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Contingency Analysis using Synchrophasor Measurements

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***Abstract* – This paper presents a new algorithm, using phasor measurements, that allows real-time analysis and correction of contingencies in power systems. The focus is specifically on overloaded lines. Contingency indicative phasor limits are investigated using current magnitude and voltage angle. These limits are applied to a rotating phasor chart. An algorithm which predicts sensitivity is applied to an off-line system in order to determine the buses that need to be monitored. An online system with available phasor measurement unit (PMU) data is used to verify the phasor chart obtained using off-line data. The chart is completed for on-line PMU data, and compared with the off-line chart for further verification.**

I. INTRODUCTION

One of the most important aspects of power system operation is manipulating the network to supply the power demands of all loads efficiently. During daily periods of high demand, loading of the network peaks. This can cause the power flow on lines to increase, driving the line to run in overloaded conditions. One outage can cause power flow to redistribute within the network and may overload transmission lines as a consequence. Certain lines within the system are very sensitive to changes in power flow, these are critically loaded, but others may be able to handle excess flow. In many cases forcing outages of specific lines within the system will properly redistribute power flow in these more lightly loaded lines and allow alleviation of the overload while still supplying the power demanded by all loads. Phasor measurements are examined within large scale systems both pre- and post-contingency. Load flow is observed in an off-line system and a predictive phasor pattern is determined using the phasors at each bus in the system. The pattern is then tested against online phasor measurement unit (PMU) data of a large scale system provided by a local utility for verification.

The importance of finding a predictive pattern in phasor measurements for an online system lies in the time needed to analyze contingencies in the system. Current practice is to run a load flow on the system whenever a contingency occurs to identify any problems that may have been introduced. These load flows are not suitable for online applications because the power flow calculations depend on state estimators, which normally take minutes to update snapshots of the power system [1]. This is more than enough time for the system to collapse while waiting for a solution. PMUs, on the other hand, along with high speed communications networks make it possible to create wide-area monitoring systems that can update a snapshot of the power system within as little as one second [2,3]. This allows for much faster correction of any problems created by overloads or contingencies.

The phasor data in this case comes from PMUs installed at selected buses. PMUs are power system devices that provide synchronized measurements of real-time phasors of bus voltage and line currents. Many PMUs are currently installed in systems around the world for various applications [4]. Phasors, in general, are currently being greatly researched in the power industry. Phasors seem to give a good indication of system state in many different areas, and are being used in security [4], to create voltage stability limits [1], in state estimation [5-6], and many other applications including model validation [7], system monitoring [8], fault recording [7], and control and protection [9-11].

This paper determines a new method for contingency analysis. In section II an off-line study is performed to determine which phasors are most indicative of overload. From these phasors, a predictive pattern is found to apply to limit thresholds. An operator display is proposed using a rotating pie chart. In section III a new sensitivity equation is proposed and tested off-line in different loading conditions. Section IV investigates the applicability of the pie chart to an on-line system with PMU data. This data includes a three-phase bus fault. The on-line system is reduced for analysis, and the reduced system is compared and verified using PMU data. It is determined that limit thresholds for the pie chart could be created in the on-line system if the given system was operating in a heavier loading condition. In section V, the sensitivity equation is applied to the on-line system and consistency is examined. Section VI provides comparisons between the off-line and on-line data, and section VII concludes the study.

II. OFF-LINE STUDY

The off-line system analyzed in this paper is a 37-bus system with a base of 138kV, 100MVA. All bus and line data is specified. The system is simulated using PowerWorld simulation software. This software is chosen above other simulators for its user interface and data configurations. The base case is built and all voltage and line capacity constraints are verified. Phasor measurements are taken at all buses in the base case loading condition and after the overall system load is increased by 20% (maximum allowable). A load increase of 25% forces some lines to operate above 100% of their capacity, which is not allowed. In the maximum allowable load increase condition, six lines are operating between 80%

and 100% capacity, which is allowed for short periods of time during heavily loaded conditions. These six lines are observed before and after overload to determine a pattern in phasor measurements which allows limit setting.

This limit indicates when a line is crossing from a permissible capacity to an overloaded capacity. Specifically, the limits observe when a line crosses both 80% and 100% capacity thresholds. The percent increase limits in voltage angle difference (δ_{ij}) and current magnitude ($|I_{ij}|$) are found to give the most consistent pattern. The percent increases are found using Equation (1)

$$\% \text{ inc} = \left| \frac{(x_{capacity} - x_{initial})}{x_{initial}} * 100 \right| \qquad (1)$$

Voltage magnitude, current angle, and real power are all observed, and determined to be inconsistent. To obtain the limits, voltage angle and current measurements are taken at the initial line loading condition. The system is then manipulated until the selected lines are operating at 80% capacity and 100% capacity. The percent increases from the initial capacity to 80% and from the initial capacity to 100% are calculated. The results are shown in Figures 1 and 2 for lines operating around 73% initial capacity. Clearly a limit can be set, as the largest base capacity to 80-% capacity is smaller than the smallest base capacity to 100% increase. Similar charts are created for lines operating at different initial capacities (all > 60%), and an average limit is found over the system.

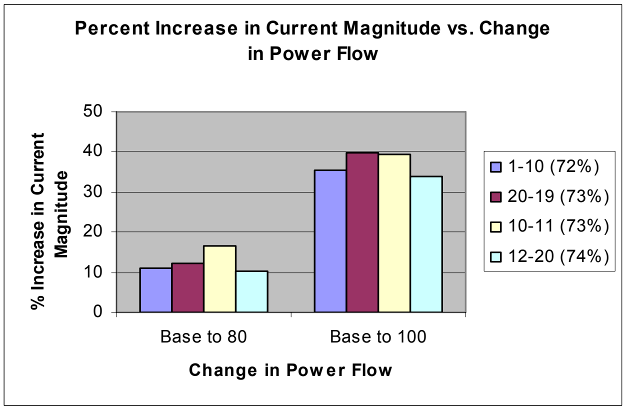


Figure 1 - % Increase in |Iij| for Lines around 73% Capacity

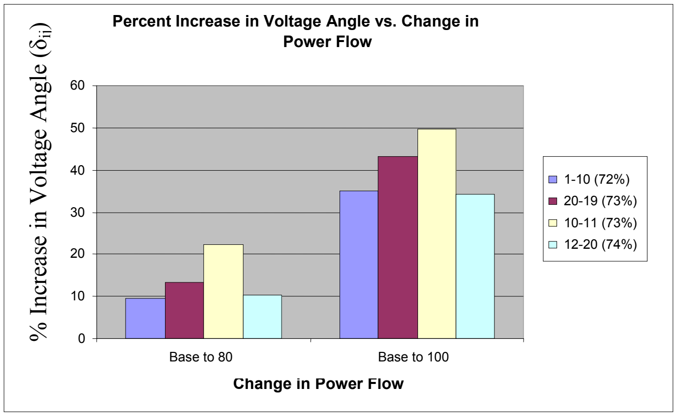


Figure 2 - % Increase in δ_{ij} for Lines around 73% Capacity

Once the limits are established for the system, an interface is created to allow the operator to observe the limits and violations. The proposed pie chart is shown in Figure 3.

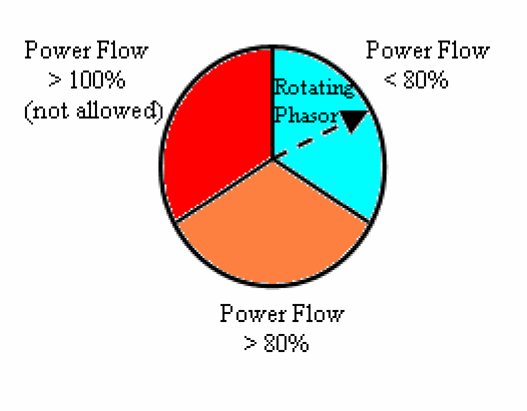


Figure 3 - Generalized Rotating Phasor Pie Chart

The current and voltage angle limits must work jointly to indicate overload, meaning both limits must be reached for the line to cross the overload threshold. For this reason, the operator display will consist of two pie charts, one for voltage angle, and one for current magnitude. These pie charts provide the operator with the proximity of a line to overloaded operation.

III. STEADY-STATE SENSITIVITY ANALYSIS

Sensitivity analysis methods are very important when analyzing a large power system. Systems with older protection schemes may not have PMU data available at all locations in the system. Sensitivity methods help to determine the most efficient placement of PMUs. Newer protection devices come equipped with the ability to obtain phasor measurements. Now data processing, rather than location, is an issue. Multiple phasor measurements are taken by each PMU. If many PMUs are located in the system, the amount of resulting data will be enormous. Operators need an efficient way to process only the data that correlates strongly to system state. This reduces both the communication required, as there is less data to move from the PMU to the operator, and also the time necessary for data evaluation.

The equation for determination of sensitivity in this paper is developed using a premise from the previous section. The current magnitude and voltage angle are shown to be good indicators of system state, and are therefore used in determining the sensitivity to overload as well. The final sensitivity equation uses the partial derivative of the line current with respect to the line voltage angle difference, as shown in Equation (2). The proof of this equation is shown in the appendix.

$$\frac{\partial \left| I_{ij} \right|}{\partial \delta_{ij}} = \frac{V_i V_j \sin(\delta_{ij}) * \left| Y_{ij} \right|}{\sqrt{V_i^2 + V_j^2 - 2 V_i V_j \cos(\delta_{ij})}} \tag{2}$$

Equation (2) is applied to the off-line system for verification and the results are shown in Table 1.

Line	Sensitivity - Base Loading	Sensitivity – 20% Increased Loading
29-33	776.25	772.45
19-34	424.56	418.81
12-17	181.38	180.74
12-20	121.18	119.77
19-20	101.54	100.33

Table 1 – Application of Sensitivity Equation in Off-line System

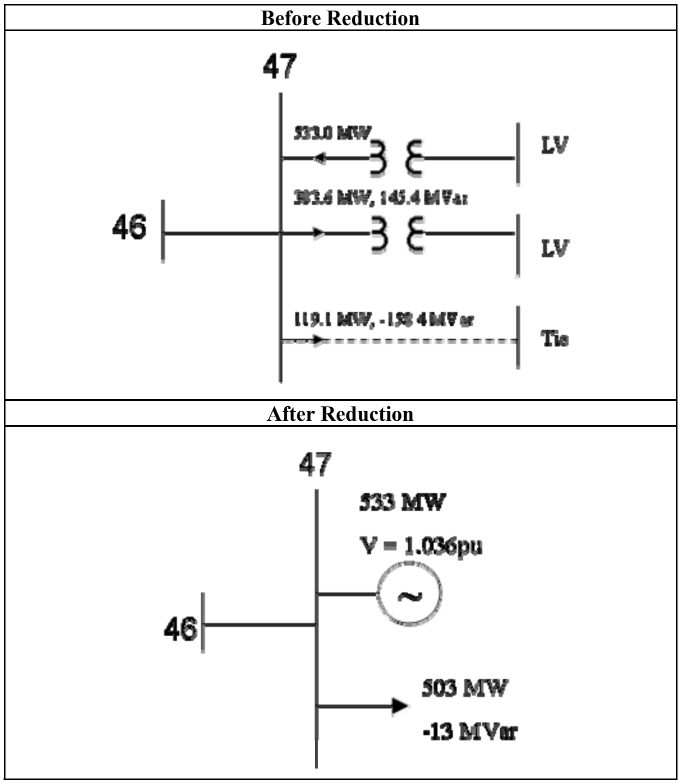
As this equation represents a scaling factor, lines above 100 are determined to be sensitive. Table 1 shows consistency in the determination of sensitive lines in different loading conditions.

IV. ON-LINE PHASOR STUDY

On-line analysis is a vital component to this research. While off-line systems are useful in determining and verifying patterns, these patterns will be applied to on-line systems and must be further verified for use in real-time. The on-line system is provided by a local utility and contains 7476 buses. It is divided into forty-seven areas, with the area of interest containing 1289 buses. Data from nine PMUs is available for this area, with most of the PMUs located on the 500kV transmission system. Due to space limitation and confidentiality agreement, the system data cannot be included here. The window of PMU data includes a three phase bus fault occurring in the low voltage side of the system.

Data for this system is provided in table format, with no visual aid. In order to verify the PMU data and system behavior, the system configuration is necessary. Attempting to determine this configuration solely by tracing out line and bus connections, found in the data table provided by the utility, requires an enormous amount of time. The system must be reduced to a manageable size for analysis. The approach used in this paper is power injection equivalents. Most of the PMUs in the on-line system are on the major transmission lines. As this paper focuses on transmission, it is decided that the transmission system will be separated from medium and low voltages. Power flow injections are observed for lines connecting high voltage to lower voltage, and for tie lines connecting 500kV buses in the area of study to 500kV buses in other areas. Any buses that are incident to the above lines are referred to as “boundary buses”. Buses in lower voltage or other areas are “external buses”, and 500kV buses in the area of study that are connected only to other 500kV buses within the area are “internal buses”.

When the equivalent is complete, the network consists of internal and boundary buses, along with the 500kV transmission lines connecting them. Any power injection going out of a boundary bus is modeled as a load, any power injection coming into a boundary bus is modeled as a generator, and if there is a comparable exchange of power (i.e. in and out) the boundary bus has both a generator equivalent and load equivalent. An example of this concept is shown in Figure 4.



Before Reduction

After Reduction

Figure 4 – Example of Power Injection Equivalents

The reduced system now contains only 47 buses, allowing for more straightforward visual analysis. System reduction is performed for three cases. The first is the pre-fault condition, the second is the faulted condition, and the third is the post-fault condition. System changes occur on the entire system and the power injections are recalculated for each individual case. As the fault occurs on the low voltage side, this provides the most accurate reduction. In all cases, the total generation is verified to be larger than the total load. The pre-fault slack bus is the bus with the highest generation, numbered bus 1 here, and remains the slack bus in all cases.

The line current magnitudes and δ_{ij}'s are determined in each reduced case. These results are then compared to the actual PMU data for verification. The PMU data for Bus 5, which is the closest 500kV bus to the fault, is shown in Figures 5 and 6. The data for the remaining high voltage PMUs is similar, with the affect of the fault decreasing in proportion to the distance of the PMU bus from the fault.

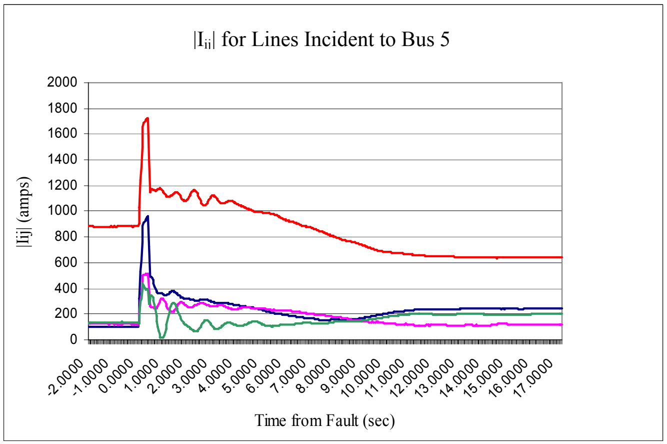


Figure 5 – Bus 5 PMU $|I_{ij}|$ Data

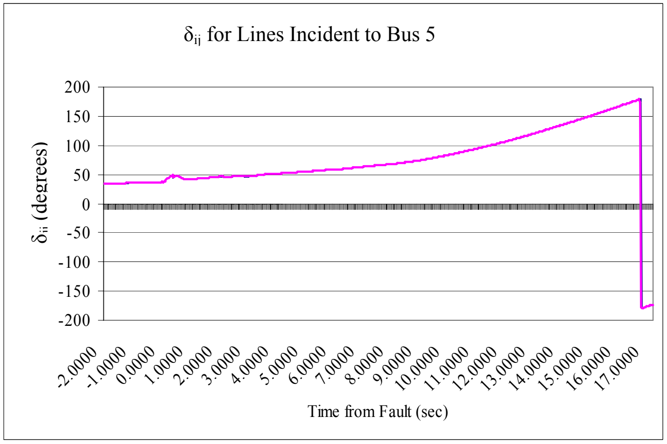


Figure 6 – Bus 5 PMU δ_{ij} Data

A three-phase bus fault occurs at time 0.00 on the time scale of the PMUs. The fault is cleared from the system after 29 cycles (about half a second) when the two lines incident to the faulted bus are outaged. It is clearly seen that the current magnitude spikes in either an upward or downward direction at the time of the fault, and a transient decay in current occurs after the fault is cleared. The current eventually settles at a value slightly higher or lower than the pre-fault value. The PMUs at a distance further away from the fault have less of a response to the fault, which is expected. The effect of the fault on voltage angle difference can most clearly be seen in the PMUs closest to the fault as well. At Bus 5 the voltage angle differences increase when the fault occurs. The post-fault angle difference then returns to a level comparable to the pre-fault difference. Due to the length of the fault, however, the angular difference continues to increase after the fault is cleared. This may lead to a stability problem in the system even though no lines are overloaded.

The reduced system data follows closely with the PMU data, with the main sources of error provided by the transient nature of the system after the fault. PowerWorld models the system in steady state, and therefore shows no transient values. The reduced post-fault system current values match best with the PMU current data after transient decay, and the continuous increase in voltage angle difference cannot be observed. This highlights the importance of monitoring actual system data in real time as opposed to simply simulating events on the system, as well as understanding the capabilities of the chosen simulation program.

The purpose of the pie chart limit display is to give the operator a real-time indication of system state, with particular emphasis on overloading. As the rotation approaches a limit threshold, the operator is given some warning that a line is about to become overloaded. In the case of a fault, however, the change is instantaneous. This has been seen in the previous PMU measurements. The operator has no warning that the fault is about to occur. In this regard, the PMU data cannot be used to verify the rotation in the direction of overload. The PMU data can, however, support the indication of system state both during and after the fault.

In the faulted case, only one line becomes overloaded. This line is operating at 49% capacity in the pre-fault case. For this line, there is a percent increase of 90.72% between the pre-fault and faulted current magnitude, and a percent increase of 98.53% between the pre-fault and faulted voltage angle difference. These percentages, along with the initial capacity of the line, will clearly pass the limit thresholds in the pie chart. In the post-fault case, when the line is operating at 47% capacity, the percent increases are 4.89% for the current magnitude and 4.16% for the voltage angle difference. These values will not cross the limit thresholds, indicating the line is no longer overloaded.

It is important to note that the pre-fault case given is very lightly loaded. The most heavily loaded line is operating at 58% capacity. This indicates that the fault occurred when the system was not at peak loading conditions. The result is that the system is more tolerant to increases in power through the lines when the fault is introduced, and fewer overloads occur. In order to set the limits for the pie chart, the system needs to be more heavily loaded. If the limits are set when the system is lightly loaded, a high percentage change may not indicate an overload, but simply an increase in power flow through a line. To verify that limits can be established if the system given had included heavier loading, the overall system load in the reduced case is increased by 25%. This brings the highest line capacity to 70%. Lines operating between 60% and 70% capacities are manipulated as in the offline study. Current magnitude and voltage angle measurements are obtained for the line in the initial state, when the line is operating at 80% capacity, and at 100% capacity. The results are shown in Figure 7.

It is seen that limits can again clearly be established, as the percent increases are easily divided into independent subsets. The allowable percent increases are largest for the line with the lowest initial capacity, as expected. It is also noticed that lines with lower capacity restrictions change loading percentages faster than lines with larger capacity restrictions. This indicates the charts should not only be observed on the lines that are heavily loaded initially, but also on the lines with the lowest MVA capacity restrictions. This study shows the pie chart limit can be established for the online system if the operator observes a higher loading condition than is given in pre-fault data. Note that the limits established above do not actually apply to the given pre-fault system, but are merely mentioned to provide assurance that limit setting is still achievable.

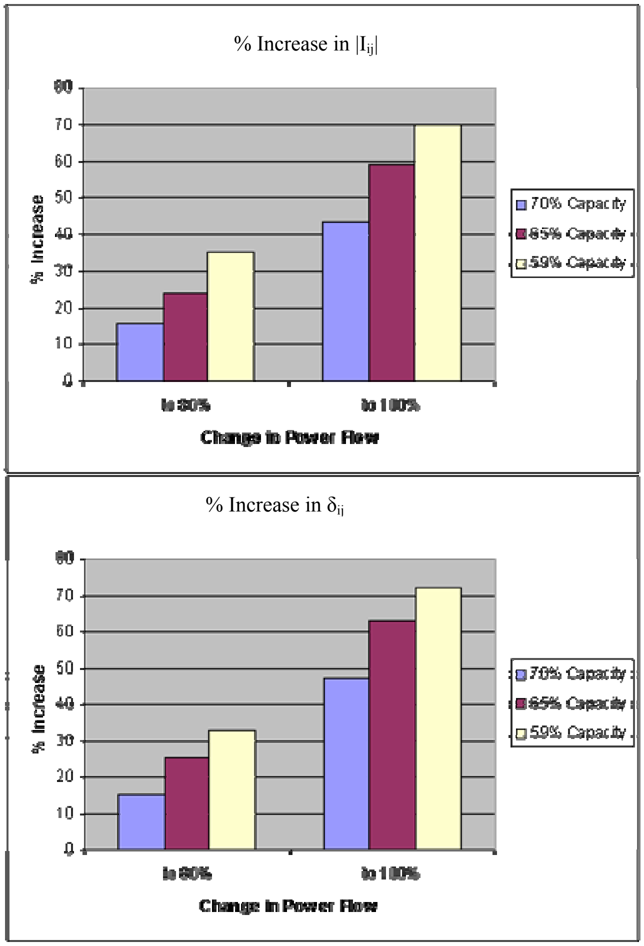


Figure 7 – On-line System Limit Study

Even though only one case of overload can actually be observed in the given data, additional values of the phasor limit chart can be seen in the faulted case. The percent increases in current at the time of the fault are very high while the voltage angle limits are not as significant. As mentioned previously in the paper, the thresholds for both current magnitude and voltage angle must be crossed jointly to indicate an overload. However, if one threshold is crossed while another is not, a system disturbance is still indicated. In the case of the fault, the current magnitude limit is crossed during the fault. The PMUs observe the transients as well, allowing the limits to be crossed repeatedly for a short period of time. Depending on the time sampling of the PMUs and the phasor pie chart, transients can be roughly observed. Once the fault is cleared, the voltage angle difference slowly continues to increase. The percent increase in the current is not significant any longer, but the percent increase in voltage angle approaches and eventually crosses limit thresholds. Now the phasor pie chart indicates problems with system stability. These additional indications lead to the conclusion that the phasor pie chart has multiple uses for the operator.

V. ON-LINE SENSITIVITY

The sensitivity equation ($\partial|I_{ij}| / \partial\delta_{ij}$) is applied to all lines in the reduced system. The results of the sensitivity study are comparable across all system cases. Many of the lines that are determined to be sensitive in the pre-fault case are sensitive in the post-fault case as well. Even though many of these same lines are sensitive in the faulted case, this case provides the most discrepancies. These discrepancies from the other cases are due to the behavior of the voltages and currents in the faulted case. This supports the conclusion that an overall system sensitivity analysis should not be performed when the system is in a faulted condition, and is only valid in steady state analysis.

VI. OFF-LINE AND ON-LINE COMPARISONS

In the course of this research, a phasor pie chart is developed. Limit thresholds are created using percent increases in voltage angle and current magnitude. These limits are developed off-line and then applied to an on-line system with PMU data. It is determined that limits can be established both in off-line and on-line systems, as long as the system is in a moderate to heavy loading condition. Tendencies to overload are observed in both off-line and on-line systems. In the off-line system, the line capacities are all similar values. In this system it is seen that the lines that start out heavily loaded are the first to overload. In the on-line system this is not always the case. The tendencies to overload here depend on a number of different factors, such as line capacities, initial line loading conditions, and locations of increased loads. This is more realistic for real applications, and indicates that the operator must observe not only heavily loaded lines, but those with strong capacity restrictions as well.

Limit thresholds must be crossed jointly to indicate an overload, but can indicate disturbances when crossed individually. This is seen in the off-line study and in the on-line study. It is important to observe when one parameter has large changes and the other does not, supporting the display proposed previously. This is also the main premise for sensitivity analysis. The lines most likely to overload cannot be constantly predicted, as the tendency to overload may change with different system configurations. The potential of a line to have large individual parameter changes with small changes in another parameter, however, is predicted using the sensitivity equation. The results of this sensitivity equation are shown to be consistent in both the off-line and on-line systems across different system loading conditions and configurations.

VII. CONCLUSIONS

This research examines many important manipulations of the power system. Increased load can become a serious problem for a sensitive power network because of overloaded lines. The increased loading can cause lines to become heavily loaded and even to fail. Many times, alleviation of overload can be accomplished by simply forcing outages, allowing power flow to be redistributed to lines which can handle the increase. The applications of this research are numerous in that every power distributor will have to deal with excess loading as well as unintentional power line outages.

Using the limits on percent increase of current magnitude and voltage angle along with PMU readings from the system and the chart shown previously, it can be determined when lines are approaching overload or instability. Operators can readily see results of forced outages, and can determine whether a change in operation has helped to redistribute the load more evenly within the system. This allows for the most efficient and effective method of transmission for the power company. The developed chart is very important in examining a system after any change in operating conditions occurs, allowing a method of correction without running load flow. This leads to a faster solution of a problem that may cause a blackout.

Future research in this area could include dynamic and stability studies, further development of the solution algorithm, a system implementation study, an application study in the unbalanced distribution system, and continued research in sensitivity analysis, specifically in bus sensitivity.

APPENDIX

The current phasor through line i-j could be expressed as

$$I_{ij}\angle\partial = (V_i\angle\delta_i - V_j\angle\delta_j)Y_{ij}\angle\theta_{ij}\,.$$

Take the norms on both sides of this equation, it equals to

$$\left|I_{ij}\right| = \left|V_i\angle\delta_i - V_j\angle\delta_j\right|\left|Y_{ij}\right|.$$

According to cosine law, the norm of the variation between voltage phasor i and j could be expressed by a quadratic term.

$$\left|I_{ij}\right| = \sqrt{V_i^2 + V_j^2 - 2V_iV_j\cos(\delta_{ij})}\cdot\left|Y_{ij}\right|$$

Differentiating the above equation with δ_{ij}, the following equation could be concluded.

$$\frac{\partial\left|I_{ij}\right|}{\partial\delta_{ij}} = \frac{V_iV_j\sin(\delta_{ij})*\left|Y_{ij}\right|}{\sqrt{V_i^2 + V_j^2 - 2V_iV_j\cos(\delta_{ij})}}$$

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