Superluminal Frames and the Group of Generalized Lorentz Transformations in Four Dimensions.

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1. – In a recent letter (1), while answering to a previous comment by RAMACHANDRAN *et al.* (2), we introduced a new group G of Lorentz transformations (LT) in four dimensions (*) generalized for both subluminal ($\beta < 1$) and superluminal ($\beta > 1$) velocities.

But in paper I—in order to limit its length—we could do nothing but scarcely mentioning our group G of generalized Lorentz transformations (GLT). Analogously, some other details were exploited not enough.

The aim of this further letter is to cast more light on the new group G, and to clarify a few other points of paper I.

Such problems—as well as many related other ones—will be extensively dealt with in a forthcoming paper, to be published elsewhere.

Let us call S the reference frames travelling faster than light with respect to the usual class of inertial frames s. The philosophical investigation developed in paper I showed that—if standard space-time measurements must be performable by S—then a « symmetry » between frames s and S must hold. In the sense that particles behaving as tachyons with respect to observers s will behave as bradyons with respect to observers S, and *vice versa* (*principle of duality*). Actually, the words bradyon (B), tachyon (T), frame s, frame S have only a *relative* meaning (1). The velocity of light c preserves of course its character of invariant quantity for both s and S frames (4).

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(1) E. RECAMI and R. MIGNANI: *Lett. Nuovo Cimento*, **4**, 144 (1972). This paper will be referred to in the following as « paper I ».
(2) G. RAMACHANDRAN, S. G. TAGARE and A. S. KOLASKAR: *Lett. Nuovo Cimento*, **4**, 141 (1972).
(*) Our extended LT's happened indeed to generalize Parker's (3) ones from the bidimensional to the four-dimensional case.
(3) L. PARKER: *Phys. Rev.*, **188**, 2287 (1969).
(4) See paper I, and references therein.

We shall call « inertial » all the (physical) frames with relative speeds both $u < c$ and $u > c$. Due to the « principle of duality », frames S are supposed to have at their disposal exactly the same physical objects as frames s have.

In paper I, it has been argued that a « principle of relativity » must hold for the whole class $\{I\}$ of « inertial » frames, since the physical laws (when generalized also for tachyons) are to be covariant (1) for GLT's of the whole class $\{I\}$. Actually, if both s and S observe *the same object* (as required in relativity), bradyonic laws will transform into tachyonic laws under a superluminal LT, and *vice versa*. Therefore, the totality of *relativistic* physical laws (written in the form valid for both B's and T's) will be « G-covariant » (1).

In these senses, we may say that all our inertial frames are *equivalent*.

2. – The condition for having the « principle of duality » satisfied is the following (4): *When passing from s to S, spacelike intervals must transform into timelike intervals*, and *vice versa.*

When in paper I we spoke (not too correctly) about « change of metric » or « metric inversion », we meant nothing but this fact. Such an inversion operates indeed on the symmetry with respect to the light-cone. Mathematically, the GLT's must be such that

$$c^2 t'^2 - \boldsymbol{x}'^2 = \pm (c^2 t^2 - \boldsymbol{x}^2) \qquad \text{for } u \lessgtr c . \tag{1}$$

The linear transformations, connecting inertial frames and satisfying eq. (1), are, roughly speaking, i) the usual, orthochronous (homogeneous) Lorentz transformations $\Lambda_<$ (and the ones $-\Lambda_< \equiv (PT)\cdot\Lambda_<$) for $u < c$, ii) the generalized Lorentz transformations $\pm i\Lambda_> \equiv \pm i\mathbf{1}\cdot\Lambda_>$ for $u > c$ (1), where (*)

$$\Lambda_< \equiv \Lambda(|\beta| < 1) , \qquad \Lambda_> \equiv \Lambda(|\beta| > 1) . \tag{2}$$

For example, in paper I we have shown that—in the simple case of collinear motion along the x-axis—condition (1) is satisfied by

$$\begin{cases} x' = \dfrac{x - ut}{\sqrt{|1-\beta^2|}} , & t' = \dfrac{t - ux/c^2}{\sqrt{|1-\beta^2|}} , \\ y' = \sqrt{\dfrac{1-\beta^2}{|1-\beta^2|}}\, y , & z' = \sqrt{\dfrac{1-\beta^2}{|1-\beta^2|}}\, z , \end{cases} \qquad [\beta^2 \lessgtr 1] , \tag{3}$$

for relative speed *both* $u < c$ and $u > c$. The GLT's, eq. (3), are precisely of the forms $\Lambda_<$ and $i\Lambda_>$ for $\beta^2 < 1$ and $\beta^2 > 1$, respectively.

In general, let us consider a universe free of charges and represent the Λ's by 4×4 matrices. Since matrices $\Lambda_>$ are formally identical with usual LT's, but corresponding to values $|\beta| > 1$, it is immediate to see that

$$\Lambda_<^{-1}(\beta) \equiv \Lambda_<(-\beta) , \qquad [i\Lambda_>(\beta)]^{-1} = -i\Lambda_>(-\beta) \equiv -i\Lambda_>^{-1}(\beta) . \tag{4}$$

Thence

$$[i\Lambda_>(\beta)]\cdot[-i\Lambda_>^{-1}(\beta)] = \mathbf{1} , \tag{5a}$$

(*) Let us explicitly recall (1) that the matrices $\Lambda_>$ are complex.

but

$$[i\Lambda_>(\beta)]\cdot[i\Lambda_>^{-1}(\beta)] = -\mathbf{1} \equiv PT\,, \tag{5b}$$

so that our generalized [1] group G will contain the *total-inversion* operator as an element. Precisely, by considering successive applications of GLT's of the types $\Lambda_<$ and $i\Lambda_>$, it is easy to realize that the group G consists of four subsets:

$$G \equiv (SU^{\uparrow}_{<}) \cup (SU^{\downarrow}_{<}) \cup (iSU^{\uparrow}_{>}) \cup (iSU^{\downarrow}_{>})\,, \tag{6}$$

where

$$SU^{\uparrow}_{<} \equiv SO^{\uparrow}_{<} \equiv SO^{\uparrow}_{+}(1,3;\,\mathbf{R};\,|\beta|\leqslant 1)\,, \qquad SU^{\uparrow}_{>} \equiv SU^{\uparrow}_{+}(1,3;\,\mathbf{C};\,|\beta|>1),$$
$$iSU^{\uparrow}_{>} \equiv \{L\colon l = i\Lambda_>;\ \Lambda\in SU^{\uparrow}_{>}\}\,,$$

and so on. All the elements L of G are rotations in the four-dimensional space-time, *i.e.* the transformations L are unimodular (with $\det L = +1$).

The structure of G will be clarified in a forthcoming article. Here let us simply mention that a correspondence exists between subluminal LT's from a frame s_0 to a frame s, moving with velocity $\boldsymbol{u}$ $(0 < u < c)$, and superluminal GLT's connecting s_0 to a frame S travelling in the same direction with speed $U = c^2/u$ $(u > c)$. Such a bi-univocal correspondence between frames s and S is the particular *conformal mapping* (*inversion*)

$$u \Leftrightarrow c^2/u\,. \tag{7}$$

In the case of a charged universe, interesting observations may be made about the CPT covariance.

3. – Afterwards, it is worth-while to clarify the following. When generalizing [1] physical laws for tachyons $(\beta > 1)$, one should pay attention that *a priori* $\sqrt{\beta^2-1} = \pm i\sqrt{1-\beta^2}$. Always (*) we consistently choose the sign *minus*, in order, *e.g.*, to get positive values of the relativistic mass (see eq. (4′) of paper I). It is understood that $\sqrt{1-\beta^2}$ represents, for $\beta > 1$, the upper–half-plane solution.

Lastly, let us notice that relativistic laws may be easily generalized for tachyons. In fact, from our discussion about the « equivalence » of *all* the inertial frames, it is immediate to get the following *tachyonization principle*: « The relativistic laws (of mechanics and electrodynamics) for tachyons follow by applying the GLT's to the corresponding laws for bradyons » (**).

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(*) A misprint occurred in the second eq. (7) of paper I, which ought to read $v'_{y,z} = +v_{y,z}\sqrt{1-\beta^2}/(1-uv_x/c^2)$ for $u > c$.

(**) After the completion of paper I, we became aware of the existence of papers [5,6], which approached our problem too. Criticizing ref. [5] is the substantial content of ref. [6].

[5] J. G. GILSON: *Mathem. Gazette*, **52**, 162 (1968).
[6] S. NARANAN: *Lett. Nuovo Cimento*, **3**, 623 (1972).