Non-stationary Demand Side Management Method for Smart Grids

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ABSTRACT

Demand side management (DSM) is a key solution for reducing the peak-time power consumption in smart grids. The consumers choose their power consumption patterns according to different prices charged at different times of the day. Importantly, consumers incur discomfort costs from altering their power consumption patterns. Existing works propose *stationary* strategies for consumers that myopically minimize their short-term billing and discomfort costs. In contrast, we model the interaction emerging among self-interested consumers as a repeated energy scheduling game which foresightedly minimizes their long-term total costs. We then propose a novel methodology for determining optimal *nonstationary* DSM strategies in which consumers can choose different daily power consumption patterns depending on their preferences and routines, as well as on their past history of actions. We prove that the existing stationary strategies are suboptimal in terms of long-term total billing and discomfort costs and that the proposed strategies are optimal and incentive-compatible (strategy-proof). Simulations confirm that, given the same peak-to-average ratio, the proposed strategy can reduce the total cost (billing and discomfort costs) by up to 50% compared to existing DSM strategies.

Index terms—Smart Grids; Demand Side Management; Critical Peak Pricing; Consumer Discomfort; Non-stationary Policies; Repeated Games; Incentive Design.

1. INTRODUCTION

Smart grids aim to provide a more reliable, eco-friendly, and efficient power system. *Demand Side Management (DSM)*, a key mechanism in smart grids [1], refers to the programs adopted by utility companies to directly or indirectly influence the consumers' power consumption behavior in order to reduce the *Peak-to-Average Ratio (PAR)* of the total load in the smart grid system.

Direct Load Control (DLC) and *Smart Pricing (SP)* are two popular approaches for implementing DSM. DLC refers to the program in which the utility company can remotely manage a fraction of consumers' appliances to shift their peak-time power usage to off-peak times [2]. Alternatively, SP [5]-[16] provides an economic incentive for consumers to manage their power usage. Examples are *Real-Time Pricing (RTP)* [5], *Time-Of-Use Pricing (TOU)* [12], *Critical Peak Pricing (CPP)* [14]-[16], etc. The above works [2][5]-[7][14]-[16], however, do not consider the consumers' discomfort costs induced by altering their power consumption patterns.

Some recent works aim to jointly minimize the consumers' billing and discomfort costs (referred to subsequently as the total cost) [3][4][8]-[13][18][21], and can be divided into two categories. One category assumes that the consumers are price-taking (i.e., they do not consider how their consumption will affect the prices). Based on the price-taking assumption, a *single* consumer is *foresightedly* minimizing the long-term total cost by solving a stochastic control problem [8]-[9]. In [10][11], *multiple cooperative* consumers are *myopically* minimizing their current total costs by solving static optimization problems. Conventional distributed algorithms are proposed to find the optimal prices. The second category assumes consumers being *price-anticipating* and *myopically* minimizing their costs(i.e., they consider how their consumption will affect the prices). These works [5]-[7][13] model the interactions among myopic consumers as one-shot games and studied the *Nash equilibrium (NE)* of the game. Existing works with multiple consumers [2]-[7][10]-[14] assume that the myopic consumers aim to minimize their current costs. The optimal DSM strategies in these works are *stationary*, i.e., all consumers adopt fixed power consumption patterns as long as the system parameters (e.g., the consumers' desired power consumption patterns) do not change. However, as we will show later in the paper, the stationary DSM strategies are suboptimal in terms of the long-term total cost.

In this paper, we also model the consumers as price-anticipating. However, in our model, since the foresighted consumers stay in the system for a long time and interact with each other repeatedly, we formulate the consumers' interactions as a repeated game. Although the proposed methodology can be applied to improve the performance of stationary DSM strategies for any SP scheme, we illustrate our approach using the CPP scheme, which has been widely used for residential consumers and is shown to work well in practical scenarios [15]-[17]. CPP defines peak days in a year or peak times in a day, and charges higher prices during these peak hours if CPP events, such as system load warning, extreme weather conditions, and system emergencies, occur [16].

We propose the nonstationary DSM strategy in the repeated energy scheduling game framework, where the consumers may adopt different power consumption patterns (e.g., a consumer may shift its peak-time consumption today but not tomorrow) even if the system parameters remain the same. The strategy recommends different subsets of consumers (referred to as the *active set*) to shift their peak-time consumption each day, based on their preferences and the past history of consumption pattern shifts. These consumers purposely incur their current discomfort costs to minimize the price. In return, they will enjoy in the future lower billing costs without incurring discomfort costs when other consumers are chosen in the active set. In this way, the proposed strategy minimizes the long-term total cost while ensuring fairness among the consumers. In addition, the proposed strategy is *Incentive-Compatible (IC)*, namely the self-interested consumers will find it in their self-interest to follow the recommended strategy.

2. SYSTEM MODEL

2.1. Energy Scheduling Game

A smart grid system consists of a utility company and multiple consumers, as shown in Fig.1. Time is divided into periods $t = 0,1,2,\cdots$, and each period is divided into $H \in \mathbb{N}_+$ time slots

The material is based upon work funded by the US Air Force Research Laboratory (AFRL). Any opinions, findings, and conclusions or recommendations expressed in this article are those of the authors and do not reflect the views of AFRL.

with equal length. We denote the set of time slots by $\mathcal{H}=\{1,2,...,H\}$. Note that we use "period" to denote each stage of the interaction among consumers and use "time slot" to denote the discrete time to schedule power usage within a period. In this paper, we consider a period to be one day as in [5]-[8][10]-[14], and each slot can be one or multiple hours.

We denote the set of consumers by $\mathcal{N}=\{1,2,...,N\}$. The action of consumer $i\in\mathcal{N}$ in period t is its power consumption pattern, denoted by $a_i^t=(a_{i,1}^t,\dots,a_{i,H}^t)$, where $a_{i,h}^t\in\mathcal{A}_i$ is the power consumption at each time slot and $\mathcal{A}_i$ is the possible power consumption set.

The power consumption $a_{i,h}^t$ at time slot h consists of non-shiftable load and shiftable load, denoted by $b_{i,h}^t\geq 0$ and $s_{i,h}^t\geq 0$, respectively. The non-shiftable load, such as lighting, cooking, watching TV, is not controllable by smart meters, while the shiftable load, such as dish and clothes washing, heating and cooling systems, can be controlled by the smart meters [5]-[7].

We denote by $\sum_{h=1}^{H}a_{i,h}^t=A_i$ the daily total power consumption for residential consumer i, where A_i is either a constant or slowly varying as in [5]-[7][11]-[14]. We denote by $\boldsymbol{a}^t=(a_1^t,a_2^t,\cdots a_N^t)\in\mathcal{A}$ the power consumption profile of all consumers, where $\mathcal{A}=\times_{i=1}^{N}\mathcal{A}_i^H$. The *total load* at time slot h, denoted by $l_h^t=\sum_{i=1}^{N}a_{i,h}^t$, is the sum of all consumers' power consumption.

The desired power consumption pattern in each period for consumer i is denoted by $\overline{a}_i=[\overline{a}_{i,1},\overline{a}_{i,2},\cdots\overline{a}_{i,H}]\in\mathcal{A}_i^H$, which refers to its preferred daily power consumption pattern [20]. The corresponding total load is denoted by $\overline{l}_h=\sum_{i=1}^{N}\overline{a}_{i,h}$. Define $\overline{h}=\arg\max_{h\in\mathcal{H}}\overline{l}_h$ as the peak time of the day, the length of which changes according to H. Empirical studies show that, compared with industrial and commercial consumers, residential consumers have very similar peak-time shiftable loads [17], implying that $\overline{a}_{i,\overline{h}}-b_{i,\overline{h}}^t=\overline{s}_{\overline{h}}$, for each consumer i.

The cost $c_i:\mathcal{A}\mapsto\mathbb{R}$ of consumer i consists of its billing and discomfort costs:

$$c_i(\boldsymbol{a}^t)=\sum_{h=1}^{H}p_h(\boldsymbol{a}^t)a_{i,h}^t+d_i(a_i^t), \tag{1}$$

where $p_h:\mathcal{A}\mapsto\mathbb{R}_+$ denotes the price at time slot h, $d_i:\mathcal{A}_i^H\mapsto\mathbb{R}$ denotes the discomfort cost of consumer i.

In CPP scheme, the utility company charges a higher price in the critical peak time when CPP events occur [14]-[16]. We model the CPP pricing scheme with a single critical peak time and consider the CPP events triggered by the total load in the system. The time-varying price function $p_h(\boldsymbol{a}^t)$ is defined as:

$$p_h(\boldsymbol{a}^t)=p_h(l_h^t)=\begin{cases}p_{Lo}, & 0\leq l_h^t\leq l_{th}\\ p_{Hi}, & l_h^t>l_{th}\end{cases}, \tag{2}$$

where $p_{Hi}>p_{Lo}$ are the peak price and off-peak price of the pricing model and l_{th} is the threshold of the total load. When $l_h^t\leq l_{th}$, the higher price will not be triggered and p_{Lo} will be adopted. When $l_h^t>l_{th}$, the CPP event occurs and the higher price p_{Hi} will be adopted. The threshold l_{th} is set to be below the peak load and above the off-peak load, namely,

$$\overline{l}_{\overline{h}}>l_{th},\quad \overline{l}_{h'}\leq l_{th},\quad \forall h'\in\mathcal{H}\backslash\{\overline{h}\}. \tag{3}$$

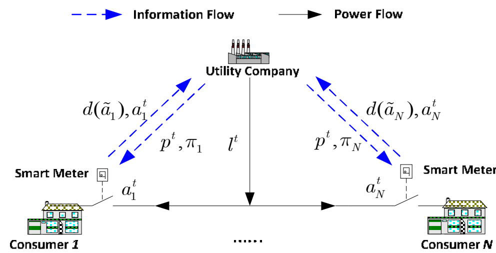

Fig. 1. Smart Grid System Model.

Given peak load reduction goal l_{th}, we further set $m=\left(\overline{l}_{\overline{h}}-l_{th}\right)/\overline{s}_{\overline{h}}$, where $m\in\mathbb{N}$ is the smallest number of consumers needed to shift their peak-time consumption such that the peak-time price is low. We denote the prices within a day by $\boldsymbol{p}^t=(p_1^t,p_2^t,\cdots p_H^t)$.

We use a discomfort cost function to model the consumers' discomfort from rescheduling their power consumption patterns, i.e., the "distance" between consumer's desired demand and actual consumption [12][18]-[21]. As in [12][18], we use a linear weighted function to model the discomfort cost:

$$d_i(a_i^t)=\begin{cases}\sum_{h=1}^{H}k_{i,h}(|a_{i,h}^t-\overline{a}_{i,h}|)+\omega_i, & a_i^t\neq\overline{a}_i\\ 0, & a_i^t=\overline{a}_i\end{cases}, \tag{4}$$

where $k_{i,h},\omega_i\in\mathbb{R}_+$ are parameters of the discomfort cost function.

Consumer i's minimum cost achievable is denoted by $\overline{c}_i=\min_{\boldsymbol{a}\in\mathcal{A}}c_i(\boldsymbol{a})=p_{Lo}A_i$, and consumer i's minimum cost achievable, when consumer i shifts all its peak-time shiftable load, is denoted by $\tilde{c}_i=\min_{\boldsymbol{a}\in\mathcal{A},\,a_{i,\overline{h}}=b_{i,\overline{h}}}c_i(\boldsymbol{a})=p_{Lo}A_i+d_i(\tilde{a}_i)$, where the last term satisfies $\tilde{a}_i=\text{argmin}_{a_i\in\mathcal{A}_i^H,\,a_{i,\overline{h}}=b_{i,\overline{h}}}d_i(a_i)$.

Based on the relationship between billing and discomfort costs, consumers with low discomfort costs only care about billing costs and always shift and choose $\tilde{a}_i$, while consumers with high discomfort costs will incur high discomfort costs when altering power consumption patterns and always choose $\overline{a}_i$. Hence, we only consider the DSM strategy for consumers with medium discomfort costs, namely,

$$\frac{(p_{Hi}-p_{Lo})\overline{l}_{\overline{h}}}{m}>d_i(\tilde{a}_i)\ \ and\ \ \omega_i>[(p_{Hi}-p_{Lo})]\overline{a}_{i,\overline{h}}. \tag{5}$$

The first inequality implies that the discomfort cost is not too large, such that the consumers are willing to shift their peak-time consumption as long as their billing costs can be greatly reduced. The second inequality implies that the discomfort cost is not too small, such that each consumer does care about its own discomfort cost and is not willing to shift its peak-time consumption every day.

The one-shot energy scheduling game can be written as $\{\mathcal{N},\{\mathcal{A}_i\}_{i=1}^{N},\{c_i\}_{i=1}^{N}\}$, where $\mathcal{N}$, $\{\mathcal{A}_i\}_{i=1}^{N}$ and $\{c_i\}_{i=1}^{N}$ denote the sets of consumers, of actions, and of cost functions, respectively.

Next, we formalize the consumers' interaction as a repeated game. In each period t, consumer i determines a_i^t based on its history, a collection of all its past power consumption patterns and the past prices made public to the consumers. The *public history* is defined as $\eta^t=\{p^0,p^1,...,p^{t-1}\}\in(\mathcal{P}^H)^t$ for $t>0$ and the initial history is defined as $\eta^0 = \varnothing$. The *public strategy* of consumer i is defined as a mapping from public history to current actions, denoted by $\pi_i : \bigcup_{t=0}^{\infty} (\mathcal{P}^H)^t \mapsto \mathcal{A}_i$, where $(\mathcal{P}^H)^0 = \varnothing$ [22]. Due to realization equivalence principle [22, Lemma 7.1.2], the operating points achieved by public strategies are equivalent to those achieved by strategies using the entire history.

Given the strategy profile of all consumers, denoted by $\boldsymbol{\pi} = (\pi_1, \pi_2, \cdots \pi_N)$, consumer i's average long-term cost is discounted by a factor $\delta \in [0,1)$:

$$C_i(\boldsymbol{\pi}) = (1-\delta)\sum\nolimits_{t=0}^{\infty} \delta^t c_i(\boldsymbol{\pi}(\eta^t)), \tag{6}$$

where $c_i(\boldsymbol{\pi}(\eta^t))$ is the cost of consumer i in period t. The discount factor represents how much the consumers care about tomorrow's costs relatively to today. A larger discount factor indicates that consumers care more about future costs. The corresponding long-term discomfort cost is defined as $D_i(\pi_i) = (1-\delta)\sum\nolimits_{t=0}^{\infty} \delta^t d_i(\pi_i(\eta^t))$.

Hence, the repeated energy scheduling game can be written as $\{\mathcal{N}, \bigcup_{t=0}^{\infty}(\mathcal{P}^H)^t, \{\pi_i\}_{i=1}^N, \{C_i(\boldsymbol{\pi})\}_{i=1}^N\}$, where $\mathcal{N}$, $\bigcup_{t=0}^{\infty}(\mathcal{P}^H)^t$, $\{\pi_i\}_{i=1}^N$ and $\{C_i(\boldsymbol{\pi})\}_{i=1}^N$ are the sets of consumers, of public histories, of strategies, and of cost functions, respectively.

2.2. Problem Formulation

The designer is the benevolent utility company that aims to minimize the total cost in the smart grid system with self-interested consumers. However, maintaining fairness among all the consumers is also essential [17]. Hence, the mechanism will ensure that the average discomfort cost of consumer i is no greater than a maximal value $D_{i,\max}$. Therefore, the optimal IC *DSM mechanism Design Problem (DDP)* can be formulated as

$$\begin{aligned}
\text{(DDP): } \underset{\boldsymbol{\pi}}{\text{minimize}} \quad & \sum\nolimits_{i\in\mathcal{N}} C_i(\boldsymbol{\pi}) \\
\text{subject to} \quad & a_{i,h}^t \ge b_{i,h}^t, \forall t \in \mathcal{T} \\
& A_i = \sum\nolimits_{h=1}^{H} a_{i,h}^t, \forall t \in \mathcal{T} \\
& D_i(\boldsymbol{\pi}) \le D_{i,\max}, \forall i \in \mathcal{N} \\
& \boldsymbol{\pi} \text{ is IC}
\end{aligned}$$

The utility company will solve this problem, then recommend the consumers with the optimal solution $\boldsymbol{\pi}^*$.

3. OPTIMAL STRATEGIES

In this section, we first discuss the performance of the one-shot and repeated energy scheduling games, and then propose the nonstationary algorithm that solves the DDP problem.

3.1. One-shot vs. Repeated Energy Scheduling Game

We formally characterize the NE of the one-shot energy scheduling game and the Pareto-optimal region (achievable cost profiles) of the repeated energy scheduling game. We state these in the two following theorems.

Theorem 1 (Nash Equilibrium of the One-shot Game): The one-shot energy scheduling game has a unique *NE*, in which each consumer chooses its desired power usage as

$$a_i^* = \bar{a}_i, \forall i \in \mathcal{N}. \tag{7}$$

Proof: See on-line proof [23].□

Theorem 2: The Pareto-optimal region of the repeated energy scheduling game is

$$\mathcal{B} = \{\boldsymbol{C} = (C_1, C_2, \cdots, C_N) \mid \sum\nolimits_{i=1}^{N} (C_i - \tilde{c}_i)/(\bar{c}_i - \tilde{c}_i) = m, C_i \ge \tilde{c}_i\}. \tag{8}$$

In addition, the stationary DSM strategies can only achieve the extreme points[1] of $\mathcal{B}$.

Proof: See on-line proof [23]. □

By adding IC and the maximum discomfort costs constraints, the *feasible* Pareto-optimal region can be written as

$$\mathcal{B}_{\bar{C}} = \{\boldsymbol{C} = (C_1, C_2, \cdots, C_N) \mid \sum\nolimits_{i=1}^{N} \frac{(C_i - \tilde{c}_i)}{(\bar{c}_i - \tilde{c}_i)} = m, C_i \ge \tilde{c}_i, C_i \le \bar{C}_i\}, \tag{9}$$

where $\bar{C}_i = \min\{C_{i,\max}, C_{i,NE}\}$, and $C_{i,\max} = \tilde{c}_i + D_{i,\max}$.

The results of Theorem 1 and 2 state that the Pareto-optimal region of the repeated energy scheduling game, except for the undesired extreme points, cannot be achieved by stationary DSM strategies for either self-interested or obedient consumers. Next, we will propose the nonstationary strategy to achieve the operating points.

3.2. Nonstationary DSM Mechanism

Given the Pareto-optimal region, we can then reformulate the *DDP* problem as a linear programming problem:

$$\underset{\boldsymbol{C}\in\mathcal{B}_{\bar{C}}}{\text{minimize}} \quad \sum\nolimits_{i\in\mathcal{N}} C_i. \tag{10}$$

We can solve this linear programming problem (10), and denote the solution by $\boldsymbol{C}^* = (C_1^*, C_2^*, \cdots C_N^*)$. Given the operating point $\boldsymbol{C}^*$, we can use the *Nonstationary DSM (N-DSM)* algorithm, described in Table I, to construct the DSM strategy. In period t, the N-DSM algorithm chooses the active set $I(t) \in \mathcal{I}$ consisting of m out of N consumers to reschedule their power consumption patterns, where $\mathcal{I}$ is the set of all possible index combination that containing m consumers out of N. The choice of active set

TABLE I. NONSTATIONARY DSM (N-DSM) ALGORITHM

Input: Target average cost vector $\boldsymbol{C}^* = (C_1^*, C_2^*, \cdots C_N^*)$, $t = 0$.
Output: Optimal strategy

1: Set $g_j(t) = (C_j^* - \tilde{c}_j)/(\bar{c}_j - \tilde{c}_j)$.
2: **Repeat**
3: **If** $p_{\bar{h}}(\tau) = p_{Hi}, \exists \tau < t$, then
4: Recommend action $\bar{a}_i$ to all consumer i.
5: **Else**
6: Find the active set $I(t) = \{i_1, i_2, \cdots i_m\}$ of m consumers, who have the m largest indices $g_j(t)$.
7: Recommend action $\tilde{a}_i$ to $i \in I(t)$, and $\bar{a}_i$ to $i \notin I(t)$.
8: Observe consumers' action a_i.
9: **If** all consumers follow the recommendation, then
10: Update $g_i(t+1) = [g_i(t) - (1-\delta)\mathbf{1}_{\{i\in I(t)\}}]/\delta$ for all i.
11: Broadcast $p_h(t) = p_{Lo}$ for all h.
12: **Else**
13: Broadcast $p_{\bar{h}}(t) = p_{Hi}$ and $p_h(t) = p_{Lo}, h \ne \bar{h}$.
14: **End if**
15: **End if**
16: $t \leftarrow t+1$
17: **End Repeat**

[1] An extreme point of a convex set is the point that is not the convex combination of any other points in this set. In our case, since $\mathcal{B}$ is part of a hyperplane, the extreme points will be the vertices of $\mathcal{B}$.

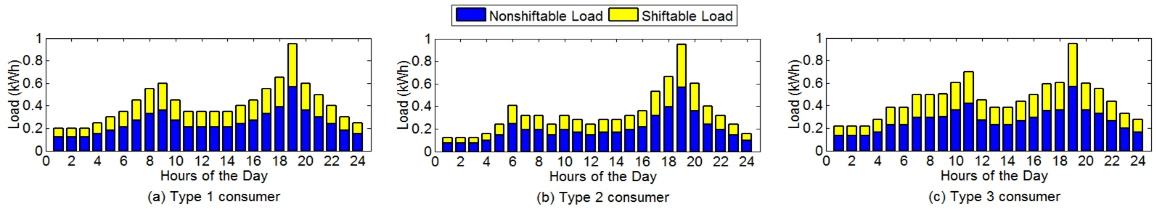

Fig.2. The Desired Power Consumption Patterns of Type 1, 2, 3 Consumers.

TABLE III. COMPARISON OF TOTAL COSTS ACHIEVED BY DIFFERENT ALGORITHMS

		Number of consumers (Homogeneous, PAR<2.280)					Number of consumers (Heterogeneous, PAR<2.359)				
		30	50	80	100	200	30	50	80	100	200
Total cost	OG-DSM	49.95	83.25	133.20	166.50	333.00	50.95	84.95	135.90	169.80	339.70
	JO-DSM	42.18	70.08	111.95	139.86	279.40	42.37	70.41	112.46	140.48	280.66
	SC-DSM	46.63	77.71	124.34	155.42	310.84	47.73	79.59	127.32	159.07	318.26
	N-DSM	30.78	50.78	80.78	100.78	200.78	26.23	43.25	68.35	84.50	168.73
Performance gain	Over OG-DSM	38%	39%	39%	39%	40%	49%	49%	50%	50%	50%
	Over JO-DSM	27%	28%	28%	28%	28%	38%	39%	39%	40%	40%
	Over SC-DSM	34%	35%	35%	35%	35%	45%	46%	46%	47%	47%

depends on "how far" they are from their target cost, which is measured by index $g_i(t)$. The consumers with m largest $g_i(t)$ will be chosen in the active set.

Theorem 3: If the discount factor δ satisfies

$$\delta \geq 1 - 1/(N - m + 1)\,. \tag{11}$$

then the three following statements hold: (1) The feasible Pareto-optimal region $\mathcal{B}_{\bar{C}}$ is achievable; (2) the optimal operating point $\boldsymbol{C}^*$ can be achieved by the N-DSM algorithm; (3) the N-DSM algorithm is IC.

Proof: See on-line proof [23]. □

Theorem 3 states that when the discount factor satisfies (11), the optimal nonstationary DSM mechanism can be constructed by the N-DSM algorithm.

4. NUMERICAL RESULTS

In this section, we compare the performance of our proposed DSM mechanism with those obtained using existing methods. We compare with the One-shot Game based stationary DSM (OG-DSM) algorithms with myopic price-anticipating consumers [5]-[7][13], the Joint Optimization (JO-DSM) algorithms with myopic price-taking consumers [10][11], as well as the Single-consumer Stochastic Control (SC-DSM) methods [8]-[9]. The OG-DSM operates at NE of the one-shot energy scheduling game, which is characterized in Theorem 1. The JO-DSM assumes that the obedient consumers jointly minimize the total cost of the system and the optimal performance of stationary DSM mechanism can be achieved by appropriate pricing schemes. The SC-DSM responds to the utility company's price $p_{\bar{h}} = p_{Hi}$ and $p_h = p_{Lo}, h \neq \bar{h}$. In this case, the consumer buys energy in advance according to its scheduled power consumption pattern a_i . We assume that renewable energy is available with probability[2] $\varepsilon = 0.8$, in which case the consumer can reschedule its power consumption pattern to the desired pattern without suffering the discomfort cost since the energy supply is abundant. The renewable energy is not available with probability $1 - \varepsilon = 0.2$, in which case the consumer must

TABLE II. PARAMETERS OF THREE TYPES OF CONSUMERS

	A_i (kWh)	$k_{i,h}(h=1\ to\ 14)$ / $k_{i,h}(h=15\ to\ 24)$ ($/kWh)	ω_i ($)	$D_{i,\max}$ ($)
Type 1	10	0.2/0.1	0.7	0.71
Type 2	8	0.1/0.05	1.5	0.91
Type 3	11	0.15/0.1	1.2	0.95

comply with its scheduled power consumption pattern and will incur discomfort cost $d_i(a_i)$.

In simulation, we set $H = 24$, $\delta = 0.995$, $p_{Hi} = 0.8$ $/kWh and $p_{Lo} = 0.1$ $/kWh[3]. We set the PAR goals (corresponding to threshold l_{th}) for homogeneous and heterogeneous scenarios, and keep them invariant over time. We simulate both the scenario with heterogeneous consumers with parameters shown in Table II and Fig. 2 and the scenario with homogeneous consumers with the same parameters as Type 1 consumers described in Table II and Fig. 2. In this experiment, the shiftable load of each consumer is set to be 40% of the consumer's total load.

Given the same PAR goal, the comparison of total costs using these algorithms is shown in Table III. We can see that when the number of consumers increases, the N-DSM algorithm significantly outperforms other three algorithms. The cost reductions compared to OG-DSM, JO-DSM and SC-DSM are 40%, 28% and 35% in homogeneous case and 50%, 40% and 47% in heterogeneous case, respectively. Note that our algorithm, which is IC, can significantly outperform the JO-DSM algorithm, even though it is not IC.

5. CONCLUSIONS

We proposed a nonstationary DSM mechanism and rigorously proved that the proposed N-DSM algorithm can achieve the social optimum in terms of the long-term total cost, and outperform existing stationary DSM strategies. Moreover, the proposed mechanism is IC, meaning that each self-interested consumer voluntarily follows the power consumption patterns recommended by the optimal DSM mechanism. Simulation results validate our analytical results on the DSM mechanism design and demonstrate up to 50% performance gains compared with existing mechanisms, especially when there are a large number of heterogeneous consumers in the systems.

[2] This comes from the uncertainty of renewable energy generation (whether it is windy in wind energy generation, whether it is shiny in solar energy generation, etc.).

[3] According to [15][17], the peak price is often at least 6 times higher than the off-peak price.

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