin pseudoinverse of $(*)$, [21]. With this notation, the following theorem gives an explicit solution to (2):

Theorem 1, [17,21]. Let (2) be a regular system and $\hat{E}, \hat{A}$ be defined in a similar way as (4). Then, every solution to (2) has the form:

$$x_k = \left(\hat{E}^D \hat{A}\right)^k \hat{E}^D \hat{E} v \quad (5)$$

for some $v \in \mathbb{R}^n$. Furthermore, the system (2) has a unique solution sequence with prescribed initial value x_0 if and only if there exists $v \in \mathbb{R}^n$ such that $x_0 = \hat{E}^D \hat{E} v$. In this case, the initial condition x_0 is said to be consistent. ■

As pointed out in [21] the products $\hat{E}^D \hat{A}$ and $\hat{E}^D \hat{E}$ do not depend on the specific choice of λ.

Directly based on this result, the solution to the switched regular descriptor system (1) is explicitly given by:

$$x_k = \Phi(k,0) \hat{E}^D_{\sigma(0)} \hat{E}_{\sigma(0)} v \quad (6)$$

$$\Phi(k,0) = \begin{cases} I & k = 0 \\ \prod_{i=1}^{k} \hat{E}^D_{\sigma(i)} \hat{A}_{\sigma(i)} & k > 0 \end{cases} \quad (7)$$

with the product (7) being performed from the left for some $v \in \mathbb{R}^n$. An important question arises when considering the existence of unique solutions to (1) due to the fact that a 'state jump' may occur at switching instants. Thus, the switching may imply that the last reached state may not be a consistent initial condition for the next active subsystem. Then, there would not be a solution to the described switching system afterwards. Hence, consider a consistent initial condition x_0 and the solution $x_k = \left(\hat{E}_{\sigma(0)} \hat{A}_{\sigma(0)}\right)^k x_0$ to

(1) corresponding to the first *k* time steps without switchings. If a switching occurs at *(k+1)* then it is necessary that $x_k \in \mathrm{Im}(\hat{E}^D_{\sigma(k+1)}\hat{E}_{\sigma(k+1)})$, where $\mathrm{Im}(*)$ denotes the image space of the linear operator (*), in order to force x_k be a consistent initial condition for the next subsystem. It is straightforward to see that this can be achieved if

$$\mathrm{Im}\left(\hat{E}^D_{\sigma(0)}\hat{A}_{\sigma(0)}\right)^k \subseteq \mathrm{Im}\left(\hat{E}^D_{\sigma(0)}\hat{A}_{\sigma(0)}\right) \subseteq \mathrm{Im}\left(\hat{E}^D_{\sigma(k+1)}\hat{E}_{\sigma(k+1)}\right)$$

The above condition can be directly generalized to give the following technical assumption guaranteeing the consistency of the switched descriptor system under an arbitrary switching policy:

Assumption 1. The switched descriptor system (1) satisfies

$$\mathrm{Im}\left(\hat{E}_i^D\hat{A}_i\right) \subseteq \mathrm{Im}\left(\hat{E}_j^{\,D}\hat{E}_j\right) \quad \text{for all } i,j \in \mathcal{I} \tag{8}$$

and x_0 is a consistent initial condition for $\hat{E}^D_{\sigma(0)}\hat{E}_{\sigma(0)}$. ■

Hence, with Assumption 1 the following theorem holds.

Theorem 2. The switched regular descriptor system (1) has a unique solution corresponding to an initial state x_0 given by (6)-(7) provided that Assumption 1 holds. ■

Remarks. 3. It can also be directly concluded from (5) that the system (E,A) (2) is asymptotically stable if and only if all the eigenvalues of $\hat{E}^D\hat{A}$ have absolute values strictly less than unity.

4. Note, in particular, that (8) can be satisfied if $\mathrm{Im}\left(\hat{E}_i^D\right) = \mathrm{Im}\left(\hat{E}_j^{\,D}\right)$ implying that $d = d_i = d_j$ for all $i,j \in \mathcal{I}$ which is the condition used in [26]. Nevertheless, equation (8) is less restrictive and it is not necessary that all the descriptor matrices have the same degree. ■

Moreover, it is appreciated from (7) that $\Phi(k,0)$ satisfies the recursive equation $\Phi(k,0) = \hat{E}^D_{\sigma(k)}\hat{A}_{\sigma(k)}\Phi(k-1,0)$ which makes (1) convert into

$$x_k = \hat{E}^D_{\sigma(k)}\hat{A}_{\sigma(k)}\Phi(k-1,0)x_0 = \hat{E}^D_{\sigma(k)}\hat{A}_{\sigma(k)}x_{k-1} \tag{9}$$

since Assumption 1 holds. Thus, the stability of the switched standard system (9) implies the stability of the switched descriptor system (1). The standard system (9) will serve as an auxiliary system in the main result below to state the conditions for stability of (1) under arbitrary switching. Note that this approach is exactly the opposite one of the selected in [26] to deal with the stability problem for discrete-time switched descriptor systems and constitutes the key point to import techniques used in standard switched systems into the analysis of switched descriptor ones, bringing the gap, therefore, between them.

The main result is established by using a simultaneous triangularization property defined in the following way.

Definition 3. Let $\mathcal{A} = \{M_i; i \in \mathcal{I}\}$ be a set of matrices. Then, $\mathcal{A}$ is said simultaneously triangularizable if there exists a single (complex) nonsingular transformation T such that $\tilde{A}_i = TM_iT^{-1}$ is upper-triangular for all $i \in \mathcal{I}$, i.e.

$$\tilde{A}_i = \begin{pmatrix} \lambda_1(M_i) & \cdots & \tilde{a}^{(i)}_{\ell j} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n(M_i) \end{pmatrix} \tag{10}$$

where $\lambda_p(M_i); p = 1,\ldots,n$ denote the eigenvalues of M_i. ■

The fundamental result presented in this paper in Section 3 is based on the simultaneous triangularization property of the set of matrices $\mathcal{A} = \left\{\hat{E}_i^D\hat{A}_i; i \in \mathcal{I}\right\}$ and in the following result, proved here for convenience.

Lemma 2. Let $\mathcal{A} = \{M_i; i \in \mathcal{I}\}$ be a set of simultaneously triangularizable matrices and $\varepsilon > 0$ be arbitrary. Then, there exists a (possibly) complex nonsingular transformation $T = T(\varepsilon)$ such that $\tilde{A}_i = TM_iT^{-1}$ is upper-triangular with $\left|\tilde{a}^{(i)}_{\ell j}\right| < \varepsilon$ for $j > \ell$ in (10) and all $i \in \mathcal{I}$.

<u>Proof</u>. Since $\mathcal{A}$ is simultaneously triangularizable, there exists a nonsingular complex transformation matrix Q and a basis $V = \{v_1,\ldots,v_n\}$ such that $\bar{A}_i = QM_iQ^{-1}$ is upper triangular for all $i \in \mathcal{I}$. Now, define a new basis $W = \{w_1,\ldots,w_n\}$ by $w_j = v_j / r^j$ with $r \in \mathbb{R}$, $r \neq 0$ for $1 \le j \le n$. The matrix of each linear transformation $\tilde{A}_i$ with respect to this basis can be calculated for all $i \in \mathcal{I}$ as:

$$\begin{aligned} \tilde{A}_i w_1 &= \frac{1}{r}\tilde{A}_i v_1 = \lambda_1 w_1 \\ \tilde{A}_i w_j &= \frac{1}{r^j}\tilde{A}_i v_j = \lambda_j w_j + \sum_{j=1}^{i-1}\tilde{a}^{(i)}_{\ell j}\frac{1}{r^{\ell-j}}w_j \end{aligned} \tag{11}$$

for $j = 2,3,\ldots,n$. It can be seen that (11) defines the elements of (10) with $\tilde{a}^{(i)}_{\ell j} = a^{(i)}_{\ell j}\dfrac{1}{r^{\ell-j}}$. It is clear that the choice of a sufficiently large r would lead to off-diagonal entries being smaller than any prescribed ε for all $i \in \mathcal{I}$. More precisely, $|r| > \max\limits_{\substack{1\le \ell,j\le n \\ i\in\mathcal{I}}} \dfrac{\left|a^{(i)}_{\ell j}\right|}{\varepsilon}$, which completes the proof.

■

Note that the transformation matrix *T* depends on the value of ε. This dependence will be omitted in the sequel for the sake of notation simplicity. A comprehensive work on simultaneous triangularizability providing conditions to check when this is possible can be found in [15]. Also, note that for the problem at hand, $rank\hat{E}_i^D\hat{A}_i \le \min\left(rank\hat{E}_i^D, rank\hat{A}_i\right) < n$ if the matrices E_i are singular which implies that the products $\hat{E}_i^D\hat{A}_i$ $i \in \mathcal{I}$ always have a number of their eigenvalues being zero. In this sense, it will be supposed without loss of generality that $\hat{E}_1^D\hat{A}_1$ possesses the largest eigenvalue from the set $\mathcal{A} = \left\{\hat{E}_i^D\hat{A}_i ; i \in \mathcal{I}\right\}$, i.e.,

$$\max_{1\le j\le d}\left|\lambda_j(\hat{E}_1^D\hat{A}_1)\right| \ge \left|\lambda_\ell(\hat{E}_\alpha^D\hat{A}_\alpha)\right|, 2 \le \alpha \le m, 1 \le \ell \le d \quad (12)$$

and that the upper-triangular matrix $\tilde{A}_1 = T\hat{E}_1^D\hat{A}_1T^{-1}$ has all its zero eigenvalues in the last positions of the diagonal entries. Now, we are capable of formulating the main result in the next Section.

III. Stability of Switched Descriptor Linear Systems

The main result of the paper follows.

Theorem 3. The switched descriptor system (1) is asymptotically stable under arbitrary switching provided that it is regular, each subsystem $(E_i, A_i), i \in \mathcal{I}$ is asymptotically stable, Assumption 1 holds and the set $\mathcal{A} = \left\{\hat{E}_i^D\hat{A}_i ; i \in \mathcal{I}\right\}$ is simultaneously triangularizable.

Proof. It will be proved that there exists a sequence $V(k) = z_k^T P z_k$, with $P = P^T > 0$ such that it is a common Lyapunov sequence for the complete family:

$$z_k = T\hat{E}_i^D\hat{A}_iT^{-1}z_{k-1} = \tilde{A}_i z_{k-1} \;;\; i \in \mathcal{I} \quad (13)$$

with $\tilde{A}_i$ defined by Lemma 2 (for sufficiently small values for ε being specified below) through the nonsingular time-invariant state transformation $z_k = Tx_k$ since (1) is regular and $\mathcal{A} = \left\{\hat{E}_i^D\hat{A}_i ; i \in \mathcal{I}\right\}$ is simultaneously triangularizable. Hence, as commented below, since Assumption 1 holds and the problem is well-posed, the asymptotic stability of (13) will imply the asymptotic stability of (1).

For proving this, we first consider $\tilde{A}_1$, which is the matrix from $\mathcal{A}$ with the largest absolute eigenvalue, and its associate discrete Lyapunov inequality

$$\tilde{A}_1^* P \tilde{A}_1 - P < 0 \quad (14)$$

where $\tilde{A}_1^*$ stands for the conjugate transpose of $\tilde{A}_1$. However, according to Lemma 2,

$$W_1 = \tilde{A}_1 - D_1 = \begin{pmatrix} 0 & \tilde{a}_{12}^{(1)} & \cdots & \tilde{a}_{1n}^{(1)} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \tilde{a}_{n-1,n}^{(1)} \\ 0 & \cdots & \cdots & 0 \end{pmatrix} \text{ with } \left|\tilde{a}_{\ell j}^{(1)}\right| < \varepsilon \quad (15)$$

where $D_1 = diag(\lambda_1(\tilde{A}_1), \ldots, \lambda_\nu(\tilde{A}_1), 0, \ldots, 0)$ with $\nu \le d_1$. Therefore, the left-hand side of (14) becomes

$$\begin{aligned} &\left(W_1^* + D_1^*\right)P\left(W_1 + D_1\right) - P \\ &= D_1^*PD_1 - P + \left(W_1^*PW_1 + W_1^*PD_1 + D_1^*PW_1\right) < 0 \end{aligned} \quad (16)$$

At this point, consider any solution to the auxiliary Lyapunov inequality $D_1^*PD_1 - P < 0$ for a diagonal matrix $P = diag(p_1, p_2, \ldots, p_n)$. It is easy to verify that $P = I_n$ satisfies the above equation since for each nonzero value of the diagonal entries $\left|\lambda_j(\tilde{A}_i)\right|^2 - 1 < 0$ for $j = 1, 2, \ldots, \nu$ and -1 for $j = \nu + 1, \ldots, n$. Hence, denoting by $\mu = \min_{1\le j\le\nu}\left|\left|\lambda_j(\tilde{A}_i)\right|^2 - 1\right|$ and $\rho = \max_{1\le i\le n}\left|\lambda_i(D_1)\right|$, the original Lyapunov inequality (14) becomes via (16) into,

$$\begin{aligned} &\tilde{A}_1^* P \tilde{A}_1 - P \\ &\le -\mu I_n + W_1^*PW_1 + W_1^*PD_1 + D_1^*PW_1 \\ &\le -\mu I_n + \varepsilon^2\Lambda + \varepsilon\rho\Delta \end{aligned} \quad (17)$$

since $\lambda_{\max}(P) = 1$ with $\Delta = \begin{pmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 1 & \cdots & 1 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & \cdots & 1 & \ddots & 1 \\ 1 & \cdots & 1 & 1 & 0 \end{pmatrix}$ and

$$\Lambda = \left(\begin{array}{c|ccccc} 0 & 0 & \cdots & & & 0 \\ \hline 0 & 1 & \cdots & & & 1 \\ & & 2 & \cdots & & 2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ & & & \ddots & n-2 & n-2 \\ 0 & 1 & 2 & \cdots & n-2 & n-1 \end{array}\right) \quad (18)$$

Δ possesses two distinct eigenvalues, namely, -1 with multiplicity $(n-1)$ and $(n-1)$ with multiplicity unity. Λ, on the other hand, has a richer eigenstructure. However, an upper-bound for its largest eigenvalue can also be obtained as follows. It is clear from (18) that the matrix posseses at least a zero eigenvalue while the remaining eigenvalues can be obtained as the eigenvalues of the (2,2) block of the partitioned matrix (18). This sub-matrix is a positive one and hence, the Perron-Fröbenius Theorem applies, [22]. Thus, the sub-matrix possesses a simple real largest eigenvalue which is upper-bounded by the largest row-sum of the entries of the matrix given by $n(n-1)/2$. Then, (17) can be upper-bounded as:

$$\begin{aligned}&\tilde{A}_1^* P \tilde{A}_1 - P \\ &\le \left(-\mu + \tfrac{1}{2}\varepsilon^2(n-1)n + \rho\varepsilon(n-1)\right) I_n\end{aligned} \tag{19}$$

so that for $\varepsilon \in \left(0, \varepsilon^*\right)$ with

$$\varepsilon^* = \frac{-\rho}{n} + \sqrt{\frac{\rho^2}{n^2} + \frac{2\mu}{(n-1)n}} > 0$$

the right-hand side of (19) is still negative definite. A value of ε satisfying this condition should be used in (15).

Finally, it will be shown that V is indeed a Lyapunov sequence for any $\tilde{A}_j, j \in \mathcal{I}$. For this, consider, (with $P = I_n$) the inequality

$$\begin{aligned}&\tilde{A}_j^* \tilde{A}_j - I \\ &= D_j^* D_j - I + \left(W_j^* W_j + W_j^* D_j + D_j^* W_j\right) < 0\end{aligned} \tag{20}$$

since $\tilde{A}_j = D_j + W_j$. Thus, from (12) and (19),

$$\begin{aligned}&\tilde{A}_j^* \tilde{A}_j - I \\ &\le -\mu I_n + \left(W_j^* W_j + W_j^* D_j + D_j^* W_j\right)\end{aligned} \tag{21}$$

since all $\tilde{A}_j, j \in \mathcal{I}$ are stable. Then, as before,

$$\begin{aligned}&\tilde{A}_j^* \tilde{A}_j - I \\ &\le -\mu I_n + \varepsilon^2 \Lambda + \varepsilon\rho\Delta\end{aligned} \tag{22}$$

Hence, for the given $\varepsilon \in \left(0, \varepsilon^*\right)$, the right hand side of (22) is still negative definite for $i \in \mathcal{I}$ since (12) holds and V is a common Lyapunov sequence for the complete set of matrices. In conclusion, since $\varepsilon^* > 0$ is arbitrary – then as small as necessary- it has been proved that (13) possesses a common Lyapunov sequence and (13) is asymptotically stable under arbitrary switching implying that the descriptor system (1) is stable under arbitrary switching. ■

In this way, the results on stability for simultaneously triangularizable standard switched systems are extended to the singular case.

Remarks. 5. If $\mathcal{A}$ satisfies the stronger property of being simultaneously diagonalizable, which can be achieved if all the matrices in $\mathcal{A}$ are pairwise commuting, then it can be selected $\varepsilon = 0$ in the formulation of Lemma 2.

6. Also, if the matrices E_i are not singular, then the classical result on stability of switched systems is recovered, [5].

7. For the case of pair-wise commuting matrices in $\mathcal{A}$, the above result implies that a reduced number of conditions than those stated in [18,26] are needed to be verified since it is not necessary to make all the matrices E_i commute with themselves and with all the matrices A_i.

8. Note that the presented approach is constructive and does not require evaluating all the minors of the matrices A_i for all $i \in \mathcal{I}$ in comparison to the method presented in [4] to obtain a solution to the Lyapunov equation. Since calculating the determinant of a $m \times m$ matrix involves the sum of $m!$ terms, the saving in computational cost with the proposed approach is apparent.

9. The construction of the matrix *T*, if desired can be performed by using the results in [15]. ■

Additionally, as a corollary, Theorem 3 allows obtaining an estimation of the worst convergence rate of the solution of the switched descriptor system. In fact, from (19), one has

$$\Delta V(k) \le (-\mu + \delta) V(k) \tag{23}$$

implying

$$V(k) \le (1 - \mu + \delta)^k V(0) \tag{24}$$

As it has been proved by Theorem 3, for any sufficiently small ε there is a sufficiently small $\delta = \delta(\varepsilon) = \tfrac{1}{2}\varepsilon^2(n-1)n + \rho\varepsilon(n-1)$ such that $\left|1 - \mu + \delta\right| < 1$. Therefore,

$$\left\|z_k\right\|_2^2 \le (1 - \mu + \delta)^k \left\|z_0\right\|_2^2$$

and,

$$\left\|x_k\right\|_2^2 \le (1 - \mu + \delta)^k \kappa(T)^2 \left\|x_0\right\|_2^2 \tag{25}$$

with $\kappa_2(T) = \left\|T\right\|_2 \left\|T^{-1}\right\|_2 < \infty$ (since *T* is nonsingular) is the condition number of *T* with respect to the 2-norm (or, spectral norm). Since μ, δ are both known, an estimation of the convergence rate can be calculated explicitly. Furthermore, the method not only allows calculating a worst-case convergence rate, but it also provides an estimation of the maximum amplitude of the transient response of the system. For this, consider the output equation $y_k = C^T x_k$ which implies,

$$\left|y_k\right| \le \left\|C\right\|_2 \left\|x_k\right\|_2 \le \left\|C\right\|_2 \kappa(T) \left\|x_0\right\|_2 (1 - \mu + \delta)^{\frac{k}{2}}$$

Since the system is exponentially stable, and the right-hand side is an strictly decreasing function, the above equation reaches its maximum at $k = 0$ whence an estimation of the maximum output amplitude (which does not depend on the specific choice for the basis) can be estimated. Moreover, a more accurate bound of such a maximum amplitude can be obtained by minimizing $k(\varepsilon) = \|C\|_2 \, \kappa(T(\varepsilon)) \|x_0\|_2$ with respect to ε. Thus, the extra knowledge of the transformation matrix $T(\varepsilon)$ provides us with this extra information.

In conclusion, the presented method allows easily obtaining an estimation of the worst-case convergence rate along with an estimation of the maximum output amplitude. Also, note that this method is based on the one introduced in [27] for continuous-time switched standard systems. Thus, this approach brings the gap between standard and descriptor systems which has not been exploited previously in the results obtained in [18,19,26].

IV. CONCLUSIONS

In this paper, the classical simultaneously triangularization condition to establish the asymptotic stability of a standard switched system under arbitrary switching is extended to the class of regular switched descriptor systems. The proposed approach is based on the explicit calculation of a common Lyapunov sequence for an auxiliary standard system whose stability implies the stability of the original descriptor system. Additionally, the construction of such a Lyapunov sequence is easier than in previous approaches providing an estimation of the worst convergence rate along with an estimation of the maximum output amplitude.

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