SINGLE PERIOD INVENTORY MODEL WITH STOCHASTIC DEMAND AND PARTIAL BACKLOGGING

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ABSTRACT

In this present scenario of world economy both the factors like salvage and stock-out situations are equally important. Continuous sources of uncertainty (stochastic demand), has a different impact on optimal inventory settings and prevents optimal solutions from being found in closed form. In this paper an approximate closed-form solution is developed using a single stochastic period of demand. Assorted level of demand is viewed in form of a special class of inventory evolution known as finite inventory process. Here the Inventory process is reviewed in form of three cases. This paper involves the study of optimality of the expected cost using the SCBZ property. Shortage cost is kept in view, in order to meet the customer demand. Finally this paper aims to show the optimal solution for three cases of finite inventory model in which the demand Q_0 is varied according to the SCBZ property. Appropriate Numerical illustrations provide a justification for its unique existence.

Keywords: SCBZ property, Newsboy problem, Inventory Model, closed form; Partial Backlogging.

1. INTRODUCTION

Inventory control is one of the most important aspects of today's complex supply chain management environment. Traditional inventory models focus on demand uncertainty and design the system to best mitigate that risk. One of the inventory models that have recently received renewed attention is that of the newsboy problem.

In some real life situation there is a part of the demand which cannot be satisfied from the inventory, leaving the system stock-out. In this system two situation are mainly considered. In order to optimize cost of inventory and profit an inventory model is developed. Newsboy problem is a special methodology in this area. In Newsboy problem, there is a

onetime supply of the items per day and the demand is probabilistic. If the order quantity is larger than the realised demand, the items which are left over at the end of period are sold at a salvage value or disposed off. In the last several years, many researchers have given considerable attention to the situation where the demand rate is dependent on the level of the on-hand inventory.

In this paper a closed form is introduced. There are many benefits of having a closed-form approximate solution. A closed-form solution clearly demonstrates the sensitivity of solutions to input parameters. It can also be embedded into more complicated models to add tractability. Closed-form approximations are also useful tools in practice, since they are easier to implement and use on an ongoing basis. When the price increase, its components is anticipated. In this situation companies may purchase large amounts of items without considering related costs. However ordering large quantities would not be economical if the items in the inventory system deteriorate. Also demand depends on the stock level. There are many benefits of having a closed-form approximate solution, such as the one we develop, for a problem which would otherwise require numerical optimization. A closed-form solution clearly demonstrates the sensitivity of solutions to input parameters. It can also be embedded into more complicated models to add tractability.

SCBZ property is defined as random variable Q having survival function Z is said to possess SCBZ property if and only if

$$\frac{z(Q+Q_0, \theta)}{z(Q_0, \theta)} = Z(Q, \theta), \quad \forall \theta \in \Theta, \quad Q, Q_0 \geq 0,$$

$$\theta^* = \theta^*(Q_0) \in \Theta \quad \forall Q_0 > 0.$$

Where θ represents the valve of the original parameter and $\theta^*(Q_u)$ represent its changed valve. In this paper three cases are discussed. In the first case, problem is studied by neglecting the salvage loss for the left over units but for the case when a significant holding cost φ_1 per unit time is incurred. In the second case, the inventory holding alone undergoes a change which is satisfying the SCBZ property. In the third case, the shortage cost with partial backlogging is discussed. In this paper a planned shortage models is studied, in which there exit time-dependent and time-independent shortage costs. Theses concept are discussed in form of case 1 and case 3

Finally the optimal solution $\psi(\hat{Z}), \psi_1(\hat{Z}), \psi_2(\hat{Z})$ is derived. This model is verified using the numerical illustration.

2. LITERATURE REVIEW

The basic Newsboy inventory model has been discussed in Hansmann.F [1]. A partial review of the Newsboy problem has been conducted in a textbook by Silver et.al [11]. Nicholas A.N et.al [3] had shown how the statistical inference equivalence principle could be employed in a particular case of finding the effective statistical solution for the multiproduct Newsboy problem with constraints.

R. Gullu et.al [17] had examined dynamic deterministic demand over finite-horizon and non-stationary disruption probabilities, and related to the optimal base-stock level of the newsboy fractal. M. Dada et.al [18] had extended the stochastic-demand newsboy model to

include multiple unreliable suppliers. L.V Snyder et.al [19] had simulated inventory systems with supply disruptions and demand uncertainty. Also his paper shows a study on how the two sources of uncertainty can cause different inventory designs to be optimal.

Traditional Newsboy models focus on risk neutral decisions makers (i.e) optimizing the expected profit or cost. But experimental finding states that the derivatives of actual quantity ordered from the optimal quantity are derived from the classical Newsboy model. Guinquing et.al [8] had considered the Newsboy problem with range information. In Jixan Xiao et.al [9] a stochastic Newsboy inventory control model was considered and it was solved on multivariate product order and pricing. P.S Sheik Uduman [10] had used demand distribution to satisfy SCBZ property and to depict the demand for the Newspaper. Also the optimal order quantity was derived in his paper.

In this paper a well known property know as SCBZ Property is used. This property was introduced by Raja Rao. B et.al [13] and it is an extension of the Lack of Memory Property. A review over his paper shows that many distributions like Exponential distribution, Linear Hazard rate distribution, Krane family, and Gompertz distribution possess this property. The author in his papers showed that SCBZ property is preserved under the formation of series systems. Furthermore, it had been shown that the mean residual life function of a random variable possessing SCBZ property takes a simple computational form.

A recent work on Newsboy model is given in Dowlath et.al [2], [15]. Here the author had discussed a models involving the problem in two levels of demand for a particular supply R_1 and R_2.

Srichandan Mishra et.al [14] had investigated the inventory system for perishable items under inflationary condition where the demand rate was a function of inflation and considered two parameter Weibull distributions for deterioration.

Finally this paper involves the optimality of the expected cost for varying demand involving either shortage or holding. These model are dealt in detail in the form of three cases.

2 Assumptions and Notations

φ_1 = The cost of each unit produced but not sold called holding cost.

φ_2 = The shortage cost arising due to each unit of unsatisfied demand.

Q = Random variables denoting the demand

Z = Supply level and $\hat{Z}$ is the optimal value of Z.

T = Total Time interval

t_1, t_2 = Time interval with respect to the shortage and holding cost

σ = Total cost per unit time

Q = Random variable denoting the individual demands.

$f(Q)$ = The probability density function.

Z = Supply level and $\hat{Z}$ is the optimal value of Z.

$f(Q, \theta_1)$ = probability density functions when $Q \le Q_0$.

$f(Q, \theta_2)$ = probability density function when $Q > Q_0$.

θ_1 = parameter prior to the truncation point Q_0

θ_2 = parameter posterior to the truncation Point Q_0.

τ = parameter of the maximum likelihood estimator

3. BASIC MODEL

Many researchers have suggested that the probability of achieving a target profit level is a realistic managerial objective in the Newsboy problem. The single period, single item newsboy problem with limited distribution information like range, mean, mode variance has been widely studied. Hansmann[1] had given a different perspective of this Newsboy model and the model discussed is given below

Total cost incurred during the interval T given in the basic model is

$$\sigma(T) = \begin{cases} \varphi_1 (Z - Q)T & , Q \le Z \\ \frac{\varphi_1}{2} Z t_1 + \frac{\varphi_2}{2} (Q - Z) t_2 & , Q > Z \end{cases} \tag{1}$$

Where t_1 and t_2 are defined as in figure 1

Therefore the expected total cost function per unit time is given in the form

$$\psi(Z) = \varphi_1 \int_0^Z (Z - Q) f(Q)\, dQ + \varphi_1 \int_Z^\infty \frac{Z}{2} \frac{t_1}{T} f(Q)\, dQ + \varphi_2 \int_Z^\infty \frac{(Q - Z)}{2} \frac{t_2}{T} f(Q)\, dQ \tag{2}$$

From Hansmann[1] we have $\quad \dfrac{t_1}{T} = \dfrac{Z}{Q}$

$$\frac{t_2}{T} = \frac{Q - Z}{Q} \tag{3}$$

To find optimal $\hat{Z}$, $\dfrac{d\psi(\hat{Z})}{dZ} = 0$ was formulated in Hansmann [1] which resulted in the following equation.

$$\psi(\hat{z}) = P(Q \le \hat{Z}) = \frac{\varphi_2}{\varphi_1 + \varphi_2} \tag{4}$$

Given the probability distribution of the demand Q using the expression $\psi(\hat{Z})$, the optimal $\hat{Z}$ can be determined. This is the basic Newsboy problem, as discussed by Hanssmann[1].
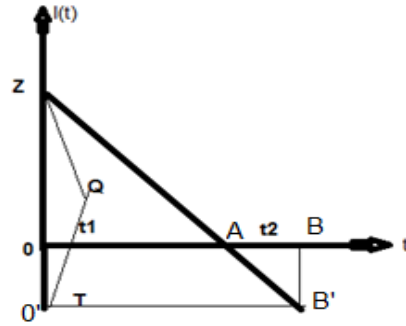
Figure (I)

The uncertainty here is related to a well known property called as SCBZ Property. The distribution here satisfies the SCBZ property.

Accordingly *SCBZ* property is defined by the pdf as

$$f(Q) = \begin{cases} \theta_1 e^{-\theta_1 Q} & ; \quad Q \leq Q_0 \\ e^{Q_0(\theta_2-\theta_1)} & \theta_2 e^{-\theta_2 Q}; \quad Q > Q_0 \end{cases}$$

(5)

where Q_0 is constant denoting truncation point .

The probability distribution function is denoted as

$$f(Q,\theta_1) \ if \ \ Q \leq Q_0$$

$$f(Q,\theta_2) \ if \ \ Q > Q_0$$

(6)

Sathiyamoorthy and Parthasarthy [13] introduced the concept of SCBZ property in inventory. The probability distribution function defined above satisfies the SCBZ property under the above assumptions and the optimal Z is derived.

Now, the total expected cost is written as

$$\psi(Z) = \varphi_1 \int_0^z (Z-Q) f(Q) \ dQ + \varphi_1 \int_z^\infty \frac{Z}{2} \frac{t_1}{T} f(Q) dQ + \varphi_2 \int_z^\infty \frac{(Q-Z)}{2} \frac{t_2}{T} f(Q) dQ$$

(7)

Equation (7) can be rewritten as

$$\psi(z) = \varphi_1 k_1 + \varphi_1 k_2 + \varphi_2 k_3 \tag{8}$$

Where

$$k_1 = \int_0^z (Z - Q) \, f(Q) \quad dQ$$

$$k_2 = \int_z^\infty \frac{Z}{2} \frac{t_1}{T} f(Q) \, dQ$$

$$k_3 = \int_z^\infty \frac{(Q - Z)}{2} \frac{t_2}{T} f(Q) \, dQ$$

$$\tag{9}$$

In this paper equation (9) is dealt in form of three cases due to the complexity involved while using the limit.

4. MODEL I

4.1 CASE 1: OPTIMALITY OF TOTAL EXPECTED COST USING SCBZ PROPERTY

When the optimality similar to the one discussed by Srichandan et.al [14] is studied. During the interval $(0, t_1)$, the inventory level neither decreases, nor increases due to the demand of the customers being fully satisfied which is shown in the figure(I). During the interval (t_1, t_2) the inventory decreases till B' due to the aspect of inflation of demand for a particular product. After t_2, the level of inventory reaches t after which shortage is allowed during (t_2, T) where T is from (0', B'). Srichandan et.al [14] had investigated the situation when shortage are allowed and partially backlogged. Here in this stage truncation point Q_0 satisfies the SCBZ Property. Then the optimality involving the holding cost is as follows,

$$\psi(z) = \varphi_1 \left(\int_0^{Q_o} (Z - Q) \, f(Q, \theta_1) dQ + \int_{Q_o}^z (Z - Q) \, f(Q, \theta_2) dQ \right)$$

$$+ \varphi_1 \left(\int_z^{Q_o} \frac{Z}{2} \frac{t_1}{T} f(Q, \theta_1) dQ + \int_{Q_o}^\infty \frac{Z}{2} \frac{t_1}{T} f(Q, \theta_2) dQ \right)$$

$$+ \varphi_2 \left(\int_0^{Q_o} \frac{(Q - Z)}{2} \frac{t_2}{T} f(Q, \theta_1) dQ + \int_{Q_o}^z \frac{(Q - Z)}{2} \frac{t_2}{T} f(Q, \theta_2) dQ \right)$$

$$\tag{10}$$

From (5) we substitute value for f (Q ,θ_1) and f (Q ,θ_2) in (9) we get

$$\psi(Z) = \varphi_1\left(\int_0^{Q_o} (Z-Q)\,\theta_1 e^{-\theta_1 Q} dQ + \int_{Q_o}^z (Z-Q)e^{Q_0(\theta_2-\theta_1)}\theta_2 e^{-\theta_2 Q} dQ \right)$$

$$+ \varphi_1\left(\int_Z^{Q_o} \frac{Z}{2}\frac{t_1}{T}\theta_1 e^{-\theta_1 Q}\, dQ + \int_{Q_o}^\infty \frac{Z}{2}\frac{t_1}{T} e^{Q_0(\theta_2-\theta_1)}\theta_2 e^{-\theta_2 Q}\, dQ \right)$$

$$+ \varphi_2\left(\int_0^{\varrho_0} \frac{(Q-Z)}{2}\frac{t_2}{T}\theta_1 e^{-\theta_1 Q}\, dQ + \int_{Q_o}^Z \frac{(Q-Z)}{2}\frac{t_2}{T} e^{Q_0(\theta_2-\theta_1)}\theta_2 e^{-\theta_2 Q}\, dQ \right)$$

(11)

$$\psi(Z) = \varphi_1\left(\begin{array}{l} \int_0^{Q_0}\theta_1 Z\, e^{-\theta_1 Q} dQ - \theta_1 \int_0^{Q_o} Q\, e^{-\theta_1 Q} dQ + \theta_2 \int_{Q_o}^Z Z e^{Q_0(\theta_2-\theta_1)} e^{-\theta_2 Q} \\[2mm] -\theta_2 \int_{Q_o}^Z Q e^{Q_0(\theta_2-\theta_1)} e^{-\theta_2 Q} dQ \end{array} \right)$$

$$+ \varphi_1\left(\frac{Z}{2}\frac{t_1}{T}\theta_1 \int_Z^{Q_o} e^{-\theta_1 Q}\, dQ + \theta_2 \frac{Z}{2}\frac{t_1}{T} \int_{Q_o}^\infty e^{Q_0(\theta_2-\theta_1)}\, e^{-\theta_2 Q}\, dQ \right)$$

(12)
$$+ \varphi_2\left(\begin{array}{l} \frac{t_2}{T}\int_0^{Q_0}\theta_1 e^{-\theta_1 Q}\, dQ - \frac{t_2}{T}\theta_1 Z \int_0^{\varrho_0} \frac{1}{Q} e^{-\theta_1 Q}\, dQ \\[2mm] + \frac{t_2}{T}\theta_2 \int_{Q_o}^Z e^{Q_0(\theta_2-\theta_1)}\, e^{-\theta_2 Q}\, dQ - \frac{t_2}{T}\theta_2 \int_{Q_o}^Z \frac{Z}{2} e^{Q_0(\theta_2-\theta_1)}\, e^{-\theta_2 Q}\, dQ \end{array} \right)$$

$$\psi(Z) = \varphi_1\left(z\left(1 - e^{-\theta_1 Q_0}\right) - \frac{1}{\theta_1}\left(1 + Q e^{-\theta_1 Q} - e^{-\theta_1 Q_0}\right) + \frac{Z\theta_2}{Q_0}\left(e^{-Q_0\theta_1}\right) \right.$$
$$\left. - e^{-Q_0(\theta_2-\theta_1)}e^{-\theta_2 Z} + \frac{e^{-Q_0(\theta_2-\theta_1)}}{\theta_2}\left[e^{-Q_0\theta_2} - Q\theta_2 e^{-\theta_2 Q} - e^{-\theta_2 Z}\right]\right)$$

$$+ \varphi_1\left\{\frac{Zt_1}{2T}\left[e^{-\theta_2 Z}\right]\right\} + \varphi_2\left\{ \begin{array}{l} \frac{t_2}{T}\left(1 - e^{-\theta_1 Q_0}\right) - \frac{z}{2}\left(\frac{(1-e^{-\theta_1 Q})}{\theta_2}\right)^n \\[2mm] + \frac{z}{2}\left(\frac{(1-e^{-\theta_1 Q})}{\theta_2}\right)^n + \left(e^{-\theta_1 Q_0}e^{-\theta_2 Z}\right) \end{array} \right\}$$

(13)

Therefore we get

$$\psi(Z) = \varphi_1\left[z\left(1 - e^{-\theta_1 Q_0}\right) - \frac{1}{\theta_1}\left(1 + Q e^{-\theta_1 Q} - e^{-\theta_1 Q_0}\right) + \frac{Z\theta_2}{Q_0}\left(e^{-Q_0\theta_1}\right) \right.$$
$$\left. - e^{-Q_0(\theta_2-\theta_1)}e^{-\theta_2 Z} + \frac{e^{-Q_0(\theta_2-\theta_1)}}{\theta_2}\left[e^{-Q_0\theta_2} - Q\theta_2 e^{-\theta_2 Q} - e^{-\theta_2 Z}\right]\right]$$

$$+ \varphi_1 \left\{ \frac{Z t_1}{2T} \left[e^{-\theta_1 Z} \right] \right\} + \varphi_2 \left\{ \frac{t_2}{T} \left(1 - e^{-\theta_1 Z} \right) \right\}$$

(14)

The following assumption is considered while solving (14) that the lead time is zero and single period inventory model will be used with the time horizon considered to as finite. When the supply and the level of inventory is same and all the other cases are considered zero. Since analytical solutions to the problem are difficult to obtain, Equation (14) is solved using the Maple 13. Final solution of the above equation is given as,

$$\psi(\check{Z}) = z - \frac{1}{\theta_1} - \frac{Z}{e^{\theta_1 Q_0}} + \frac{1}{\theta_1 e^{\theta_1 Q_0}} + \frac{Q_0}{e^{\theta_1 Q_0}}$$

(15)

Now, the optimal solution here is obtained using the numerical illustration by substituting the value for θ_1, Q_0, Z. $\psi(\check{Z})$ is obtained in form of Numerical illustration1 shown below.

The Table1 and Figure1, shows the numerical illustration for the case 1 when the shortage cost is permitted in the interval $(0, Z)$.

4.1.1. Numerical illustration

In this Numerical illustration the value for Z= 1, 2,..6, $\theta_1 = 0.5$ to 3, and $Q_0 = 5,10,15...,30$ is evaluated and the graph representing these values are given below which is obtained to get the optimal expected cost $\psi(\check{Z})$. This Numerical illustration provides a clear idea of the increased profit form curve.

| Z	θ_1	Q_0	$1/\theta_1$	e^{θ}_1	$e^{\theta 1}Q_0$	$e^{\theta}_1 Q_0$	$Z/e^{\theta}_1 Q_0$	$1/\theta_1 * e^{\theta}_1{}^{Q}_0$	$Q_0/e^{\theta}_1{}^{Q}_0$	$\psi(\check{Z})$
1	0.5	5	2	1.648721	12.18249	8.243606	0.121306	0.16417	0.410425	-0.54671
2	1	10	1	2.718282	22026.47	27.18282	0.073576	4.54E-05	0.000454	0.926924
3	1.5	15	0.666	4.481689	5.91E+09	67.22534	0.044626	1.13E-10	2.54E-09	2.288707
4	2	20	0.5	7.389056	2.35E+17	147.7811	0.027067	2.12E-18	8.5E-17	3.472933
5	2.5	25	0.4	12.18249	1.39E+27	304.5623	0.016417	2.88E-28	1.8E-26	4.583583
6	3	30	0.333	20.08554	1.14E+26	401.7107	0.014936	2.92E-27	1.75E-25	5.651731

**Table 1**
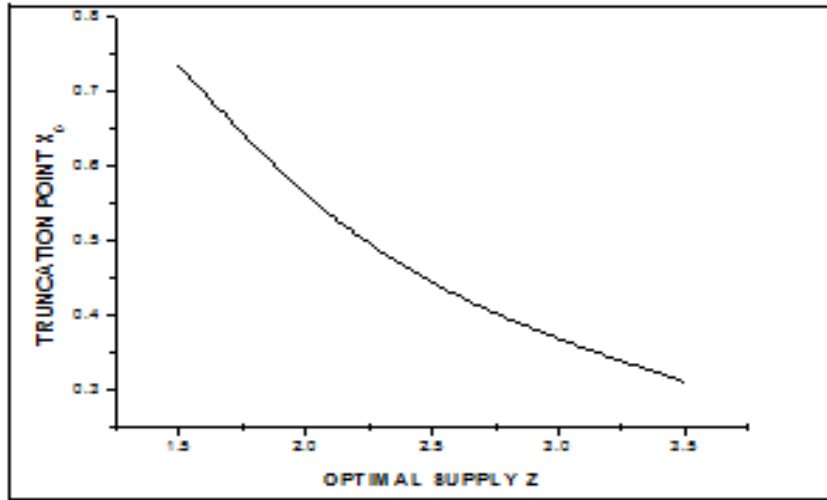
Figure 1

FIGURE1

Supply against the $\psi(\check{Z})$ is shown in the Figure1. When the supply size is increased according to the demand then there a profit or otherwise instantaneous increase in the $\psi(\check{Z})$ is noted. This model shows how a sharp increase in the cost curve is obtained.

4.1.2 : Numerical Illustration

A comparative study was carried out with the existing data value available from P.S Sheik Uduman [10] to check the optimality if the cost curve unique. Table 2 and Figure 2 shows the existence of the profit curve. The shortage graph is seen below. When there is shortage in the supply size then there is a decrease in the expected cost which leads to the profit loss for the company.

Z	θ_1	Q_0	e^{θ_1}	$e^{\theta_1 Q_0}$	$e^{\theta_1}*Q_0$	$1/\theta_1$	$Z/e^{\theta_1}*Q_0$	$1/\theta_1*e^{\theta_1 Q_0}$	$Q_0/e^{\theta_1 Q_0}$	$\psi(\check{Z})$
0.5	0.5	5	1.64872	12.18249	8.243606	2	0.060653	0.164169997	0.410425	-0.98606
0.7	1	10	2.71828	22026.47	27.18282	1	0.025752	4.53999E-05	0.000454	-0.32525
0.9	1.5	15	4.48168	5.91E+09	67.22534	0.6666	0.013388	1.12793E-10	2.54E-09	0.219946
1.1	2	20	7.38905	2.35E+17	147.7811	0.5	0.007443	2.12418E-18	8.5E-17	0.592557
1.3	2.5	25	12.1824	1.39E+27	304.5623	0.4	0.004268	2.87511E-28	1.8E-26	0.895732
1.5	3	20	20.0855	1.14E+26	401.7107	0.3333	0.003734	2.91884E-27	1.75E-25	1.162933

Table 2
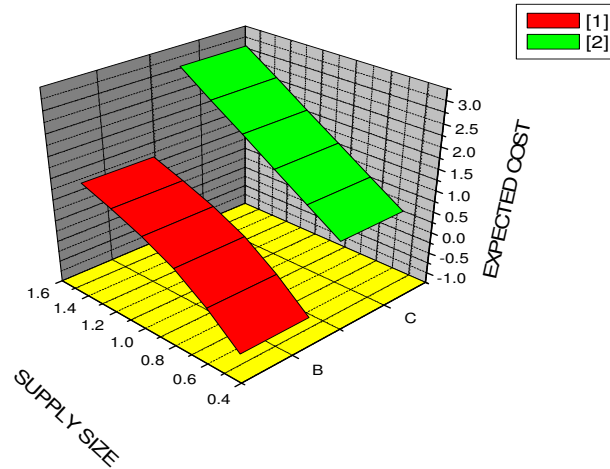
Comparative graph
Figure 2

[1] Curve as in the Existing model P. Sheik Uduman [10]
[2 New curve for model I

FIGURE 2

A comparative study on the optimal expected profit curve with that of a P.S Sheik Uduman [10] is shown in form of the Figure 2. It observed that there is an an increase in the profit from negative to positive value leading to an instantaneous increase in supply size and increased profit. The curve shows the optimality and validity of this model.

4.2 CASE 2: OPTIMALITY FOR HOLDING COST USING SCBZ PROPERTY

The units of items unsold at the end of the season if any are removed from the retail shop to the outlet discount store and are sold at a lowest price than the cost price of the item which is known as the salvage loss. A model is developed to study this problem. In Case 2, a situation is discussed when there is a holding cost occurred and there is no shortage allowed. This constitute an exponential family. In this case the attention can be restricted to the consideration of the part when the holding cost φ_1 is involved, in which case there is an immense loss to the organisation leading to the setup cost and the cost of holding the item.

Now the expected holding cost is given as follows and this cost is truncated before and after the particular event in the interval (0,Q)

$$\psi_1(Z) = \varphi_1\left(\int_0^{Q_o} (Z - Q) \; f(Q, \theta) \, dQ + \int_{Q_o}^{Q} (Z - Q) \, f(Q, \theta) \, dQ \right)$$

$$+ \varphi_1\left(\int_0^{Q_o} \frac{Z}{2} \frac{t_1}{T} f(Q, \theta) \, dQ + \int_{Q_o}^{Q} \frac{Z}{2} \frac{t_1}{T} f(Q, \theta) \, dQ \right)$$

$$\psi_1(Z) = \varphi_1\left(\int_0^{Q_o} (Z - Q) \theta_1 e^{-\theta_1 Q} \, dQ + \int_{Q_o}^{Q} (Z - Q) e^{Q_0(\theta_2 - \theta_1)} \theta_2 e^{-\theta_2 Q} \, dQ \right) \tag{16}$$

$$+ \varphi_1\left(\frac{Z}{2} \frac{t_1}{T} \int_0^{Q_o} \theta_1 e^{-\theta_1 Q} dQ + \frac{Z}{2} \frac{t_1}{T} \int_{Q_o}^{Q} e^{Q_0(\theta_2 - \theta_1)} \theta_2 e^{-\theta_2 Q} \, dQ \right)$$

$$= \varphi_1\left(\theta_1 \int_0^{Q_o} (Z - Q) e^{-\theta_1 Q} \, dQ + e^{Q_0(\theta_2 - \theta_1)} \theta_2 \int_{Q_o}^{Q} (Z - Q) e^{-\theta_2 Q} \, dQ \right)$$

$$+ \varphi_1\left\{ \frac{Z}{2} \frac{t_1}{T} \left(\theta_1 \int_0^{Q_o} e^{-\theta_1 Q} dQ + e^{Q_0(\theta_2 - \theta_1)} \theta_2 \int_{Q_o}^{Q} e^{-\theta_2 Q} \, dQ \right) \right\} \tag{17}$$

$$\psi_1(Z) = \varphi_1\left(\theta_1 \int_0^{Q_o} Z e^{-\theta_1 Q} dQ - \theta_1 \int_0^{Q_o} Q e^{-\theta_1 Q} dQ + \theta_2 \int_{Q_o}^{Q} Z e^{Q_0(\theta_2 - \theta_1)} e^{-\theta_2 Q} \, dQ - \theta_2 \int_{Q_o}^{Q} Q e^{Q_0(\theta_2 - \theta_1)} e^{-\theta_2 Q} dQ \right)$$

$$+ \varphi_1\left(\frac{Z}{2} \frac{t_1}{T} \theta_1 \int_0^{Q_o} e^{-\theta_1 Q} \, dQ + \theta_2 \frac{Z}{2} \frac{t_1}{T} \int_{Q_o}^{Q} e^{Q_0(\theta_2 - \theta_1)} e^{-\theta_2 Q} \, dQ \right)$$

$$\psi_1(Z) = \varphi_1\left(\theta_1 (\frac{Z}{\theta_1}(e^{-\theta_1 Q_0} - 1)) - Qe^{-\theta_1 Q_0} - \frac{1}{\theta_1}(e^{-\theta_2 Q_0} - 1) + e^{-Q_0\theta_1} Q \right)$$

$$\psi_1(Z) = \varphi_1\left((Z(e^{-\theta_1 Q_0} - 1)) - \frac{1}{\theta_1}(e^{-\theta_2 Q_0} - 1) \right) \tag{18}$$

4.2.1 Numerical Illustration:

In this example a load of items is taken from 1tonnes to 6 tonnes were the θ_1 is varied accordingly and the result shows a increase of the expected profit curve which is given by Table 3 and Figure 3

| Z	θ_1	$1/\theta_1$	Q_0	e^{θ_1}	$e^{\theta_1 Q_0}$	$Z(e^{\theta_1 Q_0}-1)$	$(e^{\theta_1 Q_0}-1)$	$(e^{\theta_1 Q_0}-1)1/\theta1$	$\psi_1(\hat{Z})$
1	0.05	20	5	1.051271	1.284025	0.284025	0.284025	5.680508	-5.39648
2	0.03	33.33333	10	1.030455	1.349859	0.699718	0.349859	11.66196	-11.3121
3	0.07	14.28571	15	1.072508	2.857651	5.572953	1.857651	26.53787	-24.6802
4	0.09	11.11111	20	1.094174	6.049647	20.19859	5.049647	56.10719	-51.0575
5	0.08	12.5	25	1.083287	7.389056	31.94528	6.389056	79.8632	-73.4741
6	0.09	11.11111	20	1.094174	6.049647	30.29788	5.049647	56.10719	-51.0575

Table 3
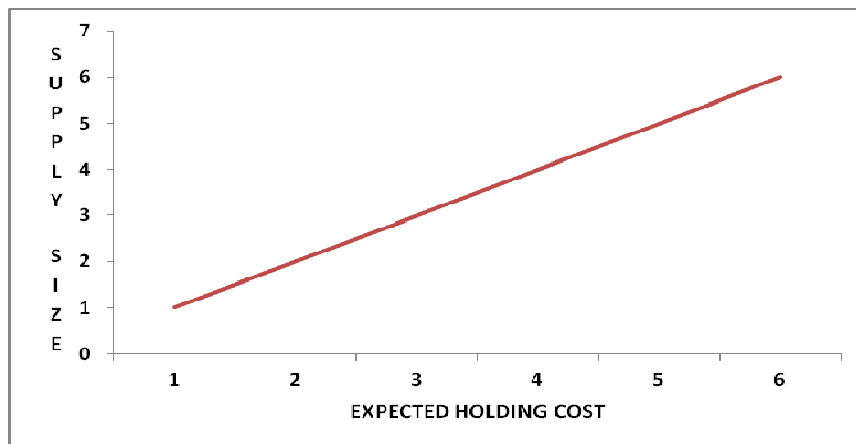
Figure 3
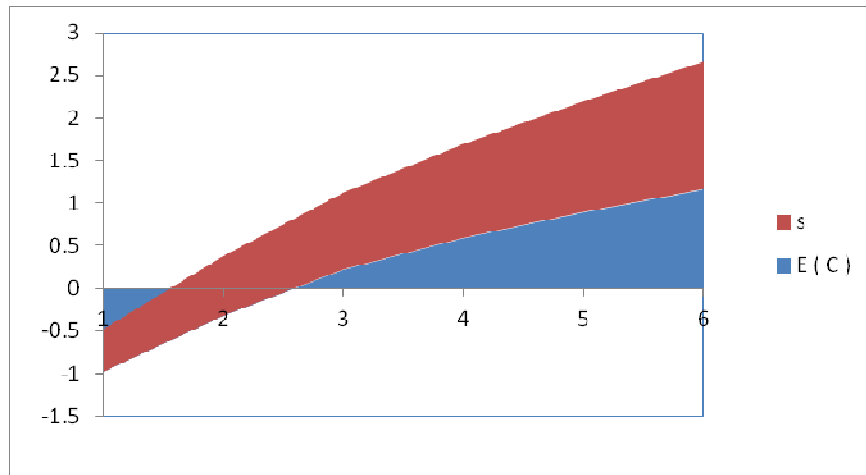
Figure 4

A diagrammatic representation of the state when the supply size is excess then it rules out the total expected cost and this state of condition aroused is shown in the above diagram.

4.3 CASE 3: OPTIMALITY FOR SHORTAGE COST CURVE

Economic Order Quantity for Planned Shortages (EOQB) model is illustrated in very few text books (Anderson et.al [23], Gupta et.al [22], Narsimhan et.al[24], Pannerselvam, [27] Sharma S.D[26] and Vora N.D [25]). In literature, few authors use term "*back ordering*" while many authors prefer *"planned shortages"* to describe this model. Both terms carry the meaning but back ordering is more preferable as it deal with the cost of back ordering. For the model, replenishment is done at a point when stock reaches maximum planned shortages (negative inventory).

Notable work is observed in partial backordering. The backlogging phenomenon is modeled without using the backorder cost and the lost sale cost since these costs are not easy to estimate in practice. Abad P [28], San-José et.al [29] had studied a continuous review inventory control system over a infinite-horizon with deterministic demand where shortage is partially backlogged.

Jhuma et.al [20] had discussed the state of condition in which a single period imperfect inventory model with price dependent stochastic demand and partial backlogging was considered. In many of the articles in literature discussed so far not allowed shortage or if occurred was considered to be completely backlogged. In a highly competitive market providing varieties of product today to the customers due to globalization, partial backorder is more realistic one. For fashionable items and high-tech products with short product life cycle, the willingness of a customer to wait for backlogging during the shortage period decreases with the waiting time.

During the stock-out period, the backorder rate is generally considered as a non-increasing linear function of backorder replenishment lead time through the amount of shortages. The larger the expected shortage quantity is, the smaller the backorder rate would be. The remaining fraction of shortage is lost. This type of backlogging is called time-dependent partial backlogging. Mainly there are two types of shortages inventory followed by shortages and shortages followed by inventory. Shortage may occur either due to the presence of the defective items in the ordered lot or due to the uncertainty of demand. The shortage charge is assumed proportional to the area under the negative part of the inventory curve.

Let us formulate the assumptions as given in Vora N.D [25]

1. The demand for the item is certain, constant and continuous
2. Lead time is fixed
3. The replenishment for order quantity is done when shortage level reaches planned shortage level in one lot
7. Stock outs are permitted and shortage or backordering cost per unit is known and is constant.

From (1) and (3) we obtain the following in the interval (Z, ∞)

$$\psi_2(Z) = \varphi_2\left(\int_z^{Q_0} \frac{(Q-Z)}{2} \frac{t_2}{T} \, f(Q, \theta_1) \, dQ + \int_{Q_0}^{\infty} \frac{(Q-Z)}{2} \frac{t_2}{T} \, f(Q, \theta_2) \, dQ \right) \quad (19)$$

$$\psi_2(Z) = \varphi_2\left(\int_z^{Q_0} \frac{(Q-Z)}{2} \frac{t_2}{T} \theta_1 e^{-\theta_1 Q} \, dQ + \int_{Q_0}^{\infty} \frac{(Q-Z)}{2} \frac{t_2}{T} \, e^{Q_0(\theta_2-\theta_1)} \theta_2 e^{-\theta_2 Q} \, dQ \right) \quad (20)$$

Using (3) in (20) we get

$$\psi_2(Z) = \varphi_2 \left(\int_z^{Q_0} \frac{(Q-Z)^2}{2Q} \, \theta_1 e^{-\theta_1 Q} \, dQ + \int_{Q_0}^{\infty} \frac{(Q-Z)^{2^2}}{2Q} \; e^{Q_0(\theta_2-\theta_1)} \, \theta_2 e^{-\theta_2 Q} \, dQ \right)$$

$$\psi_2(Z) = \varphi_2 \left(\frac{1}{2Q} \left[\int_z^{Q_0}(Q-Z)^2 \, \theta_1 e^{-\theta_1 Q} \, dQ + \int_{Q_0}^{\infty}((Q-Z))^2 \; e^{Q_0(\theta_2-\theta_1)} \, \theta_2 e^{-\theta_2 Q} \, dQ \right] \right) \tag{21}$$

$$\psi_2(Z) = \varphi_2 \left\{ \left(\sum_z^{Q_0} \left(-2 \frac{e^{-\theta_1 Q}}{Z^2} \left(Q + \frac{1}{\theta_1} \right) \right) + 2 \frac{e^{-\theta_1 Q}}{Z} \left(-Q - \frac{1}{\theta_1} \right) \right) - e^{-\theta_1 Q} \right\}$$

$$\tag{22}$$

Adding (18) and (22) we have the following result

$$\Psi(\check{Z}) = \psi_1(Z) + \psi_2(Z) \tag{23}$$

The solution of (23) requires the basic property Z=□ and equation (4) suggest a general principle of balancing the shortage and overage which shall have an occasion to be applied repeatedly. By recalling the standard notations let us generally introduce a control variable Z and a random variable Q with known density and two functions $\psi_1(Z,Q) \geq 0$ and $\psi_2(Z,Q) \leq 0$ which may be interpreted as overage and shortage levels respectively. Let us assume the fundamental property of linear control:

$$\frac{d}{dQ}[\psi_1(Z) + \psi_2(Z)] = \alpha \tag{24}$$

Where ψ denotes the expected value and α is a constant. To minimize a cost of the form

$$\Psi(\check{Z}) = \varphi_1 \psi_1(Z) - \varphi_2 \psi_2(Z) \tag{25}$$

We differentiate with respect to Q and the use (24), thus obtaining

$$\frac{d\Psi(Z)}{dQ} = \varphi_1 \frac{d\psi_1(Z)}{dQ} - \varphi_2 \left[-\frac{d\psi_1(Z)}{dQ} + \alpha \right] \tag{26}$$

This leads to the following condition for the optimal valve □ :

$$\frac{d\psi_1(Z)}{dQ} = \frac{\alpha\varphi_2}{\varphi_1 + \varphi_2} \tag{27}$$

In other words the derivative of the expected overage must be equal to the characteristics cost ratio in the equation (27).

Now accordingly the SCBZ Property satisfies the existence of the solution hence when

$$\psi_1(Z) = (Z - Q)^+ \tag{28}$$

Then when $\alpha = 1$ and by computing the general principle of balancing shortage and overage we compute $\frac{d\psi_1(Z)}{dQ} = \int_0^Z f(Q,\theta)dQ + Z \int_Z^{\infty} \frac{f(Q,\theta)}{Q}dQ = \Psi(\check{Z})$ $\tag{29}$

Thus the optimal solution is given by the following $\Psi(\check{Z}) = 1 - e^{-\theta_1 Q}\left(\frac{\alpha\varphi_2}{\varphi_1+\varphi_2}\right)$ $\tag{30}$

Now this shows that we have a linear control with α=1

Therefore it leads to the following result $\Psi(\check{Z}) = 1 - e^{-\theta_1 Q}\left(\frac{\varphi_2}{\varphi_1+\varphi_2}\right)$ $\tag{31}$

5. CONCLUSIONS

Thus the SCBZ property seems to be a useful concept in reliability theory and needs further attention. In the most realistic setting the variability of benefit in stochastic inventory models cannot be ignored. The news boy problem is treated in this paper as an example of an item under inflation has to be ordered according to the variability of the profit. Our model framework is extended for three cases. Where as in extending the other case the expression was more complex and it had a complex probability functions and integrations.

This model is examined in form of time point occurring before the truncation point, and the time point occurring after the truncation point. Thus the demand may be illustrated by three successive time periods that classified time dependent ramp type function as, first phase the demand increases with time after which it attains steady state towards the end and in the final phase it decreases and becomes asymptotic. Thus the demand may be stock dependent up to certain time after that it is constant due to some good will of the retailer. This model can be considered in future with deteriorating items. The necessity of storage of items cannot be ignored and emphasis should be given whether the storage is needed or not in the context of deteriorating items and allowing shortages.

Appendix

$$restart\ A = expand(int((Z - Q).\theta[1].exp(-\theta[1]Q), Q = 0 \dots Q[0]));$$

$$Z - \frac{1}{\theta_1} - \frac{Z}{e^{\theta_1 Q_0}} + \frac{1}{\theta_1 e^{\theta_1 Q_0}} + \frac{Q_0}{e^{\theta_1 Q_0}}$$

$$B = expand(int\left((Z - Q.exp(Q[0].(\theta[2] - \theta[1]))).(\theta[2].exp(-\theta[2].Q)), R = Q[0] \dots Z\right));$$

$$-ZQ_0 - \frac{R}{e^{\theta_1 Q_0}} - Z^2 + \frac{Re^{\theta_1 Q_0}}{e^{\theta_1 Q_0} e^{\theta_2 z}}$$

$$e = expand(A + B);$$

$$Z - \frac{1}{\theta_1} - \frac{Z}{e^{\theta_1 Q_0}} + \frac{1}{\theta_1 e^{\theta_1 Q_0}} + \frac{Q_0}{e^{\theta_1 Q_0}} - ZQ_0 - \frac{R}{e^{\theta_1 Q_0}} - Z^2 + \frac{Re^{\theta_1 Q_0}}{e^{\theta_1 Q_0} e^{\theta_2 z}}$$

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