

Fig. 1 Area-mean exit temperatures for fluid *a* and fluid *b* (narrow lines)

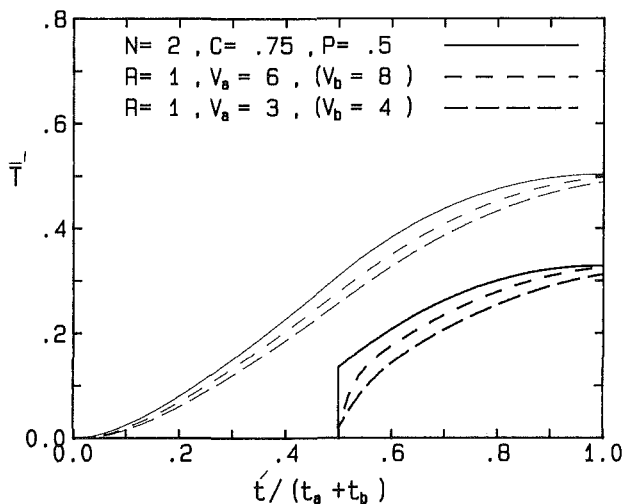


Fig. 2 Area-mean exit temperatures for fluid *a* and fluid *b* (narrow lines). Continuous lines are for zero core capacitance, broken lines for finite core capacitance.

a response exhibits a step when $t = P$ and thus $t' = t_a$. The first cross-sectional fluid *a* lamina that enters at time zero with a temperature of unity sees fluid *b* at zero temperature. (The wall is "transparent" when its thermal capacitance is zero.) This lamina therefore has a temperature of $\exp(-Nx)$. When it reaches its exit plane ($x = 1$) at time t_a its temperature is $\exp(-N)$, which is the magnitude of the jumps seen in Fig. 1.

In order to compare solutions with zero core capacitance with solutions with finite core capacitance, two additional parameters, R and V_a , must be specified. If the values of C and P are specified, then, using parameter definitions, $V_b = V_a(1/P - 1)/C$. Figure 2 gives a comparison of the Spiga and Spiga solution with the solution of this note. The comparison uses $V_a = 6$ and 3 with $N = 2$, $C = 0.75$, $R = 1$, and $P = 0.5$. It can be observed that at $t = 1$ ($t' = t_a + t_b$) the transient is effectively completed with $V_a = 6$ and almost completed with $V_a = 3$. With finite core capacitance, the first fluid *a* cross-sectional lamina sees the wall at zero temperature during its transit of the exchanger and its temperature is therefore $\exp(-N_a x)$ in which $N_a = N(1 + R)/R$. The jump at $t' = t_a$ is $\exp(-N_a)$, which, for Fig. 2, is 0.018.

The solution with zero core capacitance is much easier and faster to compute than the Spiga and Spiga solution. It is therefore of interest to estimate the upper range of V_a or V_b

that is reasonable for an exchanger. Suppose that fluid *a* flows inside tubes. Then for thin wall tubes $V_a \approx Dv/(4\Delta)$ in which D and Δ are the tube diameter and wall thickness and v is the ratio of the volumetric heat capacities of the fluid and tube materials. This ratio has the exceptionally large value of 1.7 for water and aluminum. Using this material combination gives $V_a \approx 0.4D/\Delta$. Thus an aluminum tube with 25 mm diameter and 1 mm wall thickness gives $V_a = 10$. The parameters V_a and V_b can, of course, vary over a wide range, but this illustration indicates that the simplicity of the zero core capacitance solution can sometimes be enjoyed with acceptable accuracy.

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Transport Correlations for Laminar Aiding Mixed Convection Over a Vertical Isothermal Surface

K. S. Manning¹ and Z. H. Qureshi²

Nomenclature

- $f''(0)$ = dimensionless shear stress parameter for natural convection
- $F(\text{Pr})$ = Prandtl-number-dependent function in heat transfer correlations
- g = acceleration due to gravity, m/s^2
- Gr = Grashof number = $g\beta x^3(T_o - T_\infty)/\nu^2$
- h = local heat transfer coefficient, $\text{W/m}^2\text{-K}$
- \bar{h} = surface-averaged heat transfer coefficient, $\text{W/m}^2\text{-K}$
- k = fluid thermal conductivity, W/m-K
- n = exponent in the mixed convection correlations
- Nu = Nusselt number = hx/k
- $\bar{\text{Nu}}$ = Nusselt number based on average heat transfer coefficient = $\bar{h}x/k$
- Pr = Prandtl number = ν/α
- Re = Reynolds number = $u_\infty x/\nu$
- T = temperature, K
- u = streamwise velocity component, m/s
- x = streamwise coordinate from the leading edge, m

¹Department of Mechanical Engineering, Norwich University, Northfield, VT 05663; Mem. ASME.

²Westinghouse Savannah River Company, Aiken, SC 29808; Mem. ASME. Contributed by the Heat Transfer Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received by the Heat Transfer Division June 1993; revision received January 1994. Keywords: Mixed Convection. Associate Technical Editor: Y. Jaluria.

y = coordinate normal to the surface, m
 α = final thermal diffusivity, m^2/s
 β = coefficient of thermal expansion, K^{-1}
 μ = absolute viscosity, $N\cdot s/m^2$
 ν = kinematic viscosity, m^2/s
 ξ = mixed convection parameter = Gr/Re^2
 τ = local shear stress, N/m^2
 $\bar{\tau}$ = surface-averaged shear stress, N/m^2

Subscripts

f = forced convection
 m = mixed convection
 n = natural convection
 o = conditions at the surface
 x = local value
 ∞ = free-stream fluid conditions

Introduction

This note focuses on the transport in aiding laminar mixed convection flow adjacent to a vertical isothermal surface. The direction of forced flow is taken to be upward for the heated surface. This situation causes the flow to be predominantly forced at the leading edge, primarily natural far downstream, and mixed in the middle. In the intermediate region of mixed convection Merkin (1969) reported a finite difference solution for $Pr = 1$. Acrivos (1966) outlined a method to analyze mixed convection based upon the limiting solutions for $Pr \rightarrow 0$ and $Pr \rightarrow \infty$. A local similarity method was employed by Lloyd and Sparrow (1970) to determine the buoyancy effects on a basic forced convection flow. This method facilitated results for relatively large values of Gr/Re^2 in the range $0.003 < Pr < 100$. However, the method was shown to be inapplicable for a free convection dominated mixed flow. An implicit difference scheme was employed by Oosthuizen and Hart (1973) to analyze the entire mixed convection regime over a vertical surface. Based upon these analyses Wilks (1976) proposed a heat transfer correlation. This correlation does not accurately predict the natural and forced convection limits. Raju et al. (1984) produced a numerical analysis of the entire mixed convection spectrum, culminating in a correlation that is very simple, but is, by their own admission, of limited (20 percent) accuracy.

Experimental investigations of mixed convection over a vertical surface in air are reported by Kliegel (1959), Oosthuizen and Bassey (1973), Gryzagoridis (1975), and Ramachandran et al. (1985).

Although an extensive effort is evidenced in the above-mentioned literature, various aspects of mixed convection from a vertical surface are either missing or not reported in an easy-to-use form. This note unifies available results and suggests correlations for local and average heat transfer rates, and for local and average shear stress over the entire spectrum of mixed convection, and for a wide range of Prandtl numbers. Additionally, the limits between natural, mixed, and forced convection are defined and mapped.

Development of the Transport Relations

Any transport correlation describing the entire mixed convection spectrum must accurately predict the forced convection regime as Gr/Re^2 approaches zero, and the natural convection regime as Gr/Re^2 approaches infinity. Furthermore, the asymptotic limits of very small and very large Prandtl number must also be approached. Before presenting the laminar transport equations a few comments on the extent of the mixed regime are in order. It has been shown by Carey and Gebhart (1983) that an aiding free stream has a stabilizing effect on the basic buoyancy-induced flow. Similarly, for a basic forced flow aiding buoyancy has been found to stabilize the flow (Mucoglu and Chen, 1978). Based upon these studies it is proposed here

that, under aiding flow circumstances, the mixed flow remains laminar as long as $Re_x < 5 \times 10^5$ for forced convection, and $Gr_x Pr < 10^9$ for natural convection. These limits are well established as the limits of laminar flow.

Transport phenomena where the transport rates vary from one asymptotic behavior to another are frequently encountered in nature and in practice. Churchill and Usagi (1972) proposed a method for describing such phenomena. This method, which is used here, involves the asymptotic solutions and one additional parameter that describes the intermediate region. In the present study, this parameter is found to be Prandtl number dependent. In the following sections the heat transfer correlations will be established first, and then the shear stress correlations.

Heat Transfer Correlations. Transport correlations for pure forced or natural convection based upon similarity analyses have been substantiated with abundant experimental data.

In the forced convection limit, as $\xi \rightarrow 0$, the local heat transfer rate is given by:

$$Nu_f = \frac{h_f x}{k} = F_f \cdot Re_x^{1/2} \quad (1)$$

The Prandtl number dependent coefficient, F_f , exhibits the following asymptotic behavior (Schlichting, 1979):

$$F_f \rightarrow 0.564 Pr^{1/2} \text{ as } Pr \rightarrow 0 \quad (2)$$

and

$$F_f \rightarrow 0.339 Pr^{1/3} \text{ as } Pr \rightarrow \infty \quad (3)$$

For intermediate values of Prandtl number Schlichting (1979) has tabulated F_f . Following the method suggested by Churchill and Usagi (1972) F_f can be represented over the entire range of Prandtl number by:

$$F_f = 0.339 Pr^{1/3} [0.100 Pr^{-3/4} + 1]^{-2/9} \quad (4)$$

This correlation yields results very close to those tabulated by Schlichting (1979), or those calculated using the correlation of Churchill and Ozoe (1973). Disagreement with the same type of correlation presented by Chen et al. (1986) seems to be due to a misprint of the leading coefficient in that paper. Printed is 0.399 when the number should be 0.339. [Editor's Note: This coefficient has been corrected in an Errata published in the JOURNAL OF HEAT TRANSFER, Vol. 116, 1994, p. 324.]

In the natural convection limit, as $\xi \rightarrow \infty$, the local heat transfer rate is given by:

$$Nu_n = \frac{h_n n}{k} = F_n \cdot Gr_x^{1/4} \quad (5)$$

The coefficient, F_n , has the following asymptotic behavior (Ede, 1967):

$$F_n \rightarrow 0.6 Pr^{1/2} \text{ as } Pr \rightarrow 0 \quad (6)$$

and

$$F_n \rightarrow 0.503 Pr^{1/4} \text{ as } Pr \rightarrow \infty \quad (7)$$

For the entire range of Prandtl number, F_n can be represented by:

$$F_n = 0.503 Pr^{1/4} [0.670 Pr^{-9/16} + 1]^{-4/9} \quad (8)$$

Churchill and Usagi (1972) suggested correlations similar to those given by Eqs. (4) and (8). However, with the Prandtl number dependency built in the equations here are slightly simpler to use, yet as accurate. Values for F_f and F_n generated by Eqs. (4) and (8) are within 1 percent of those values tabulated by Schlichting (1979) and Ede (1967).

After establishing the asymptotic correlations we now present the correlation for mixed convection. Local heat transfer rates for mixed convection may be written as:

$$\frac{\text{Nu}_m}{\text{Re}_x^{1/2}} = [F_f^n + (F_n \xi^{1/4})^n]^{1/n} \quad (9)$$

The average heat transfer rates for mixed convection are given by:

$$\frac{\overline{\text{Nu}}_m}{\text{Re}_x^{1/2}} = \left[(2F_f)^n + \left(\frac{4}{3} F_n \xi^{1/4} \right)^n \right]^{1/n} \quad (10)$$

The only unknown parameter is n . It was selected here in such a way that the heat transfer results predicted by Eq. (9) agree closely with those obtained under mixed convection conditions by Merkin (1969), and Lloyd and Sparrow (1970). We found that no single value of n yielded the best agreement for all values of Prandtl number. Instead, the best value of n increases with Prandtl number. Thus, for each value of Pr the exponent, n , was determined that gave the best agreement between Eq. (9) and the available numerical results. The exponent n was found to be:

$$n = 3.5 \text{Pr}^{0.075} \quad (11)$$

Churchill (1977) proposed, and Ruckenstein (1978) confirmed (for very large Pr), a correlation similar to Eq. (9); however, the exponent n was given as equal to 3 for all Prandtl numbers. Using the exponent as found by Eq. (11) offers greater flexibility and accuracy over a broad range of Prandtl numbers.

The average Nusselt number, in Eq. (10), cannot be found by integrating the local Nusselt number from Eq. (9). Instead, it is constructed using the asymptotic average heat transfer correlations. The parameter n in Eq. (10) was determined by comparing Eq. (10) with a numerical integration of Eq. (9). The same Prandtl number dependence of n shown in Eq. (11) was found to hold for the average heat transfer correlation as well.

Shear Stress Correlations. In a fashion similar to that used to develop the heat transfer correlations, the shear stress correlations for mixed convection conditions were found using the asymptotic behavior under the pure forced and natural convection conditions.

For forced convection flow, as $\xi \rightarrow 0$, the boundary layer analysis yields the following correlation for the local shear stress, τ_f :

$$\tau_f = 0.664 \text{Re}_x^{-1/2} \left(\frac{\rho u_\infty^2}{2} \right) \quad (12)$$

Under natural convection conditions, as $\xi \rightarrow \infty$, the local shear stress can be found from the similarity solution of Gebhart (1971):

$$\tau_n = \frac{4\mu\nu}{x^2} \left(\frac{\text{Gr}_x}{4} \right)^{3/4} \cdot f''(0) \quad (13)$$

where $f''(0)$ is the Prandtl number dependent dimensionless shear stress parameter. For $0.7 \leq \text{Pr} \leq 100$ this parameter is very nicely described by a form of Hoerl's equation (Daniel and Wood, 1971):

$$f''_{(0)} = 0.6398 \text{Pr}^{-0.1783} e^{-0.00111 \cdot \text{Pr}} \quad (14)$$

It is interesting to note here that the local heat transfer rate, h , under both the forced and natural convection conditions decreases in a downstream direction, as seen in Eqs. (1) and (5). However, the local shear stress behavior is different. For a forced flow it decreases as $x^{-1/2}$, whereas it increases in a buoyancy induced flow as $x^{1/4}$. The latter is due to an increasing characteristic velocity downstream.

Equations (12) and (13) may be integrated to give the average shear stress under the forced and natural convection limits. This yields

$$\bar{\tau}_f = 2\tau_f \quad \text{and} \quad \bar{\tau}_n = \frac{4}{5}\tau_n \quad (15)$$

In a similar way to that used for the heat transfer correlations the shear stress, τ_m , in a mixed convection flow may be represented in terms of τ_f and τ_n as:

$$\tau_m = [(\tau_f)^n + (\tau_n)^n]^{1/n} \quad (16)$$

The exponent, n , is determined by comparing the values for τ_m from Eq. (16) with the available values of τ_m . Only the analysis of Merkin (1969) tabulates the values of τ_m for $\text{Pr} = 1$, over the entire range of the convection parameter ξ . Graphic representation of τ_m is given by Afzal and Banthiya (1977) for $\text{Pr} = 0.7$ and for $0 < \xi < 40$. Based upon these analyses a value of $n = 6/5$ resulted in the best agreement. Since the results for other values of Prandtl number are not available, dependence of n on Pr could not be determined for the correlations. Using this value of n and substituting Eqs. (12), (13), and (14), the local correlation becomes

$$\frac{\tau_m}{\left(\frac{1}{2} \rho u_\infty^2 \right)} \text{Re}_x^{1/2} = [0.612 + (2.828 \cdot f''(0) \cdot \xi^{3/4})^{6/5}]^{5/6} \quad (17)$$

A correlation for the average shear stress under mixed convection is proposed here by combining $\bar{\tau}_f$ and $\bar{\tau}_n$ in a way similar to that used in Eq. (16). The correlation parameter, n , is determined by comparing τ_m with the values calculated by integrating Eq. (17) numerically. The final correlation for τ_m becomes:

$$\frac{\bar{\tau}_m}{\left(\frac{1}{2} \rho u_\infty^2 \right)} \text{Re}_x^{1/2} = [1.392 + (2.263 \cdot f''(0) \cdot \xi^{3/4})^{7/6}]^{6/7} \quad (18)$$

We note that these correlations approach the forced convection limits when $\xi \rightarrow 0$, and the natural convection limits when $\xi \rightarrow \infty$. The limits, shown in Eq. (15), are valid for any value of Pr. Although the best value of n in Eq. (16) is determined for the values of Prandtl number closer to unity, we believe that the shear stress correlations may be applied to other values of Pr as a first approximation.

Discussion

Values of the local heat transfer rates and shear stress rates for $\text{Pr} = 1$ were reported by Merkin (1969) for $0 < \text{Gr}/\text{Re}^2 < \infty$. The discrepancies between these results and those obtained from the proposed correlations, Eqs. (9) and (16) are less than 3 percent over the entire mixed convection regime. For other values of Prandtl number the results are compared with the analysis of Lloyd and Sparrow (1970), whose results were shown to be in excellent agreement with Kliegel's (1959) data in air. The local heat transfer rates as predicted by Eq. (9) deviate from those of Lloyd and Sparrow (1970) by less than 1, 3, and 2 percent for $\text{Pr} = 0.72, 10$, and 100 , respectively. For extremely low values of $\text{Pr} = 0.003$ the deviations are within 8 percent.

Agreement with results from other analyses (Oosthuizen and Hart, 1973; Afzal and Banthiya, 1977), and experimental results (Gryzagoridis, 1975) was also very good. Based upon experimentation in air, Oosthuizen and Bassey (1973) suggested a correlation for the average heat transfer rates. The discrepancy between that correlation and the proposed correlation in Eq. (10) is within 1 percent for the entire mixed convection regime. Correlations similar to those in Eqs. (9) and (10) were suggested by Churchill (1977) with a maximum discrepancy here of about 5 percent over the entire range of Pr.

We now characterize the limits between the forced, mixed, and natural convection regimes. Since the transport rates vary gradually as the flow regime changes from pure natural to pure forced convection some arbitrary criterion is needed to mark the end of one regime and the beginning of the other. The criterion for mixed convection limits used here is when the transport rates deviate by more than 5 percent from those of

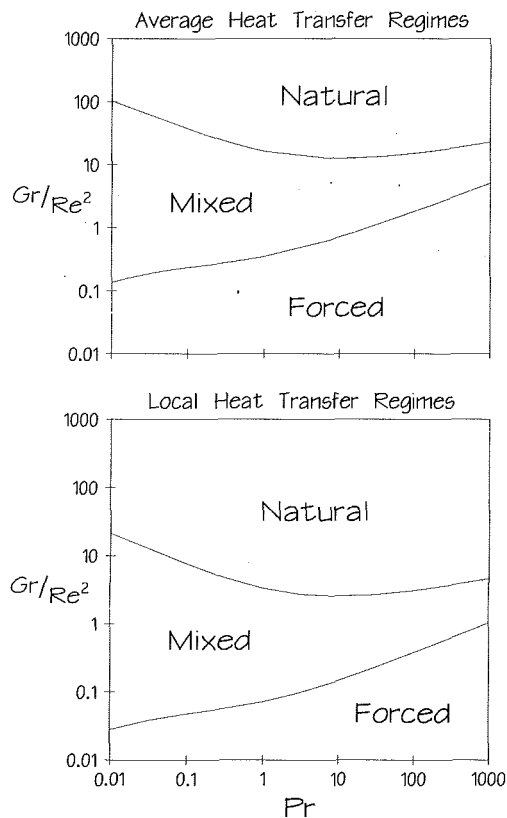


Fig. 1 Heat transfer regimes in aiding laminar mixed convection from a vertical isothermal surface based on a 5 percent limit between the pure convection and the mixed convection modes

the limiting regimes. Figure 1 shows the limits of transport regimes based on both the local and the average heat transfer rates. In both cases, values of Gr/Re^2 marking the transport regimes depend upon the value of the Prandtl number. It is also interesting to note that, in a given flow circumstance, the local transport regime at a given downstream location may become mixed while the average transport is still dominated by one of the limiting regimes, and vice versa. Thus, in practical circumstances where the total transport is of primary interest, characterization of the flow regimes based on average heat transfer is more realistic.

Since the proposed correlations are developed using their asymptotic behavior under pure natural and forced convection conditions their use is recommended for all three regimes without worrying about the presence of more than one regime present on the surface. Only the restriction of laminar aiding flow applies.

Acknowledgments

The authors acknowledge the support provided by the National Science Foundation under Grant No. MEA 82-04361.

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Correlations for the CHF Condition in Two-Phase Crossflow Through Multitube Bundles

M. K. Jensen¹ and H. Tang¹

Nomenclature

- A = dimensionless parameter
 Bo = boiling number = q''/Gh_{fg}
 D = tube diameter, m
 G = mass flux based on minimum flow area, kg/m²s

¹Department of Mechanical Engineering, Aeronautical Engineering, and Mechanics, Rensselaer Polytechnic Institute, Troy, NY 12180-3590.

Contributed by the Heat Transfer Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received by the Heat Transfer Division June 1993; revision received January 1994. Keywords: Boiling, Heat Exchangers, Multiphase Flows. Associate Technical Editor: L. C. Witte.