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Selecting Wavelet Transforms Model in Forecasting Financial Time Series Data Based on ARIMA Model

S. Al Wadi[a], Mohd Tahir Ismail[b]

[a,b]School of Mathematical science, Universiti sains Malaysia
11800 Minden, Penang, Malaysia
sadam_alwadi@yahoo.co.uk, mtahir@cs.usm.my

M. H. Alkhahazaleh[c]

Faculty of Science and Technology
University Kebangsaan Malaysia
43600 Ukm Bangi, Selangor D. E. Malaysia
ahmed_2005kh@yahoo.com

Samsul Ariffin Abdul Karim[d]

[c] Universiti Teknologi Petronas
31750 Bandar Seri Iskandar, Tronoh, Perak, Malaysia
samsul_ariffin@petronas.com.my

Abstract

Recently, wavelet transforms have gained very high attention in many fields and applications such as physics, engineering, signal processing, applied mathematics and statistics. In this paper, we present the advantage of wavelet transforms in forecasting financial time series data. Amman stock market (Jordan) was selected as a tool to show the ability of wavelet transform in forecasting financial time series, experimentally. This article suggests a novel technique for forecasting the financial time series data, based on Wavelet transforms and ARIMA model. Daily return data from 1993 until 2009 is used for this study.

Keywords: Wavelet transform, ARIMA model, financial time series, forecasting

1 Introduction

Time series forecasting is very popular in many fields such as economics, statistics, etc. In recent years, stock markets forecasting is required for the investors and it has got very high attention in financial time series and financial researchers. The accurate forecasting of financial prices is an important issue in investment decision making. However, financial time series data appears noisy and non-stationary [17,10]. The noise characteristic indicates the unavailability of complete information from past behavior of financial markets to fully capture the dependency between future and past prices. The information that is excluded in the forecasting model is considered as noise while the non-stationary characteristic indicates the distribution of financial time series changing over time. Therefore, financial time series forecasting is considered as one of the most challenging tasks of time series analysis.

There are many forecasting models that have been used in the forecasting literature, such as; simple moving average, linear regression, neural network, ARMA model and ARIMA model. In order to provide estimates for the future, these models analyze the historical data. Usually time series are not deterministic series. In fact, in many cases the researchers considered the series to be stationary time series. One way to model any time series is to consider it as a deterministic function plus white noise. The white noise in any time series process can be minimized by some procedures which are called the de-noising. Then a better model can be obtained. Consequently, to obtain a good de-noising, there are some mathematical models that can be applied such as Fourier transform and Wavelet transform.

Wavelet transforms have been used in many fields of mathematical forecasting, [15] in 2008 Sanjeev Kumar et al. decomposed the historical price data into wavelet domain constitutive sub series using wavelet transform, and then combined with the other time domain variables to perform the set of input variables for the proposed forecasting model. Based on statistical analysis the behavior of the wavelet domain constitutive series has been studied. It has been observed that forecasting accuracy can be improved by the use of wavelet transforms in a forecasting model. Rumaih M. and Mohammad A., in 2002[14] used Saudi stock index to illustrate that wavelet transform is better than the other forecasting technique in predicting the de-noising of the financial time series. Aggarwal et al., 2008 [9] suggested that Forecasting performance of the wavelet transforms based mixed model has been compared with the other three models. The proposed model was found to be better. Performance evaluation for different wavelets were performed, and it has been observed that for improving forecasting accuracy using WT, Daubechies

wavelet of order two, gives the best performance.
The fundamental and novel contribution of the paper is to use the wavelet transform, to decompose the return Amman stock market into a set of better-behaved approximation series. The forecasting results based on wavelet transform and ARIMA model will compare with the forecasting values based on ARIMA model by using some statistical criteria. MATLAB 2008a and SAS 9.1 programs have been used to get significant results and fair comparison.
This paper is organized as follows. The next section describes the principle of the mathematical framework. Section 3 provides a description of data set. Section 4 provides the methodology. In Section 5 the experimental results are presented to demonstrate the effectiveness of wavelet transform in the forecasting methods. In Section 6 we summarize our contributions and mention the conclusion. And finally we mention the acknowledgement.

2 Mathematical Frameworks

2.1 Wavelet Analysis

Wavelet analysis is a mathematical model that transforms the original signal (especially with time domain) into a different domain for analysis and processing [18,22, 27]. This model is very suitable with the non-stationary data, i.e. mean and autocorrelation of the signal are not constant over time, that is well know, most of the financial time series data is non- stationary, that is why we applied wavelet transform.
In mathematical literature, Fourier transforms decomposed the original signal into a linear combination as a sine and cosine function whereas by wavelet transform the signal is decomposed as a sum of a more flexible function called wavelet that is localized in both time and frequency. The wavelet transforms were used to adopt a wavelet prototype function (mother wavelet). Temporal analysis is constructed with a contracted, high-frequency version of prototype wavelet, whereas frequency analysis is performed with a dilated, low frequency version of the prototype wavelet. Because the function can be represented in terms of a wavelet expansion (using coefficients in a linear combination of the wavelet functions), data decompositions can be constructed by just using the corresponding wavelet coefficients. There are several types of wavelet transforms. Depending on the applications, regarding the continuous input signal, the time and scale parameters can be continuous, leading to the continuous wavelet transform (CWT). On the other hand, the discrete wavelet transform (DWT) can also be used for discrete time signals.
In the wavelet transforms case, consider that the time domain is the original domain. Although, wavelet transforms is the transformation process from time domain to time scale domain, these processes are known as signal decomposition because a given signal is decomposed into several other signals with deferent levels of resolution. These processes allow recovering the original time domain signal without losing any information. Wavelet

transforms has reverse process which is called the inverse wavelet transform or signal reconstruction [8].

The wavelet transform is implemented using a multiresolution pyramidal decomposition technique. In fact, a recorded digitized time signal *S* (*n*) can be analyzed into its detailed *cD* 1(*n*) and smoothed (approximations) *cA*1 (*n*) signals using high-pass filter (HiF-D) and low-pass filter (LoF-D), respectively. High-pass filter has a band-pass response. Consequently, the filter signal *cD* 1(*n*) is a detailed coefficient of *S* (*n*) and contains higher frequency components. While the approximation signal *cA*1 (*n*) has a low-pass frequencies filter response. The decomposition of *S* (*n*) into *cA*1 (*n*) and *cD* 1(*n*) is the first scale decomposition. Inversely, that is possible to perform the original signal from the approximations and details coefficients.

In this paper we will focus in the most famous types of discrete wavelet transform which are Haar wavelet transform and Daubechies wavelet transform. The wavelets having compact support or narrow window function are suitable for local analysis of the signal. Daubechies wavelets and Haar wavelet are compactly supported orthonormal wavelets and are the most appropriate for treating a non-stationary series [1].

Definition: [2,11] discrete Wavelet transform can be defined by the following function:

$$\psi_{j,k}(t) = 2^{\frac{j}{2}}\psi(2^j t - k),\ j,k \in Z;\ z = \{0,1,2,....\}.$$

Where ψ is a real valued function having compactly supported, and $\int_{-\infty}^{\infty} \psi(t)dt = 0$ Generally, the wavelet transforms were evaluated by using dilation equations, given as:

$$\phi(t) = \sqrt{2}\sum_k l_k \phi(2t-k),$$

$$\psi(t) = \sqrt{2}\sum_k h_k \phi(2t-k).$$

Father and mother wavelets were defined by the last two equations where $\phi(2t-k)$ represents the father wavelet, and $\psi(t)$ represents the mother wavelet. Father wavelet gives the high scale approximation components of the signal, while the mother wavelet shows the deviations from the approximation components. This is because the father wavelet generates the scaling coefficients and mother wavelet evaluates the differencing coefficients. Father wavelet defines the lower pass filter coefficients (h_k). High pass filters coefficients (l_k) are defined as [7].

$$l_k = \sqrt{2}\int_{-\infty}^{\infty} \phi(t)\phi(2t-k)dt,$$

$$h_k = \sqrt{2}\int_{-\infty}^{\infty} \psi(t)\psi(2t-k)dt.$$

Haar wavelet transform is the oldest and simplest example in the wavelet transforms and is defined as:

$$\psi^H(t) = \begin{cases} 1, & 0 \le t \le \frac{1}{2} \\ -1, & \frac{1}{2} \le t \le 1 \\ 0, & Otherwise \end{cases}$$

For the Haar wavelets transform:

$$l_k = \sqrt{2}\int_{-\infty}^{\infty} \phi(t)\phi(2t-k)dt = \begin{cases} 1, & 0 \le \text{t} \le 1 \\ 0, & \text{otherwise} \end{cases},$$

And for N = 2, $l_k = \{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\},\ h_k = \{\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\}.$

Note: the mother wavelet satisfies the following two conditions:

$$\int_{-\infty}^{\infty} \psi(t)dt = 0,\ \int_{-\infty}^{\infty} |\psi(t)| < \infty,\ \int_{-\infty}^{\infty} \frac{|\psi_1(\omega)|^2}{|\omega|} d\omega < \infty.$$

Where $\psi_1(\omega)$ presents the wavelet transform.

Daubechies wavelet transforms: Since Haar wavelet is the simplest and oldest wavelet transform; it was improved by Daubechies in 1992[14]. He developed the frequency – domain characteristics of the Haar wavelet. However, we do not have a specific formula for this method of wavelet transform. So, we tend to use the square gain function of their scaling filter, the square gain function was defined as [13].

$$g(f) = 2\cos^l(\pi f)\sum_{l=0}^{\frac{l}{2}-1} \begin{pmatrix} \frac{l}{2} - 1 + l \\ l \end{pmatrix} \sin^{2l}(\pi f).$$

l: Positive number and represents the length of the filter, for more details and examples see [2,12,13,25,26].

2.2. ARIMA model

Application of nonlinear regression to price forecasting has not been reported so far. Other approaches of econometric modeling are univariate time series methods like auto regressive moving average (ARMA) [3,9]. ARMA is a suitable model for the stationary time series data, although most of the software uses least square estimation which requires stationary. To overcome this problem and to allow ARMA model to handle non-stationary data, the researchers investigate a special class for the non-stationary data. This model is called Auto-regressive Integrated Moving Average (ARIMA). This idea is to separate a non-stationary series one or more times until the time series

becomes stationary, and then find the fit model. ARIMA model has got very high attention in the scientific world. This model is popularized by George Box and Gwilym Jenkins in 1970s [4]. There are a huge number of ARIMA models; generally there are ARIMA (p, q, d) where: P: order of autoregressive part (AR), d: degree of first differentiation (I) and q: order of the first moving part (MA). Note that, if there is no differencing been done (d = 0), Then ARMA model can be got from ARIMA model.
The general mathematical ARIMA model can be defined as [16]:

$$W_t = \mu + \frac{\beta(v)}{\varepsilon(v)} a_t.$$

Where:

t : Indexes time.

W_t : The response series Y_t or a difference of the response series.

μ : The mean term.

v : The backshift operator; that is, $vX_t = X_{t-1}$.

$\varepsilon(v)$: The autoregressive operator, represented as a polynomial in the backshift operator:

$$\varepsilon(v) = 1 - \varepsilon_1(v) - ... - \varepsilon_p v^p.$$

$\beta(v)$: The moving-average operator, represented as a polynomial in the backshift operator:

$$\beta(v) = 1 - \beta_1(v) - ... - \beta_p v^p.$$

a_t : The independent disturbance, also called the random error.

The model building process involves the following steps; Model identification,
Model parameter estimation, Model Diagnostics and Forecasting. For more details refer to [20].

2.3. Daily return data

The daily logarithmic return, r_t for all market prices can be calculated using the definition of historical volatility as [20]:

$$r_t = \ln(p_t) - \ln(p_{t-1}).$$

Where p_t indicates to the price information at time t.

3 Data Description

In order to illustrate the effectiveness of Haar wavelet transforms and Daubechies wavelet transform, the Amman Stock Market data sets are selected for discussion. We consider a daily return data for the time period from April 1993 (the days when stock exchanges were open) until December

2009 with a total of 4096 observations. The total number of observations for mathematical convenience is suggested to be divisible by 2^j. It means that the data should satisfy the condition of observations= 2^j. For more details refer to [24,11].
In order to apply ARIMA model, the data should be stationary. Therefore, the return data can be considered for this comparison. Because in the financial literature, it is well known that the return series is stationary. Moreover, this result can be checked empirically, provided, it has a sufficient number of historical returns available [21]. Moreover, the returns are serially uncorrelated which means that the data is stationary and suitable to apply the ARIMA model with the return data, directly without any treatments [23]. The approximation series data has been considered in this comparison since it contains the main component of the transform and it shows all the information about the original series.

4 Methodologies

This section consists of two subsections. Firstly, we will present the criteria which have been used to make a fair comparison, and then the framework comparison will be presented with more details.
4.1 Prediction accuracy criteria
We have been adopted to compare the performance of the models within three types of accuracy criteria [19]:

1- Mean square error (MSE).
2- Root mean squared error (RMSE).
3- Mean absolute error (MAE).

MSE can be defined by:

$$MSE = \frac{\sum_{i=1}^{N} (\text{actual value-predicted value})^2}{N}.$$

RMSE can be defined by:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (\text{actual value-predicted value})^2}{N}}.$$

And

MAE can be defined by:

$$MAE = \frac{1}{N}\sum_{i=1}^{N} \left| \frac{\text{actual value-predicted value}}{\text{actual value}} \right|.$$

Where N represents the number of observations used for analysis.

4.2. Comparison framework:

The wavelet transform converts the return data series into two sets; approximation series (CA1 (n)) and details series (DA1 (n)). These two series present a better behavior. i.e. More stable in variance and no outliers than the original price series, then, they can be predicted more accurately. The reason for the better behavior of these two series is the filtering effect of the wavelet transform. In this paper the Approximation series has been used since this series behave as the main component of the transform, while the detail series provides “small” adjustments. The procedure explained in this paper is as follows:

Firstly, Decompose through the wavelet transform (Haar wavelet transform and Daubechies wavelet transform (db2)) the available historical return data.

Secondly, Use a specific ARIMA model fitted to each one of the Approximation series to make the forecasting.

Thirdly, this technique is compared with an ARIMA model used directly to forecast the return data series by using the above criteria.

5 Experimental Results

In this paper, the minimum value of MSE, RMSE and MAE is considered to select the best ARIMA model of the daily return data. All choices of ARIMA models for the return data are included in this test between (0,0,0) and (2,2,2). If we choose more than two, then there are more complicated conditions that should be satisfied. Also, if p and q are more than two, then Autocorrelation function (ACF) and partial Autocorrelation function (PACF) will be presented as an exponential decay. This means that ARIMA model becomes worthless and there is no importance.

Table1. Shows the statistical criteria for the ARIMA (p,d,q) model

Statistical fit	Value after transform via Daubechies	Value after transform via Haar	Value before transform(original return data)
Mean square error (MSE)	0.00001221	5.67541E-6	0.00001886
Root mean square error(RMSE)	0.0034987	0.0023823	0.0043386
Mean absolute error (MAE)	0.0002364	0.0002198	0.00294

The return data for Amman stock market has been used as a case study. Price forecasting is performed using daily data. Moreover, for the sake of fair comparison the same sample data is selected. (From 1993-2009). The fit ARIMA model for the original return data is considered as ARIMA(2,0,2) with root mean square error equal to 0.0043386 as presented in table 1, while the fit ARIMA model for the transform data by using Haar wavelet transform is selected as ARIMA (1,0,1) with root mean square error equal to 0.0023823 as presented in table 1 also. Although the fit ARIMA model for the transform data using Daubechies wavelet transform is selected as ARIMA (2,2,0) with root mean square error equal to 0.0034987,Table 1 shows some other criteria about these results. All of these criteria explain that the Wavelet - ARIMA model is better than the ARIMA model. Moreover, the Haar wavelet transform gives more sufficient result and better than Daubechies wavelet transform in the forecasting. However, in some statistical literature, Daubechies wavelet transform is better than Haar wavelet in the decomposition, but in this paper we found a negative result, the reason is related to the data set since just the approximation series have used in the comparison.

Moreover, Results in Table 1 indicate that ARIMA model for the returns data after wavelet transforms produce smaller forecast error as compared to the ARIMA model for actual returns data. Furthermore, the standard errors which measure the variation between returns data after wavelet transforms are also small. All of these criteria explain that the Haar wavelet transform gives more sufficient result and better than Daubechies wavelet transform in the forecasting.

6 Conclusions

As conclusion for this article, if the Wavelet transform is used for the return data, then there are no outlier, seasonal effects and other irregular effects. Generally the result of the approximation series under the wavelet transforms (rather by using Haar or Daubechies) is better than the original return data and more stable in variance, mean and no outliers. Furthermore, the forecasting using ARIMA (p, d, q) under the transformed series is better than forecasting directly, and also it gives more accurate results.

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References

[1] A.J. Rocha Reis and A.P. Alves da Silva, Feature extraction via multiresolution analysis for short term load forecasting, IEEE Transactions on Power Systems, 20 (2005), 189-98.

[2] Chang Chiann and A. Moretin, A Wavelet Analysis for Time Series, Nonparametric Statistics, 10 (1998), 1-46.

[3] D.J. Swider and C. Weber, Extended ARMA models for estimating price developments on day-ahead electricity markets, Electric Power Systems Research, 77(2007), 583-93.

[4] G.E.P.Box, and G.M. Jenkins, Time series analysis: Forecasting and control, San Francisco, 1970.

[5] G. Janacek and L. Swift, Time series forecasting, simulation and applications, Ellis Hoewood limited, England, 1993.

[6] H. Zareipour, C.A. Canizares and K. Bhattacharya, Application of public-domain market information to forecast Ontario’s wholesale electricity prices, IEEE Transactions on Power Systems, 21 (2006), 1707-17.

[7] I. Daubechies, Ten Lectures on Wavelets,SIAM and Philadelphia, 1992.

[8] In-Keun Yu; Chang-Il Kim and Y. H. Song, A Novel Short-Term Load Forecasting Technique Using Wavelet Transform Analysis, Electric Machines and Power Systems, 24 (2000), 537–549.

[9] J. Contreras, R.Espinola, F.J. Nogales and A.J. Conejo, ARIMA models to predict next-day electricity prices, IEEE Transactions on Power Systems, 18(2005), 1014-20.

[10] G.J. Deboeck, Trading on the Edge: Neural, Genetic and Fuzzy Systems for Chaotic Financial Markets, Willey, New York, 1994.

[11] Philippe Masset, Analysis of Financial Time-Series Using Fourier and Wavelet Methods, University of Fribourg (Switzerland) - Faculty of Economics and Social Science, 2008.

[12] P.Manchanda, J.Kumar and A.H.Siddiqi, Mathematical methods for modeling price fluctuations of financial time series, Journal of Franklin institute, 344 (2007), 613-363.

[13] R. Gencay, F. Seluk and B. Whitcher, An Introduction to Wavelets and Other Filtering Methods in Finance and Economics, Academic Press, New York, 2002.

[14] M. Alrumaih and A. Al- Fawzan, Time series forecasting using wavelet denoising an application to Saudi stocks index, J.king Saudi Univ,14 (2002), 221-234.

[15] Sanjeev Kumar Aggarwal, Lalit Mohan Saini and Ashwani Kumar Electricity Price Forecasting in Ontario Electricity Market Using Wavelet Transform in Artificial Neural Network Based Model, International Journal of Control, Automation, and Systems (2008) 639-650.

[16] SAS/ETS® 9.22 User's Guide, SAS Institute Inc, 2010.

[17] S.A.M. Yaser and A.F. Atiya, Introduction to financial forecasting, Applied Intelligence, 6 (1996) 205–213.

[18] S.J. Yao, Y.H. Song, L.Z. Zhang and X.Y. Cheng, Wavelet Transform and Networks for Short-term Electrical Load Forecasting, Energy Conversion and Management, 41(18) (2000), 1975-1988.

[19] S.K. Aggarwal, L.M. Saini and Ashwani Kumar, Price forecasting using wavelet transform and LSE based mixed model in Australian electricity market. International Journal of Energy Sector Management, 2 (2008), 521-546.

[20] Spyros Makridakis, C. Wheelwright and J. Hyndman, Forecasting methods and applications, Third edition, Wiley& Sons, Inc, New York, 1998.

[21] S.Ruey, analysis of financial time series. Second edition, Wiley & Sons, Inc, 2005.

[22] S. Skander, On the Use of the Wavelet Decomposition for Time Series Prediction, Neurocomputing, 48(1-4) (2002) 267–277.

[23] Tim Bollerslev, Steve Lange, Forecasting financial market volatility: Sample frequency vis-a`-vis forecast horizon, Journal of Empirical Finance 6(1999), 457–477.

[24] Todd Wittman, Time-Series Clustering and Association Analysis of Financial Data, CS 8980 Project, 2002.

[25] Y. Motohiro, Measure Business Cycle: A Wavelet Analysis of Economic Time Series. Economics Letter, 100 (2008), 208-212.

[26] R.Struzik, Wavelet Methods in (Financial) Time Series Processing, Physica A, 296 (2001), 307-319.

[27] Z.Dong, X. Guo and J.Zheng, Calculation of Noise Resistance by Use of the Discrete Wavelets Transform, Electrochemistry Communications 3(2001), 561-565.

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