Power Electronic Voltage Regulation In LV Distribution Networks

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***Abstract*—Distribution networks are becoming more stressed by new connection of large single phase loads, namely heat pumps and plug in electric vehicles, as well as connection of distributed generation. These low carbon technologies cause the voltage profile to stray outside of regulator imposed standards, hence limiting the capacity for installation of these technologies. Here it is shown, by detailed simulation of a generic UK network, the effect of increasing load and generation upon the network. Four LV based power electronic topologies for voltage regulation are presented and then incorporated into the network. Results indicate the ability of power electronics installed at LV to increase capacity and reduce network losses. Finally, a comparison of these topologies is given.**

I. Introduction

As loading in LV networks grows due to electrification of domestic space heating via heat pumps (HP) and due to the increasing numbers of plug in electric vehicles (PEV) the variation of voltage profile on distribution networks grows [1]. Whilst heavy loading acts to reduce network voltage, distributed generation (DG), typically in the form of photovoltaic (PV), can cause reverse power flows thus increasing the voltage profile along the feeder [2].

As the distribution network operators (DNOs) are required to supply all LV customers within the voltage range defined in EN50160 (230V$\pm$10% in the EU [3] and +10%, -6% in the UK [4]), if thermal ratings are not reached prior to voltage straying outside of tolerance bands the network is voltage constrained. This limits the capacity for further low carbon technologies (LCT), so the DNO must then upgrade their equipment to alleviate this problem [5], [6]. These upgrades are however very costly, in particular cable upgrades cost £80/m [5].

Distribution networks in the UK do not include voltage regulation downstream of the 11kV primary substations [6]–[9]. As a result, the DNOs operate the network voltage towards the upper end of the appropriate voltage band to account for voltage drop along the HV (11kV) and LV (0.4kV) feeders [10]. Other work [10] has presented measured substations voltages and found almost half of the readings from the secondary substations were in the range 6-8% above the system nominal voltage. Continuing this strategy leads to frequent overvotlage problems as the amount of DG installed increases.

Fig. 1 shows the structure of a typical UK HV network, it is clear there will be variation of voltage profile not only in the LV network but also the HV network. Currently, the primary substations employ line drop compensation (LDC) in which locally measured power flow is monitored and controls the primary substation taps to regulate the substation voltage [7], [11].

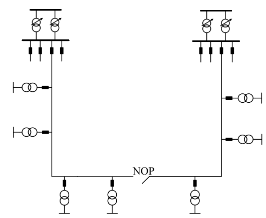

Fig. 1. Typical Arrangement of HV radial network with interconnected normally open point (NOP) for security of supply

Regarding DG in the LV network; PV is the most common and installations have risen rapidly since 2010 [12]. Small scale PV (<4kWp) installed by domestic customers in the LV network accounts of 33% of the 5GW PV capacity in the UK [13]. If voltage rise exceeds the G83/1 limits, PV will typically stop exporting power to the grid [14]. This means the PV owner will not benefit from local feed-in tariffs. The frequency and severity of over voltages is related to the PV point of common coupling (PCC), with PV at the remote feeder end being the most venerable. To alleviate overvotlage and PV disconnection, voltage control via PV power curtailment has been considered as a solution [15] but is not attractive as it still reduces the PV output. Reactive power control has been shown to improve voltage profile on German LV networks [16] and local control optimisation schemes have been presented in [17], [18].

In the previous work [16]–[18], centralised control schemes for PV control were not required. This is a key point to consider as the existing communications infrastructure and presently installed PV inverters are not readily able to facilitate centralised communication with thousands of small generators. Also for fast action to mitigate voltage problems local control are seen as more viable [18] and also achieved 80% of the reduction in network losses compared to a centralised control schemes. The costs associated with the ICT and communication required for centralised control account for a large portion of the total costs for devices with centralised control [19]; making decentralised control attractive.

II. Voltage Drop and Power Flow in LV Networks

Voltage regulation at transmission and HV levels in distribution network can be effectively controlled by reactive

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compensation, such as shunt connected capacitor banks [20] or FACTS devices such as the STATCOM [21]. For the simple two bus network shown in Fig. 2(a) the associated phasor diagram is shown in Fig. 2(b).

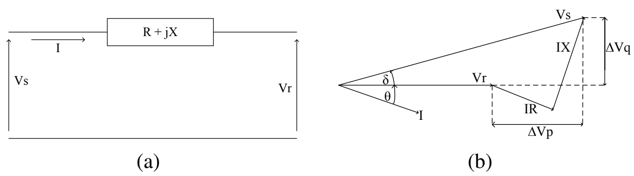

Fig. 2. Network power flow (a) and associated phasor digram (b)

With reference to Fig. 2(b) the voltage drop components [22] across the feeder are:

$$\Delta V_p = IR\cos(\theta) + IX\sin(\theta) \tag{1}$$

$$\Delta V_q = IX\cos(\theta) - IR\sin(\theta) \tag{2}$$

Noting that, $P = VI\cos(\theta)$ and $Q = VI\sin(\theta)$, we can rewrite (1) and (2) as:

$$\Delta V_p = \frac{RP + QX}{V_r} \tag{3}$$

$$\Delta V_q = \frac{XP - RQ}{V_r} \tag{4}$$

Given $\Delta V_q \ll V_r + \Delta V_p$ will hold in all practical scenarios, (3) and (4) are:

$$\Delta V \approx \frac{RP + XQ}{V_r} \tag{5}$$

$$\Delta V \approx \frac{RP + XQ}{V_r} \tag{6}$$

$$\frac{d\Delta V}{dP} \propto R \tag{7}$$

$$\frac{d\Delta V}{dQ} \propto X \tag{8}$$

The key point from (7) and (8) is that the magnitude of X and R (line parameters) impact the relative effect of P and Q flows upon the system voltage. Where the X/R ratio is high, it is the reactive power flow that plays the larger part in the voltage change along the network. Whilst this is true in HV networks where the X/R ratio is high ($\geq$3), it is not so in LV networks where the ratio is typically below unity [23]. For this reason voltage control via reactive compensation on LV networks is ineffective, inefficient, and expensive. Use of reactive compensation for noteworthy voltage control in LV networks would lead to increased cable losses (I^2R) and require a large compensator rating.

III. Power Electronic Solutions

The devices considered for improvement of voltage profile and capacity in LV networks are:

- Power Electronic Substation (PES)
- On-line Tap Changer (OLTC)
- Active Power Filter (APF)
- Mid Feeder Compensator (MFC)

A. Power Electronic Substations (PES)

Power electronic substation for use in LV networks are receiving greater attention [24]. Some particular advantages offered are its ability to offer continually variable output voltage across each feeder. From Fig. 3 a modular output converter can be used to enable per feeder control and the possibility of LVDC networks. For the purposes of this work the PES will be used to offer a continually adjustable magnitude of balanced set of voltages on each LV feeder.

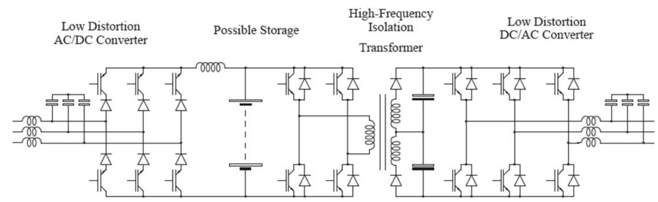

Fig. 3. Possible Configuration of a PES

The PES control scheme can utilise LDC employed individually for each feeder to control the magnitude of the PES voltage. Individual control of each feeder has clear advantages if the substation had both commercial and residential feeders, which have different load/generation profiles.

B. On-line tap changers (OLTC)

The secondary substations in the UK are already fitted with off-line taps on the primary winding, these transformers have a nominal ratio of 11/.433 kV [6], [9], aligned with taps set at the nominal value. This results in high substation voltages [10], limiting DG hosting capacity [25].

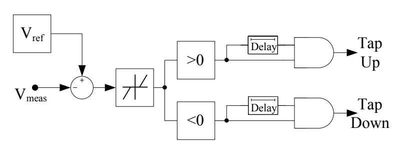

Fig. 4. AVC Type Control Logic of an OLTC

Herein, we suppose, OLTC functionality is added to 11/0.4kV transformers, where tap selection is controlled as in Fig.4. As with the PES, LDC is used for generation of the reference voltage. Taps are considered to have a range of $\pm 5\%$ and a resolution of 2.5%; this corresponds to the ratios of the off-line taps presently [6].

C. Active Power Filters (APF)

Load rebalancing and harmonic / reactive compensation are well know objectives of APFs, but these objectives can also act to improve voltage regulation [26]. A high level control circuit is shown in Fig. 5, where the P_{calc} block can be used to calculate only the active power of the load; ideally eliminating reactive and harmonic currents [27].

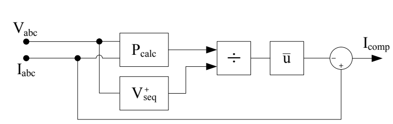

Fig. 5. Configuration of a APF Control Scheme

Work regarding location of APF [28] focuses on voltage quality issues and network losses utilising LV lines with X/R ratios above 1. Considering location of a D-STATCOM for improved voltage profile and network losses [29], [30] will be ineffective in LV networks due to (7) and (8). The rating of the APF will increase as it placed closer to the location of the secondary substation. As only the fundamental component is to be considered and the method used to calculate reference currents is perfect harmonic cancellation (PHC) [27] it can be considered as a phase balancer (PB).

D. Mid Feeder Compensation (MFC)

The MFC in topology is similar to that of a UPFC (or UPQC). It comprises shunt and series inverters tied to a common DC link (Fig. 6). The MFC control scheme used is as described in [31] under the term UPQC-P.

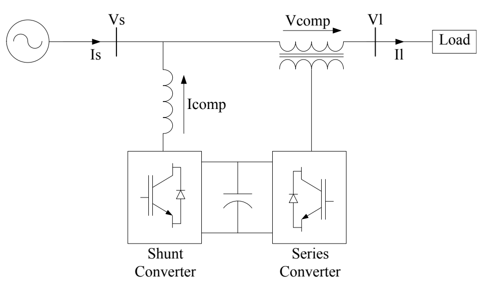

Fig. 6. Configuration of a UPQC

For optimal location of a single MFC it should maintain voltage profile as tightly as possible, this will be achieved if the MFC is located where the voltage drop is half that of total voltage drop on the feeder. Consider a single phase line of length L and impedance Z. If the load is evenly distributed along the whole length of the feeder and the total line current is I_t the line current per unit length is:

$$di = \frac{I_t}{L} \tag{9}$$

The line current (i_x) and gradient of voltage (dV_x) at position x on the feeder are given by:

$$i_x = I_t - x\ di \tag{10}$$

$$dV_x = Z\ i_x\ .dx \tag{11}$$

The total voltage drop (ΔV_x) at x is then:

$$\Delta V_x = \int_0^x dV_x\ .dx \tag{12}$$

Finally, the voltage drop is half that of the remote feeder end voltage drop (ΔV_L) when x is:

$$\Delta V_x = \frac{\Delta V_L}{2} \quad \Rightarrow \quad x = L\ \frac{2-\sqrt{2}}{2} \tag{13}$$

The solution indicates the voltage drop is half the maximum value when x equals $0.29L$, or 29% of the total feeder length.

IV. Test Networks and Load Profiles

The range and diversity of LV networks means each network topology will face different stresses dependant upon its configuration. For this reason a representative test network is employed to act as the base for comparison of the devices shown in section III. The network structure consists of a primary substation with two 15MVA transformers serving seven 11kV feeders each of which has six connected secondary substations across the feeder length of 3km. From each substation 386 customers are severed across four feeders of length 300m. Domestic customers are supplied by 30m of service cable. In total the network serves over 18,000 domestic LV customers. Detailed network information can be found in [32].

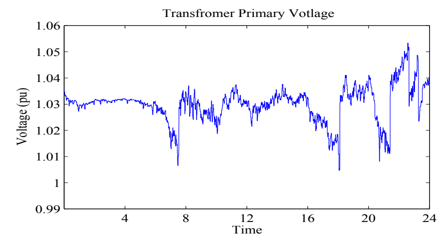

Fig. 7. Voltage Profile at Primary Side of the LV Substation

Some changes are implemented to the detailed part of the UK generic network to make it more suitable for unbalanced 3 phase 4 wire simulation. Firstly, a single feeder is considered in greatest detail with the other feeders being lumped with 2/3 and 1/3 of the load connected at 1/4 and the end of the feeder, respectively. Compared to a perfectly distributed line, this will give the same power losses and voltage drop [33]. Secondly, the cable model (originally phase and neutral RL parameters) is changed to the mutual impedance matrix of “wavecon” cable [34] as is the preferred cable of UK DNOs. Secondly grounding is specified as TN-S or TN-CS (PME) where the grounding impedances are also defined in the latter. Herein results presented are for a TN-S network with 150m of 185mm^2 tapering to 150m of 95mm^2 wavecon cable.

The load and PV profiles build upon the work in [35] where high resolution data sets are generated. The load profiles are adjusted to include static voltage sensitivity and the portion of the load that will be modelled as a constant energy device, i.e. the devices which are thermally controlled. Finally for considering the use of LCT, the heat pump profiles are synthesised with a 1 minute resolution as are EV charging profiles.

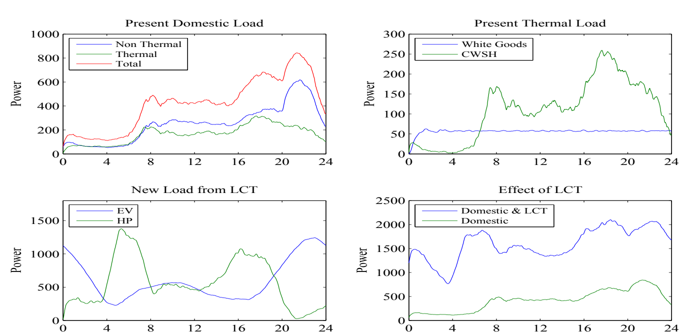

Fig. 8. Top Left: Average Domestic 24 hour demand profiles. Top Right: Thermally Controlled Loads (White Goods & Restive Based). Bottom Left: EV and HP loads. Bottom Right: Present Load Profile and with 100 % LCT use

V. Simulation Scenarios & Regulator Device Operation

The proliferation scenarios shown in Table I are considered. For each node a load profile is allocated and then (dependant upon the considered scenario) the LCT are allocated across the test network with a defined probability. A 24 hour load flow is performed with a resolution of 1 second. All load profiles have a 1 minute resolution, so grouped load profiles are linearly interpolated between time steps whilst single loads hold their value for 1 minute of the simulation. For the single loads a random circular time shift of up to 1 minute is used to avoid synchronised load changes.

TABLE I. LCT Proliferation Scenarios (%)

	PV	EV	HP
Low	-	13	-
Mid	30	33	30
High	60	71	80

After the load flow completes and the voltage and current (VI) measurements are stored and the process is repeated for a given number of trails (here 10) or for the next scenario. All of the VI measurements are then compared with relevant standards or equipment limits, with outlying results flagged. Network losses, utilisation factor, efficiency, and voltage quality measures are also found. This process is repeated for each of the technologies mentioned in section III.

• ***Base*** : This represents the case "as is" and indicates the effect of LCT use without any network changes. The transformer ratio is 11/0.415kV.

• ***PES*** : The PES is configured so that each feeder is individually regulated. The voltage at the start of each feeder is continually adjusted between 1.08pu and 0.96pu by the LDC, with the no load reference voltage as 1.02pu.

• ***OLTC*** : The OLTC uses LDC using the same settings as the PES. As the tap changers have a bandwidth of 2.5% with a maximum tap range of $\pm$5%, the limits of the supply voltage (assuming nominal HV) are 0.96pu to 1.06pu. Unlike with the PES, the secondary winding is connected to a substation busbar so all 4 feeders are regulated simultaneously.

• ***APF*** : The APF is considered to provide reactive compensation and load rebalancing. As the system is 4 wire, the neutral current is also eliminated, this in practice requires use of a 4-leg (or spilt capacitor) inverter. No limit is put on the APF rating. The control strategy used it that termed perfect harmonic cancellation (PHC), which was also shown to be the most suitable APF control strategy when non ideal supply voltages were considered [27].

• ***MFC*** : The MFC here is operated as a power balancing device. The role of the shunt converter is solely to supply the active power demand by the series converter. In practice the shunt converter can also be utilised in the same manner as the APF. The MFC is rated to inject up to 10% of the nominal voltage; which when considered with the MFC location and the cables thermal limit give a required rating for each converter of 12kVA.

VI. Results

Without consideration of any LCTs the network operates without limits being breached, as shown in Fig. 9. Some voltages outside tolerance are seen but the frequency of these are very low.

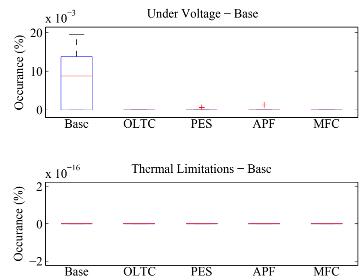

Fig. 9. Outlying Results with no LCTs Considered

A. Heat Pump Scenarios

As the amount of HP used increases under voltage measurements are seen at remote feeder ends. From Fig. 10, which show 30% installation of HP, the voltage limits are breached whist the networks thermal limits are not exceeded. This clearly indicates the network is voltage constrained in this scenario.

The PES and MFC are both able to improve voltage regulation such that the network is not voltage constrained. In the case of the OLTC the cause of the voltages which lie outside regulator limits is due to the delay time used in the tap selection process. The APF is also seen to reduce the frequency of under voltage measurements.

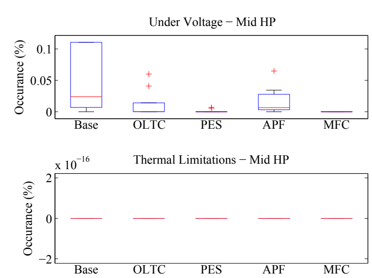

Fig. 10. Network with 30% proliferation of Heat Pumps

As the amount of HP installed is increased to 60%, network thermal limits are encountered. Here the PES and MFC compensate for under voltages such that the network can be said to be thermally limited. This scenario would therefore need further network reinforcement to be a viable operating scenario for a UK based DNO. The under voltages with the OLTC are again due to the tap selection time but also the limit placed on the maximum tap variation ($\pm 5\%$) which is not the case for the PES.

Thermal limits are reached more frequently with the MFC as compared to the OLTC and PES as the power required by the voltage injected by the series converter must be balanced by that of the shunt converter. As the HP is treated as a constant power load operation with high supply voltages acts to reduce the current drawn.

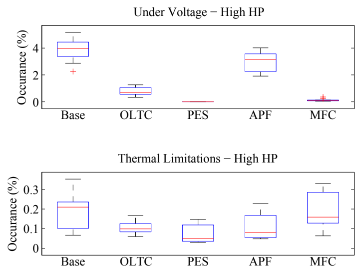

Fig. 11. Network with 80% proliferation of Heat Pumps

B. Electric Vehicle Scenarios

For brevity results for low penetration of EV are not shown, however in this scenario all regulation devices except the APF reduce under voltage occurrences to almost zero and in non of the cases is the network seen to be thermally limited.

Medium penetration of EVs are given in Fig. 12, again the PES and MFC are able to remove any voltage limitations. Thermal limitations in this scenario occur all with very limited frequency (similar to the frequency of recorded under voltage in the base scenario without voltage regulation).

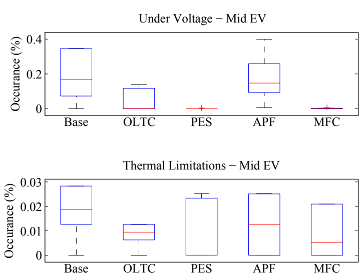

Fig. 12. Network with 33% proliferation of Electric Vehicles

When the amount of EVs increases to 71% results (Fig. 13) again show only the PES and MFC reduce voltage constraints to a level below their thermal limits. The OLTC is effective in reducing the occurrence of under voltage, where in this case the remaining under voltages are due to the limit of $\pm 5\%$ on the tap ratios.

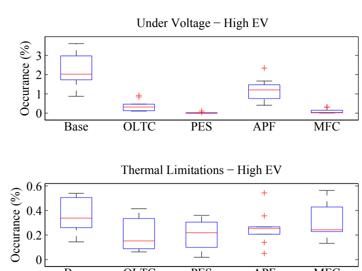

Fig. 13. Network with 71% proliferation of Electric Vehicles

In all cases thermal limits are breached and like in the case of a high penetration of heat pumps network upgrades would need to be made for this scenario to be a viable in the UK. All regulation devices are shown to reduce the occurrence of thermal limits

C. PV Generation Scenarios

In the case of PV installations it is overvotlage and/or thermal problems we would expect to encounter [2]. Hence, results presented in this section are for occurrences of over voltages and breached thermal limits.

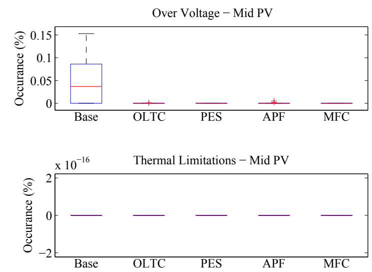

Fig. 14. Network with 30% proliferation of PV Installations

With a medium penetration of PV (Fig. 14) over voltages are seen in when no voltage regulation is present. Compared to with a medium penetration of HP (see Fig. 10) voltage problems are encountered more frequently. This in-spite of the fact that no base generation is present, unlike with the HP & EV scenarios where baseline domestic loading is present. This is a result of the high substation voltages chosen by the DNOs [10]. In this scenario all regulation devices are able to reduce network over voltages, and again like the mid HP scenario the APF is effective in reducing the occurrence of voltage outside regulator limits. In all cases the thermal limits of the network equipment are not exceeded.

As the PV penetration level reaches 60%, over voltages become more common as expected (Fig. 15). In this case the APF is not able to effectively reduce the occurrence of over voltages to an effective level. The other regulation devices are however all effective in reducing the occurrence of over voltages.

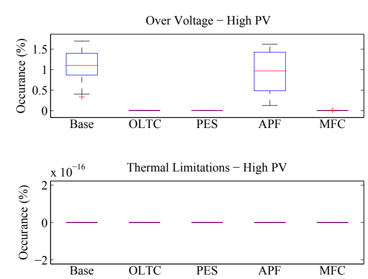

Fig. 15. Network with 60% proliferation of PV Installations

As with the medium penetration of PV scenario in no cases are the thermal limits of the network equipment breached. The difference in occurrence of thermal limits and voltage limits (over or under) being breached between the PV scenarios and the HP/EV scenarios is a direct result of network operation voltages. It is clear as the amount of LV connected PV grows and over voltage becomes more frequent (already encountered more than under voltages [10]) the DNOs must act to either upgrade network infrastructure, reduce substation voltages, or improve voltage regulation.

D. Analysis of Active Power Filters

The active power filter is used for phase rebalancing which in turn reduces network losses; its ability to regulate voltage coming as a result of this. Namely, the APF is not first and foremost a voltage regulator, this should be clear from Figs. 10-15 where it reduced voltage issues only when the occurrence (hence magnitude) was low.

TABLE II. CABLE LOSSES

Scenario	Loss (kWh/day)		Reduction
	No APF	APF	
No LCTs	16.10	14.47	1.63 (10%)
High DG	13.21	12.19	1.02 (7.7%)
High EV	60.93	55.14	5.79 (9.5%)
High HP	60.82	56.45	4.37 (7.1%)

For the three high penetration scenario and the base case the average cable losses (phase and neutral) are given in table II. It can be seen the reduction in losses is between 7-10%. In absolute terms if we take the the cost of network losses as £0.06 /kWh [16] the annual cost in reduction of losses will be £127 at most. The greater benefit of the APF lies in is ability to reduce the occurrence of thermal limitations in network equipment as is evident in Figs. 9-15, and mitigation of harmonic power quality issues should they become limiting factors.

E. Comparison of Regulation Approaches

For the 8 LCT scenarios and all voltage regulation scenarios the 75th percentile is checked for values above 0.07%, this corresponds to 1 minute of outlying results a day. Table III lists the number of LCT scenarios without these outliers, with the final column listing the percentage of the 16 considered voltage or thermal limits which were not exceeded. This metric was used instead of individual application of EN50160 to the consumers so to give an indication of the wider network performance, but results are very similar with either method.

TABLE III. SCENARIOS CONSIDERED WITHOUT LIMITS EXCEEDED

Regulation	Voltage		Thermal	Overall %
	Under	Over		
Base	1	0	6	44
OLTC	3	2	6	69
PES	6	2	6	88
APF	2	1	6	56
MFC	5	2	6	81

The reason for the OLTCs poorer voltage regulation compared to PES are: the delay time implemented in the tap selection process, the smaller range of voltages the OLTC can achieve, and the simultaneous regulation of all 4 feeders. Non of the regulation scenarios were able to remove thermal limits, but from Figs. 9-15 it is noted the frequency of them are reduced. As the LCTs considered are all assumed to be inverter connected (constant power loads) operation at higher voltage levels can reduce current drawn by these loads and in the case of the APF current limits are reduced via load balancing.

VII. CONCLUSION

Here we have presented 4 forms of power electronic based voltage regulation to improve voltage regulation in LV networks and have compared them to the case at present (base) scenario. De-regulation when utilised in conjunction with changes to the HV operating voltage or the LV substation taps could allow greater hosting capacity for all the considered LCT. The potential benefits of voltage de-regulation alongside these regulation schemes will be explored in future work.

In the cases of the MFC, OLTC, and PES, all were able to improve voltage regulation to accommodate medium penetrations of LCT. For DG (PV) hosting capacity was increased by all of the regulation devices. It is clear voltage control at either the substation or mid-feeder allows for greatly improved LCT hosting capacity when the network isn't thermally limited. In practice the advantages of the PES over the OLTC could be reduced by increasing the range (or number) of the taps, along with faster tap selection. Concern about OLTC maintenance and reliability can be addressed by use of new tap changer technologies [36]. The prominent advantages of he MFC compared to the PES and APF, is the reduced rating needed along with voltage regulation comparable to the PES.

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