

Enforcing High-Performance Operation of Multi-hop Wireless Networks With MIMO Relays

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Abstract—In multi-hop wireless networks where links are prone to be broken or degraded, it is important to guarantee the network connectivity as well as satisfy the performance requirements. Observing the promising features of Multiple-Input Multiple-Output (MIMO) techniques for improving the transmission capacity and reliability, in this paper, we make the very first attempt to deploy MIMO nodes as relays to assist weak links in wireless networks, with the aim of reducing the number of relay nodes and providing performance provisioning. We identify the specific constraints of MIMO relay nodes for assisting weak links, and take advantage of the MIMO ability to flexibly select among different transmission strategies. The constraints and flexibility, however, make the MIMO deployment problem different from conventional single-antenna deployment schemes and much more challenging. Based on the constraints, we formulate the MIMO relay deployment problem, and provide a polynomial-time approximation scheme (PTAS) algorithm, as well as a distributed heuristic algorithm. The performance of the proposed algorithms is evaluated through simulations and demonstrated to be very effective.

I. INTRODUCTION

Over the past a few years, uncoordinated multi-hop wireless networks such as ad hoc networks, wireless mesh networks, and sensor networks have gone through rapid development. They are widely used in both military and civilian applications. In wireless networks, factors such as energy depletion, harsh environmental conditions, and malicious attacks may result in node failures. An active link could thus become *broken* and a network tends to lose connectivity. Moreover, different links have different quality depending on the channel conditions, and thus have different link capacities. In a wireless environment, the channel condition may experience a significant change due to reasons such as the variation of weather and the existence of obstacles. As a result, an active link may become too weak for data transmission, and thus become a *bottleneck* in an end-to-end path. The links that fall into the aforementioned categories are called *weak links* in general throughout this paper.

In order to have the network perform properly, it is of significant importance to deal with the weak links to restore connectivity as well as to provide acceptable throughput and ensure transmission reliability in a severe environment. A group of weak links may be close to each other, as their channel degradation is caused by the same reason or they are within a heavy-loaded bottleneck region (i.e. near a data

sink in the sensor networks) thus the nodes are more likely to run out of the energy. A practical option is to deploy a small number of more powerful relay nodes to re-establish the network connectivity while meeting traffic requirements. In order to reduce the cost of relay node deployment, we would like to place as few new nodes as possible.

In wireless sensor networks, there are some studies on placing relay nodes to provide the connectivity and/or prolong the network lifetime [1]–[5]. In addition to maintaining the connectivity, in general wireless networks, it is also important to guarantee the desirable rate over data transmission paths. For example, it is critical to ensure uninterrupted monitoring of a remote site through video cameras. Different from the previous work, we notice that the traffic requirements can impact the relay placement result and significantly increase the deployment challenge. Although traffic is generally considered in routing and scheduling, a deployment algorithm that takes into account the statistical traffic information could help improve the efficiency of MAC schemes and thus the overall network performance. In the literature work, Steiner-tree-based schemes have often been exploited for relay placement, which cannot satisfy the traffic requirements of the weak links. It is therefore important to introduce a new strategy that can enable a larger network capacity and higher transmission reliability especially in a severe network environment.

Multiple-input multiple-output (MIMO) technique has been proven to be able to provide high spectral efficiency and increase channel capacity substantially through multiple spatial channels without need of the additional spectrum. With multiple antennas at the transmitter and/or receiver, a MIMO system takes advantage of multiplexing to simultaneously transmit multiple data streams to increase the wireless data rate and diversity to optimally combine signals from different transmission streams to increase the transmission reliability and range. MIMO technique is considered as one of the most promising techniques for future wireless networks, and has been adopted by 802.11n, WiMAX, and LTE. To meet the high data rate requirements, more and more wireless devices are equipped with multiple antennas. As the cost of a MIMO node is usually higher than a regular node, it may not be economically efficient to have all nodes in a network equipped with multiple antennas. However, it is beneficial to deploy a small set of MIMO nodes to assist weak links, which can

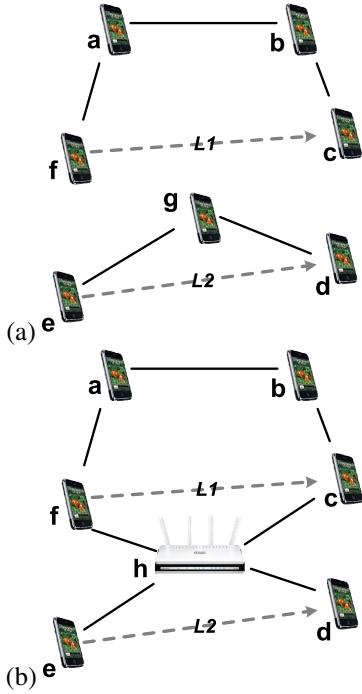


Fig. 1. Assisting weak links with relays: (a) single antenna and multihop relays; (b) MIMO relay.

significantly improve the overall network performance in a severe environment.

Specifically, MIMO relays can address issues that cannot be handled by simply adding more conventional single-antenna relay nodes. As an example, in figure 1 (a), the links L_1 from node f to node c and L_2 from node e to node d are detected to be weak links. There are two conventional ways to exploit relay nodes. One is using multi-hop intermediate nodes as relays, so the transmission for L_1 could be redirected via the route $f \rightarrow a \rightarrow b \rightarrow c$. This strategy raises the traffic load over the relay routes as well as increases the transmission latency, and it cannot be guaranteed that there are available neighboring nodes to serve as multi-hop relays. This method may temporarily facilitate transmissions of a weak link, but not if the inferiority of the link is permanent or lasts for a long period of time. Moreover, the consecutive hops cannot transmit simultaneously and the total throughput is reduced as a result. As the other way to exploit relay, a regular node with single antenna can be deployed near a weak link, for example, node g could assist the weak link L_2 . Even if node g is close to another weak link such as L_1 , it cannot simultaneously assist it as g can only communicate with one node at a time. Due to this transmission limitation, deployment of multiple single-antenna relays cannot meet the total traffic requirement of weak links as the relays cannot transmit concurrently in the same neighborhood. In such cases, MIMO relays can better assist weak links and provide a higher performance gain without extra spectrum cost. As in figure 1 (b), when placed close to nodes c , d , e and f , a MIMO node h with a multi-antenna array could simultaneously assist links L_1 and L_2 , i.e., it could receive concurrently from f and e and transmit concurrently to c and d . Thanks to spatial diversity and

multiplexing, these concurrent transmissions can enjoy higher rates than single-antenna transmissions, and h can even reach a larger area to assist more weak links by using spatial diversity to extend its communication range. It is therefore critical to develop techniques that can efficiently deploy MIMO relays and harvest their multiplexing and diversity gains for a higher network performance.

A MIMO node could provide various transmission rates and ranges thus meet different transmission needs with an appropriate configuration of the antennas. This flexibility can bring in a significant advantage. However, it also makes the deployment of MIMO nodes much more challenging than conventional relay placement. In order to fully take advantage of MIMO features, it is necessary to identify the specific constraints of MIMO transmissions, and design appropriate transmission strategy to flexibly determine the specific transmission mode of a MIMO relay node when facilitating a specific weak link according to the network topology, channel conditions and traffic requirements. As the relay deployment is made for a higher network performance over a relatively longer period, the provisioning is based on statistical channel conditions and traffic requirements over a period of time. If the relay nodes are mobile (i.e., robots) or mobile agents are available to move the relay nodes, the relay positions can be adjusted over time.

In this paper, we aim to design MIMO relay node placement algorithms to facilitate transmissions over weak links in a multi-hop wireless network so that higher performance can be achieved with the minimum deployment cost and the traffic requirements of the weak links are satisfied. Our work is distinguished from the previous work in that:

- 1) It is the first work that considers deployment of MIMO nodes in wireless multi-hop networks. Different from conventional deployment problems, MIMO nodes are with different transmission ranges and rates when different transmission strategies are configured.
- 2) The deployment problem not only guarantees the full coverage of the weak links that require assistance for connectivity, but also opts to minimize the number of MIMO nodes while considering the traffic demand of flows in the deployment to provide performance provisioning.
- 3) We perform cross-layer optimization to flexibly select MIMO transmission strategies for each of the weak links facilitated depending on the network topology, statistical channel conditions and traffic demands.

The rest of this paper is organized as follows. We discuss related work in Section II. We introduce background and describe the system model in Section III, and formulate the problem in Section IV. The centralized and distributed deployment schemes are proposed in Section V and Section VI, respectively. Simulation results are presented in Section VII. The paper is concluded in Section VIII.

II. RELATED WORK

Relay placement in wireless sensor networks (WSN) has been studied over the past several years, where the focuses

have been on improving energy efficiency [3] or minimizing the number of relays to guarantee network connectivity [1], [2], [4], [5], assuming a homogeneous transmission range for both relay nodes and sensor nodes. The limited studies [1], [2] that do not assume uniform-range often consider specific WSN architectures, i.e. tiered and/or with base-station, instead of a general multi-hop wireless network we study here. More recently, the authors in [4], [5] further extended the problem of relay deployment to heterogeneous wireless sensor networks, with the assumption that the candidate relay positions are known. The finding of candidate positions is a challenging problem itself. The objectives of these existing studies are constrained to connectivity, with little consideration of traffic requirements or throughput optimization. In an environment with severe channel conditions such as scattering and fading or with a number of weak links close by, placing simple relay nodes cannot ensure the transmission quality. In this work, we consider the deployment of MIMO relay nodes with higher capacity and various operational modes to address all these issues in an integrated and coherent manner.

Besides relay placement in WSN, the placement of Internet transit access points was studied in [6] to provide Internet connectivity in multi-hop wireless networks. Gateway placement for throughput optimization in multi-hop wireless mesh networks was addressed in [7], and joint mobile backbone node placement and regular node assignment was proposed in [8]. The solutions proposed in the above work cannot be applied to MIMO relay deployment, which is more challenging as a MIMO node can be configured flexibly with different transmission ranges and rates.

The application of MIMO technique in wireless mesh and ad hoc networks has gained increasing attention in recent years. Many efforts have been made in developing efficient MAC [9]–[12] and routing [13], [14] schemes to enable MIMO communications in ad hoc networks. However, to the best of our knowledge, there is no study on deployment of MIMO relay nodes in meshed wireless networks. The specific features of MIMO technique promise great potential to improve network performance, but also bring in new challenges in both identifying the constraints and designing proper MIMO relay placement algorithms.

III. MODEL DESCRIPTION

Deployment algorithms can be classified into two types, global and local. Based on the knowledge of complete network topology and all the flow traffic, a global scheme may achieve a better performance, but will incur a much higher cost for collecting network information and potential global network reconfiguration (i.e. change of paths). Alternatively, the *local* deployment identifies the set of broken/bottleneck links, and uses MIMO nodes to cover these links so that the broken links can be bridged and the bottleneck capacities of flows can be improved. In this paper, we focus on the local deployment for better distributed implementation.

In this work, we consider a multi-hop wireless network with two types of nodes, regular nodes (RN) with one antenna each, and MIMO relay nodes (MR) each equipped with an array of

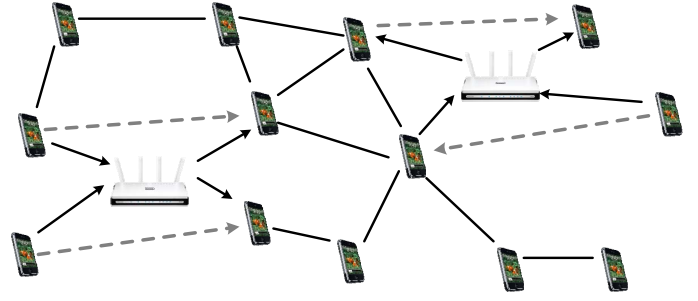


Fig. 2. Illustration of MIMO relay placement.

antennas. The locations of the regular nodes are fixed and pre-determined. We consider the problem of deploying a set of MRs to facilitate the transmissions of RNs, as illustrated in Figure 2. In addition to providing much more powerful transmission capability, MIMO nodes also have the flexibility of switching between different strategies exploiting two types of gains of MIMO, *spatial multiplexing* and *spatial diversity*. In a rich scattering environment where the transmission channels for different streams are differentiable and independent, i.e., *orthogonal*, multiple independent data streams can be transmitted between a transmission node pair through spatial multiplexing. Alternatively, different types of diversity techniques can be exploited to improve the transmission reliability or range.

The term *degree of freedom* (DoF) is widely used to describe the dimension of the space over which communication can take place. The DoF of a MIMO link is limited by the antenna array sizes of the end nodes as well as the channel conditions, and a MIMO operational mode takes up a specific number of DoFs. Generally speaking, the value of DoF for each node is close to its antenna array size in a rich scattering environment. A group of RNs can be assigned to the same MR to form a virtual MIMO node, taking advantage of multi-user MIMO [15] and spatial multiplexing to provide multiple access. Therefore, each MR can *simultaneously* assist several weak links, which not only effectively reduces the number of MRs needed but also provides a higher transmission capacity to relieve a bottleneck area. We call this strategy *MAS* for multiple access. Using this mode, an MR simultaneously receives data uplink from multiple RNs through cooperative spatial multiplexing and transmit to several RNs downlink using zero-forcing multi-user beamforming. The available DoF of the MR is thus shared by several RNs. In addition, multiple antennas can be used to improve the link capacity when an MR receives from or transmits to one RN, taking advantage of the power gain. Alternatively, we can use MIMO for an extended transmission range to achieve guaranteed reliability, denoted as mode *RANGE*, which exploits a diversity mode, i.e. uplink receiving with spatial diversity or maximal ratio combining and downlink transmission using schemes such as space-time coding and beamforming.

The *MAS* mode and the *RANGE* mode can be combined by adjusting the DoF used for each to form different transmission strategies, each with different transmission range and link capacity. For instance, an MR with 4 DoFs can assist 2 RNs

simultaneously if each of them takes 2 DoFs for *RANGE*. The strategies of MRs can be saved in a table, so that for each combination of modes and the number of DoFs to use, the corresponding rate and range can be determined easily based on the statistics of the channel and traffic conditions. As the receiving signal to noise ratio (SNR) is a function of the diversity order, the transmission power, and the channel conditions while the last one is unknown during the deployment phase, after the MRs are deployed, the power of an MR used for transmission can be adjusted to communicate with RNs at different distances.

IV. PROBLEM FORMULATION

In this section, we first introduce the notations, then identify the specific constraints for deployment, and finally mathematically formulate the problem.

A. Notations

The set of weak links in the network is denoted as \mathcal{L} and $|\mathcal{L}| = L$. For a weak link $l_j \in \mathcal{L}$, its corresponding sender and receiver nodes are denoted as $t(l_j)$ and $h(l_j)$ respectively. We consider two types of nodes in our problem, regular nodes denoted as a set \mathcal{RN} are the end nodes of links in \mathcal{L} , and MIMO nodes denoted as a set \mathcal{MR} serve as relays for the weak links. Obviously, $\mathcal{RN} = \{t(l_j), h(l_j) | l_j \in \mathcal{L}\}$. For a MIMO node $m_i \in \mathcal{MR}$, its available degree-of-freedom value is denoted as α_i and its corresponding position is denoted as p_i . A MIMO node that covers l_j is denoted as $m(l_j)$, and needs to receive data from $t(l_j)$ and forward the data to $h(l_j)$. A MIMO node m_i can choose from multiple strategies, denoted as a set \mathcal{A}_i , depending on the specific requirement of the weak link and the environmental condition. The k -th strategy in \mathcal{A}_i , i.e. $\mathcal{A}_i(k)$, is a pair $(\mathcal{A}_i^U(k), \mathcal{A}_i^D(k))$ for uplink and downlink transmissions respectively, with the corresponding receiving and transmission ranges (which could be an equivalent range based on the target received signal strength) denoted as $R_{i,k}^U$ and $R_{i,k}^D$ and the average capacity represented as $C_{i,k}^U$ and $C_{i,k}^D$ respectively. Correspondingly, it costs $O_{i,k}^U$ and $O_{i,k}^D$ DoFs for uplink and downlink transmissions respectively. In order to support the traffic demands of weak links, we denote the aggregate traffic requirement of a weak link l_j in a unit transmission period as F_j .

B. Deployment Constraints

The deployment will meet the basic traffic demands of weak links and resource constraints of MIMO relays. The sender and receiver of a weak link can be regarded as a local source and destination with the flow requirement of the aggregate link traffic. In this section, we first study the flow constraints during the packet scheduling in each time slot at the link-layer with MIMO relays, and then translate the rate variables into the deployment constraints for performance-guarantee. Although traffic of flows are not constant, and the channel conditions and thus link rate could vary, their long-term statistics are relatively stable and can serve for the resource provisioning purpose. Our deployment algorithm does not depend on specific MAC, and the per-slot traffic scheduling between weak links and MIMO

relay nodes is out of our scope. We use $x_{i,j,k}^U$ to denote the flow on the uplink transmission from $t(l_j)$ to MIMO node m_i using strategy $\mathcal{A}_i^U(k)$, and $x_{i,j,k}^D$ to denote the flow on the downlink transmission from m_i to $h(l_j)$ using strategy $\mathcal{A}_i^D(k)$.

A necessary condition to meet the traffic requirement is the existence of the link flow $\{x_{i,j,k}^U, x_{i,j,k}^D\}$ that satisfies the following *flow conservation constraints*:

$$\sum_k x_{i,j,k}^U = \sum_k x_{i,j,k}^D, \forall l_j \in \mathcal{L}, m_i = m(l_j); \quad (1)$$

$$\sum_k x_{i,j,k}^U = F_j, \forall l_j \in \mathcal{L}, m_i = m(l_j); \quad (2)$$

$$x_{i,j,k}^U = x_{i,j,k}^D = 0, \forall l_j \in \mathcal{L}, m_i \neq m(l_j); \quad (3)$$

where (1) assures that the incoming traffic equals the outgoing traffic for the link l_j which is associated with a MIMO node m_i ; and (2) guarantees that the aggregate traffic requirement of each weak link can be satisfied with the help of MIMO relay nodes. Denote the weak links that are within the range of m_i as \mathcal{L}_i , which may include the weak links that are associated with m_i , i.e. $\forall j \text{ s.t. } m(l_j) = m_i$, as well as other weak links whose transmissions interfere with the transmission or reception of the node m_i . As the *flow capacity constraint*, the total transmissions on a MIMO channel should not exceed its capacity:

$$\sum_{j:l_j \in \mathcal{L}_i} x_{i,j,k}^U \leq C_{i,k}^U, \forall \mathcal{A}_i(k) \in \mathcal{A}_i, m_i \in \mathcal{MR}; \quad (4)$$

$$\sum_{j:l_j \in \mathcal{L}_i} x_{i,j,k}^D \leq C_{i,k}^D, \forall \mathcal{A}_i(k) \in \mathcal{A}_i, m_i \in \mathcal{MR}. \quad (5)$$

While constraints (1)-(5) are conventional for flow problems, the use of MIMO technique imposes new constraints. Even though the DoF constraints are generally formulated at the MAC layer, they actually have a significant impact on resource provisioning and the deployment strategy of MIMO nodes. Let $I_{i,j,k,\tau}^U / I_{i,j,k,\tau}^D$ be the indicator variable that has value 1 if and only if $m(l_j) = m_i$ and the link from $t(l_j)$ to m_i / from m_i to $h(l_j)$ is *active* using MIMO strategy $\mathcal{A}_i^U(k) / \mathcal{A}_i^D(k)$ in the time slot τ . During the deployment phase, there is no knowledge on the group of nodes scheduled to transmit together, so the actual interference information is unknown. We assume the transmissions are over orthogonal channels and constrain the number of concurrent transmissions with the available number of DoFs, while throughput reduction due to interference beyond the transmission range or due to uncanceled interference as a result of physical decoder limit can be mitigated with a certain level of the over-provisioning of resources. To satisfy the DoF constraint at the transmitter side, the DoF used by a node m_i for all its active downlink transmissions must be no larger than its available DoF, i.e. α_i , in each time slot τ :

$$\sum_{j:l_j \in \mathcal{L}_i} \sum_k I_{i,j,k,\tau}^D O_{i,k}^D \leq \alpha_i, \forall m_i \in \mathcal{MR}. \quad (6)$$

Similarly, corresponding to the receiver's DoF constraint, the DoF number required to decode the receiving transmissions

should not exceed the receiving capability of the node:

$$\sum_{j:l_j \in \mathcal{L}_i} \sum_k I_{i,j,k,\tau}^U O_{i,k}^U \leq \alpha_i, \forall m_i \in \mathcal{MR}. \quad (7)$$

For a time period with T time slots, define $g_{i,j,k}^U = \frac{x_{i,j,k}^U}{C_{i,k}^U}$ and $g_{i,j,k}^D = \frac{x_{i,j,k}^D}{C_{i,k}^D}$ as the *utilization* of uplink and downlink channels of MIMO node m_i using strategy $\mathcal{A}_i^U(k)/\mathcal{A}_i^D(k)$ respectively. Note that we also have $g_{i,j,k}^U = \frac{1}{T} \sum_{1 \leq \tau \leq T} I_{i,j,k,\tau}^U$ and $g_{i,j,k}^D = \frac{1}{T} \sum_{1 \leq \tau \leq T} I_{i,j,k,\tau}^D$ for all l_j . Adding equations (6) and (7) over all the T time slots and dividing by T results in the constraints:

$$\sum_{j:l_j \in \mathcal{L}_i} \sum_k \frac{x_{i,j,k}^U}{C_{i,k}^U} O_{i,k}^U \leq \alpha_i, \forall m_i \in \mathcal{MR}; \quad (8)$$

$$\sum_{j:l_j \in \mathcal{L}_i} \sum_k \frac{x_{i,j,k}^D}{C_{i,k}^D} O_{i,k}^D \leq \alpha_i, \forall m_i \in \mathcal{MR}. \quad (9)$$

C. The Deployment Problem

Based on the system model and the notations, the problem is then formulated as follows.

Objective: Find a set \mathcal{MR} of MIMO relay nodes that achieves the minimum cardinality $\min |\mathcal{MR}|$ and determine the position p_i for each MIMO node $m_i \in \mathcal{MR}$ and the values of all flows $\{x_{i,j,k}^U, x_{i,j,k}^D\}$ associated with weak links, subject to the following **constraints**:

- For each weak link $l_j \in \mathcal{L}$, a MIMO node $m(l_j) = m_i$ is assigned to assist it with the strategy set \mathcal{A}_i . Specifically, $t(l_j)$ and $h(l_j)$ are within the range of $R_{i,k}^U$ and $R_{i,k}^D$ with regard to m_i , when uplink strategy $\mathcal{A}_i^U(k)$ and downlink strategy $\mathcal{A}_i^D(k)$ are used, where $(\mathcal{A}_i^U(k), \mathcal{A}_i^D(k)) \in \mathcal{A}_i$. Equations (1)-(3) are satisfied for each l_j .
- For each deployed MIMO node $m_i \in \mathcal{MR}$, equations (4), (5), (8) and (9) are satisfied, so the deployment of MIMO nodes can provide performance provisioning.

The above formulation is relatively descriptive. If the candidate positions for MIMO nodes are known and denoted as a set \mathcal{P} , the problem can be formulated more clearly as a programming problem. Let y_i be the indicator that equals 1 if a MIMO node is placed at the position $p_i \in \mathcal{P}$, otherwise $y_i = 0$. Let z_{ijk} be the indicator that equals 1 if the weak link l_j is assigned to a MIMO node at p_i using the strategy $\mathcal{A}_i^U(k)/\mathcal{A}_i^D(k)$ for uplink/downlink transmission respectively; otherwise, $z_{ijk} = 0$. The problem is then reformulated as

$$\min \sum_{i=1}^{|\mathcal{P}|} y_i, \quad (10)$$

subject to:

$$\sum_{i=1}^{|\mathcal{P}|} \sum_k z_{ijk} = 1, \forall j = 1, \dots, |\mathcal{L}|; \quad (11)$$

$$\sum_{j=1}^{|\mathcal{L}|} \sum_k z_{ijk} \frac{F_j}{C_{i,k}^U} O_{i,k}^U \leq \alpha_i, \forall i = 1, \dots, |\mathcal{P}|; \quad (12)$$

$$\sum_{j=1}^{|\mathcal{L}|} \sum_k z_{ijk} \frac{F_j}{C_{i,k}^D} O_{i,k}^D \leq \alpha_i, \forall i = 1, \dots, |\mathcal{P}|; \quad (13)$$

$$\begin{aligned} \sum_{j=1}^{|\mathcal{L}|} z_{ijk} F_j &\leq C_{i,k}^U, \sum_{j=1}^{|\mathcal{L}|} z_{ijk} F_j \leq C_{i,k}^D, \\ z_{ijk} &\leq y_i, y_i = \{0, 1\}, z_{ijk} = \{0, 1\}, \\ i &= 1, \dots, |\mathcal{P}|, j = 1, \dots, |\mathcal{L}|, k = 1, \dots, |\mathcal{A}_i|; \end{aligned} \quad (14)$$

where (11) assigns one MIMO node to a weak link obeying constraint (a) in the above problem formulation, (12)(13) reflect constraint (b) and are together called *feasibility constraints*, and (14) ensures the flow constraints as well as the correct relation between parameter y_i and z_{ijk} . Although candidate positions are assumed to be known in many literature work, they are actually quite challenging to be found.

NP-hardness: We now briefly analyze the complexity of the deployment problem with candidate positions. Consider a simplified version of our problem where each MR node works at only one mode and has unlimited capacity. It can be shown that the NP-complete Vertex Cover problem in planar graph with maximum degree 3 is polynomial-time reducible to our simplified problem. Following the first two steps of transformation as in [16], an arbitrary planar graph G_a of maximum degree 3 can be transformed in polynomial-time to the MIMO relay deployment problem, with $u_{i,l}$ in the step 2 as the candidate positions. It is then easy to verify that G_a has a vertex cover set of size N in a planar graph with degree at most 3 if and only if the deployment problem has a solution of size $N + \frac{1}{2} \sum_{e_i \in E(G_a)} (|e_i| - 1)$, where $E(G_a)$ is the edge set of graph G_a . Therefore, the MIMO relay deployment problem with candidate positions is NP-hard.

V. CENTRALIZED DEPLOYMENT

Our problem consists of two coupled subproblems: 1) where to exactly place the MRs, and 2) the assignment of weak links to the MRs with specific transmission strategies. As seen in Section IV, the optimum position of an MR depends on the links it is assigned to cover and the transmission ranges (which depend on the strategies) it uses to cover them. On the other hand, the position of an MR impacts the number of links it can cover and the transmission strategy it needs to use. Different from the literature work, e.g. [4], [5], where the candidate positions are assumed to be known, the main difficulty in finding the optimum position is that there are an infinite number of potential locations for the MIMO nodes.

In this section, we jointly consider the two subproblems. We first discuss three possible ways of narrowing down the search space of the optimum MR positions. Then based

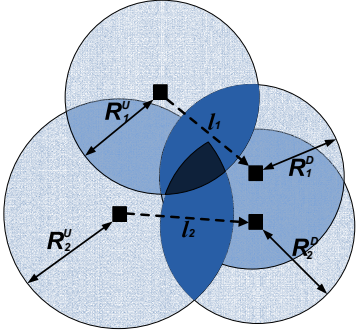


Fig. 3. Illustration of the optimum position.

on the candidate positions, we provide a polynomial time approximation scheme (PTAS) to determine the association of weak links with MRs and the transmission strategies.

A. The Candidate Positions of MIMO Nodes

We first discuss several ways of finding the candidate positions of MIMO nodes, and then compare their impact on the performance later in Section VII.

1) *Optimum positions*: The position of a MIMO node that can assist a weak link l_{j_1} with strategy k_1 should be inside the intersection area (which is a lune shape) of the circles centered at $t(l_{j_1})$ and $h(l_{j_1})$ with radiuses $R_{k_1}^U$ and $R_{k_1}^D$ respectively. Intuitively, a position at the center of the intersection area of the lune shapes formed by the weak links could be considered as an optimum position of a MIMO node to cover these weak links (Figure 3). However, the complexity is high to find the intersection of all possible lune-shapes and determine the optimum position.

2) *1-center positions*: Considering the coverage of end nodes instead of links, the candidate positions can be regarded as a subset of the candidate positions of 1-center problem. According to [8], the following fact holds for the 1-center problem.

Fact 1: The unique 1-center location of a set of nodes V , denoted as $1C(V)$, is defined by:

- 1) A pair of nodes $a, b \in V$. If this is the case, then $1C(V)$ is located at the midpoint of a, b .
- 2) A triplet of nodes $a, b, c \in V$ that form an acute triangle. If this is the case, then $1C(V)$ is located at the circumcenter of a, b, c .
- 3) A single node $a \in V$.

It is therefore reasonable to consider the candidate positions of the MIMO nodes based on the above facts.

3) *Simplified positions*: Although using the 1-center positions can significantly reduce the size of the candidate position set, searching over it may still be computationally complicated. In some circumstances, it is possible to tradeoff the accuracy with simplicity, so we can use even more simplified positions.

Vicinity Criterion: Let $\overline{(a, b)}$ be the distance between node a and b in the network. For a link l_{j_1} with the sender/receiver $t(l_{j_1})/h(l_{j_1})$ and a link l_{j_2} with the sender/receiver $t(l_{j_2})/h(l_{j_2})$, if the maximum of $\{\overline{(t(l_{j_1}), t(l_{j_2}))}, \overline{(t(l_{j_1}), h(l_{j_2}))}, \overline{(h(l_{j_1}), t(l_{j_2}))}, \overline{(h(l_{j_1}), h(l_{j_2}))}\}$ is no larger than r^* , a given parameter of the

transmission range, links l_{j_1} and l_{j_2} are considered to be in each other's vicinity under r^* .

The vicinity criterion is based on the simple fact that the distance between two links is bounded by the pairwise distance of the four endpoints. If r^* is the transmission range of an MR node, then two links that are in each other's vicinity can be guaranteed to be covered by one MR that is placed at the mid-point of any of them. Note that the distance here can be equivalent distance depending on the received signal level instead of physical distance between nodes.

B. Approximation solution

As the deployment problem is NP-hard, it is important to develop an algorithm that can provide some performance bound. In this section, we propose a polynomial time approximate solution using shifting strategy that is specifically tailored for our problem, and prove the approximate ratio through the description of the algorithm. Different from the existing work [16], [17], the problem becomes much harder given the performance provision requirements and the flexibility of MIMO modes.

We first simplify the problem formulated in (10)-(14) as follows. Assume that the candidate positions of MIMO nodes are known and denoted as a set \mathcal{P} , and the set of available operational strategies is the same for all MIMO nodes. Denote the set for all available values of the ranges as \mathcal{R} . If a MIMO node placed at position $p_i \in \mathcal{P}$, denoted as node m_i , uses the strategy k , it has the transmission range $R_k \in \mathcal{R}$ and capacity $C_k \in \mathcal{C}$. A MIMO node can activate several strategies of operation, as long as constraints (12) and (13) are satisfied.

We can then have the following theorem for our problem.

Theorem There exists a polynomial-time approximation scheme for the deployment problem with above simplification.

Proof: To approximate the deployment problem, we consider the shifting strategy [16], [17]. To facilitate finding the solution, we first introduce the following graph representation, and then present the proof along with the construction of the solution step by step.

Graph Representation. Set the maximum range $R^M = \max\{R_k\}$ of all the available MIMO modes as the distance unit in the network. Let \mathcal{R} denote the smallest rectangular region that can hold the network graph, with width \mathcal{R}_w and length \mathcal{R}_l normalized to R^M . The candidate position p_i for an MR to cover a weak link l_j using the strategy $A_i(k)$ lies in the lune region $S(j, k)$ that is the intersection of the disks centered at $t(l_j)$ and $h(l_j)$ respectively, both with radius R_k . A weak link may be covered by an MR using different transmission ranges. For the set of all the available ranges \mathcal{R} , $|\mathcal{R}|$ layers of planes can be constructed (Figure 4), where the lune shapes in each layer is formed with a specific $R_k \in \mathcal{R}$. Therefore, the deployment problem can be considered as selecting the minimum size of position set from \mathcal{P} so that each weak link is covered at least once by positions from the $|\mathcal{R}|$ layers and the feasibility constraints in (8) and (9) of MIMO nodes placed at the selected positions can be satisfied.

Shifting Strategy. For a positive integer $m > 0$, consider any even integers a, b satisfying $2 \leq a, b \leq m$, with the value of m

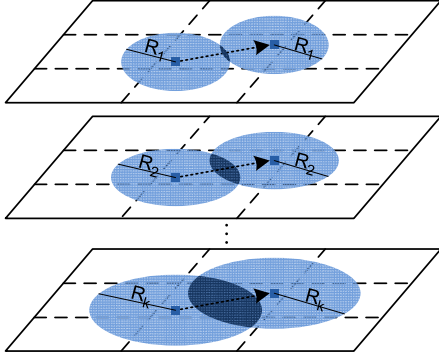


Fig. 4. The layered graph construction.

given at the end of the proof. We partition the region \mathcal{R} into $m \times m$ squares by horizontal lines at $a + k_1 \cdot m$ and vertical lines at $b + k_2 \cdot m$, where k_1 and k_2 are selected as all the possible non-negative integers such that $a + k_1 \cdot m \leq \mathcal{R}_w$ and $b + k_2 \cdot m \leq \mathcal{R}_l$. By exhaustively varying the possible values of a and b , the division of squares is shifted over the plane. Let $S_{a,b}$ denote the set of squares for a fixed pair a, b . A lune is said to belong to a square if and only if its geometric center lies in the square. For a square $s \in S_{a,b}$, let $L(s)$ denote the set of lunes belonging to the square s . The radius of a lune is at most 1 unit and the range is the same for both uplink and downlink. For different layers, the squares actually associate with a set of lunes formed by the same end points but the lunes have different sizes corresponding to different strategies of the MR nodes, as shown in Figure 4.

Local Optimum. We assume the node density of the network has an upper bound, so that for each $m \times m$ square in $S_{a,b}$, the number of available candidate positions is bounded by $O(m^2)$. Therefore, the optimum solution within a square can be found through exhaustive search over the square area on all the $|\mathcal{R}|$ layers in polynomial time using complete enumeration of all possible positions for the given constant m and $|\mathcal{R}|$, while guaranteeing that each MIMO node placed at a selected position satisfies the feasibility constraints (8) and (9). The union of positions selected from all squares in $S_{a,b}$ gives a candidate solution for a given pair of a, b , and the candidate solution for each pair of a, b can be obtained through the shifting strategy. Among all the candidate solutions, the one with the minimum cardinality is considered as our solution, denoted as a set H .

Approximation. Now we analyze the approximation ratio. Let H_o be the optimal solution, and H be the solution obtained by the shifting strategy. For a given pair of a, b , let $H_o(a, *)$, $H_o(*, b)$, $H_o(a, b)$ respectively be the vertices set in H_o that lie in lunes intersecting horizontal lines, vertical lines, and both horizontal and vertical lines. Let $H_o(s)$ be vertices in $H_o \cap L(s)$, $OPT(s)$ be the optimum MIMO nodes set to cover the weak links represented by the lunes in $L(s)$. We have

$$|H| \leq \left| \bigcup_{s \in S_{a,b}} OPT(s) \right| \leq \sum_{s \in S_{a,b}} |OPT(s)| \leq \sum_{s \in S_{a,b}} |H_o(s)|. \quad (15)$$

Note that based on the definition of the distance unit, positions in lunes that cross an active division line can be used in at

most 4 squares. Therefore,

$$\sum_{s \in S_{a,b}} |H_o(s)| \leq |H_o(a, *)| + |H_o(*, b)| + |H_o(a, b)| + |H_o|. \quad (16)$$

As the shifting step is set to be two units, all lunes that cross one horizontal(or vertical) line do not intersect with lunes that cross another horizontal(or vertical) line, regardless of their radii. Hence, a position is counted at most once in $H_o(a, *)$ with a changing as well as in $H_o(*, b)$ with b changing. Consequently, we obtain the following inequalities, $\sum_{2 \leq a \leq m} |H_o(a, *)| \leq |H_o|$, $\sum_{2 \leq b \leq m} |H_o(*, b)| \leq |H_o|$. We can find a pair of (a, b) , such that

$$|H_o(a, *)| \leq \frac{2}{m}|H_o|, |H_o(*, b)| \leq \frac{2}{m}|H_o|. \quad (17)$$

Therefore,

$$\begin{aligned} |H_o(a, b)| &= |H_o(a, *) \cap H_o(*, b)| \\ &\leq \min\{|H_o(a, *)|, |H_o(*, b)|\} \leq \frac{2}{m}|H_o|. \end{aligned} \quad (18)$$

Combining inequalities (15)-(18), we finally have

$$|H| \leq \left(\frac{2}{m} + \frac{2}{m} + \frac{2}{m} + 1 \right) |H_o| = \left(1 + \frac{6}{m} \right) |H_o|. \quad (19)$$

As a result, given any $\epsilon > 0$, let $m > 0$ be the smallest even integer such that $(6/m) \leq \epsilon$ and find a solution H using the shifting strategy with a specific value m , the solution H thus can achieve $1 + \epsilon$ approximation ratio. \square

To search for the local optimum for each pair of a, b , $O(|\mathcal{R}|m^2)$ candidate positions need to be examined and the number of shifts is bounded by $m^2/4$. Considering $(6/m) \leq \epsilon$, the time complexity of the shifting algorithm is bounded by $O(|\mathcal{R}|(1/\epsilon)^4)$.

VI. DISTRIBUTED DEPLOYMENT

As there are no actual nodes available at the candidate positions and the existing nodes themselves may not be able to communicate due to the weak links, we resort to mobile agents to determine the final deployment positions. A mobile agent can move to the candidate positions, cooperates and communicates with other mobile agents when they are within each other's transmission range as well as communicates with RN nodes closed by using all the available MIMO modes. Mobile agents collect the topology information of the network and traffic requirements of weak links independently, and each agent coordinates with its neighbors in reaching a consensus on how many MRs are required and the deployment strategy. The communications between mobile agents are in an ad hoc manner, and there is no central controller to manage them. The details of information acquisition and communications process are beyond the scope of this paper.

Generally, there are two tasks for the agents. One is to find out where the weak links and the candidate locations of MRs are, and the other is to decide where to actually place the MR nodes and how to assign weak links to the MR nodes using appropriate transmission strategy. The agents exhaustively go through the whole area twice to accomplish the tasks. First,

Algorithm 1 Distributed Deployment (for each mobile agent n in each move)

- 1: **Input:** The set of uncovered weak links \mathcal{L} in the network and the candidate MIMO node positions p_i assigned for n in the current move
 - 2: **Output:** Decide if p_i is selected as a position for an MR node, if yes, also determine the association of weak links $\{z_{ijk}\}$
 - 3: Initialize: INPROCESS = 1, $\tilde{\alpha}_i = \alpha_i$, $k = 1$
 - 4: **while** INPROCESS **do**
 - 5: $\mathcal{L}' = \emptyset$, $\mathcal{L}'' = \emptyset$.
 - 6: Use range R_k to communicate with the end nodes of weak links in \mathcal{L} , find the subset \mathcal{L}' it can cover; count the covering degree for an MR placed at p_i , i.e., m_i .
 - 7: Share the information of covering degree of p_i with the 2-hop neighboring agents if p_i is not selected.
 - 8: **if** p_i is the position with the highest covering degree among active positions within 2-hop neighborhood **then**
 - 9: Mark p_i as selected if it hasn't been marked.
 - 10: **end if**
 - 11: **if** p_i is selected and the covering degree is not zero **then**
 - 12: **if** the feasibility constraints of m_i are satisfied when covering all the weak links in \mathcal{L}' **then**
 - 13: Send out \mathcal{L}' , and $\mathcal{L} = \mathcal{L} - \mathcal{L}'$. Update $\tilde{\alpha}_i$.
 - 14: If $\tilde{\alpha}_i > 0$ and k is not the maximum, let $k = k + 1$; otherwise, send out END to other agents, and INPROCESS = 0.
 - 15: **else**
 - 16: Tentatively add in \mathcal{L}'' the weak links of \mathcal{L}' in the ascending order of their covered degrees, until the constraints of m_i cannot be satisfied.
 - 17: Send out \mathcal{L}'' , $\mathcal{L} = \mathcal{L} - \mathcal{L}''$. Also send out END to other agents. INPROCESS = 0.
 - 18: **end if**
 - 19: **else**
 - 20: Listen to other nodes' messages, update \mathcal{L} and the covering degree of weak links in \mathcal{L} .
 - 21: **if** the covering degree is 0 **then**
 - 22: If k is the maximum value, INPROCESS = 0; else $k = k + 1$.
 - 23: **end if**
 - 24: **end if**
 - 25: **end while**
-

given the number of agents, the area of the network is divided into equal-size stripes. Each agent goes over its assigned stripe to "meet" with each RN within the stripe to collect the information of weak links and aggregate traffic requirements. It can determine the candidate MR positions within the stripe based on the network topology and the collected information. In the second time, all agents move together and stay close as a group, collaboratively checking the candidate positions and determining the placement of MRs distributively, as in

Algorithm 1.

Two concepts are used here, *covering degree* of a MIMO position i is used to describe the number of weak links that a MIMO node at position i can cover, and *covered degree* of a weak link l_j is used to describe how many MIMO positions can cover l_j . Obviously, a MIMO position with higher covering degree could potentially cover more weak links, and a weak link with higher covered degree has relatively more options of MIMO positions to cover it.

In this algorithm, each agent is assigned a sequence of candidate positions and will check one of the positions p_i in each move to determine if it can be selected to deploy an MR. The parameter INPROCESS is used to indicate if the current move is completed for agent n , and $\tilde{\alpha}_i$ is the remaining DoF that can be used for the potential MR at position p_i , i.e., a potential m_i . The weak links that can be covered by m_i is determined starting from its lowest transmission range, until all its DoF is used up. An agent needs to communicate with end RN nodes of weak links to make sure it can reach them and also determine its covering degree of weak links, and share the information with its two-hop neighboring agents (which potentially cover some common weak links), as in lines 6 and 7. Generally, the positions with a higher covering degree are favored, as they potentially cover more weak links. If the feasibility constraints are satisfied for covering the weak links in its range and the remaining DoF is larger than zero, the remaining DoF of m_i is used to cover more weak links and will be preferably used; otherwise, the weak links with lower covered degrees are preferably covered. When an agent at a selected position finishes determining all possible weak links to cover, it will send an END message to inform the neighboring agents, and the ones which also have this position in their assigned candidate list learn that this position is no longer active. These are shown in lines 12-18. In lines 20-23, neighboring agents cooperatively determine the selection of MR positions and the association of weak links with appropriate strategies. When the process is completed, if the adjacent candidate positions of p_i have all been explored or the DoF of m_i is used up, agent n should move to the next position; otherwise, it should stay in the current position, as it is on the boundary of the current area and could use its remaining degree to serve some weak links of the next area to explore.

VII. PERFORMANCE EVALUATION

We evaluate the performance of the proposed algorithms through simulations. We implemented the approximate algorithm and the distributed algorithm proposed in Sections V-B and VI, as well as a reference scheme where the relay nodes are only equipped with one antenna. The performance metric is the objective of our proposed algorithms, namely the minimum number of relay nodes to fully cover the weak links in the network while satisfying the aggregate traffic requirement of each weak link.

The node locations are generated within a 1500×1500 area according to two-dimensional uniform random distribution. The regular nodes (RN) are assumed to have the same range.

Suppose that the ratio of weak links over all the connected links is γ and the weak links are randomly distributed. Each MIMO relay (MR) node is equipped with an array of antennas to facilitate MIMO transmission. For an MR node m_i with α_i available DoFs, we consider up to $2\alpha_i$ transmission strategies available to take advantage of the *MAS* and *RANGE* modes discussed in Section III, each with empirical parameters of capacity, range and the required value of DoF. For simplicity, all weak links are assumed to have the same traffic requirement, whose value is normalized to the capacity of MIMO-channel with the degree-of-freedom 1. The default values of γ and α_i are 0.5 and 4 respectively, and the network has 150 nodes if not otherwise specified. The parameter ϵ for the approximate algorithm is set as 3 so the PTAS has approximation ratio of 4. For each simulation setup we take 100 runs and the average result is reported.

In Fig. 5, we first study the impact of candidate node positions as discussed in Section V-A. As the detection of optimum positions is coupled with the selection of MR strategies and thus is prohibitive to track, we compare the 1-center positions (type 1) and the simplified positions (type 2), with two types of grid-based positions, sparsely and densely distributed with distances between adjacent nodes to be 300 (type 3) and 50 (type 4) respectively. The node density are set to 60 and 300 to represent a sparse network and dense network respectively. It can be seen that for all the three proposed algorithms, position type 1 and 2 achieve very close number of MR nodes in both network scenarios, which indicates that it is sufficient to use type 2 positions to reduce the deployment complexity while maintaining the accuracy of candidate positions. Both type 1 and 2 perform better than the grid positions, especially type 3, as it does not take the actual positions of weak links into consideration and also does not provide fine-grained candidate positions. Moreover, type 3 positions have the chance of failing to cover all the weak links, as sparsely distributed MRs may not be able to reach the end nodes of some weak links. The densely distributed grid positions are seen to have the similar performance as that of type 1 and 2 at the cost of much higher computational complexity, as the number of the type 4 positions can be quite large especially in a dense network.

We then evaluate the performance of our algorithms using the type 2 candidate positions. We compare our distributed heuristic algorithm with the approximate algorithm with approximation ratio 4, as well as a centralized reference scheme where the relay nodes are only equipped with an antenna each. Both the node density and the ratio of weak links can impact the density of weak links in the network, as in Fig. 6(a) and (b). By employing MIMO nodes as relays, where the multiplexing and diversity features are exploited, our proposed algorithms require only up to 1/3 the number of relay nodes compared with the single antenna relays. Note that the reduction of the relay number cannot be the same as the size of the antenna array, as all the antennas from the same node are constrained to the same location. With the increased aggregate traffic requirements of weak links, as in Fig. 6(c), 30% more relay nodes are required for single antenna case

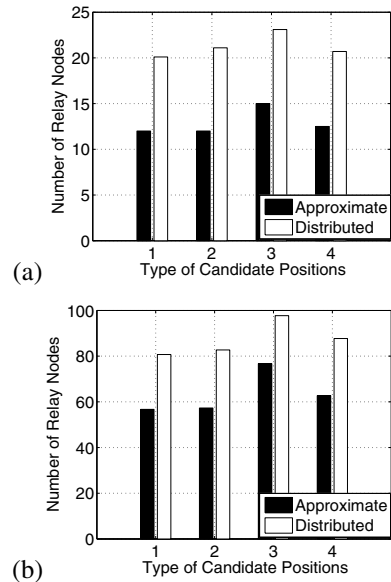


Fig. 5. Impact of the candidate positions: (a) in sparse network; (b) in dense network.

while only 16% more is needed for MIMO relay nodes, as our algorithms enable the MR nodes to flexibly select the appropriate transmission strategies based on the deployment conditions and better satisfy the traffic requirements of weak links. In Fig. 6(d), we show the performance when the DoF value of MR nodes are varied. A larger value of DoF brings the opportunity for more options of available strategies, and also provides the potential of higher capacity and longer range. Thus the number of MRs is significantly reduced. The distributed algorithm requires 61% more MR nodes compared with the approximate algorithm, however, it still significantly outperforms the single-antenna case with 67% fewer relay nodes.

As discussed in Section I, simply increasing the number of single-antenna relays cannot substitute MIMO relays due to interference. As the traffic requirements increase, there is a possibility that some of the weak links can not be successfully covered to provide performance provisioning for them. A new metric called Fail-to-Cover Ratio is introduced to denote the ratio of the links that cannot be covered over all the weak links. In Fig 7, MIMO relays achieves up to 86% lower Fail-to-Cover Ratio compared with single-antenna relays, thanks to the higher capacity of MIMO with exploration of spatial DoFs. This demonstrates that the deployment of MIMO relay is necessary in cases where single-antenna relays could not provide the sufficient coverage, and our deployment schemes are effective in achieving the coverage especially for relieving a traffic bottleneck.

VIII. CONCLUSIONS

In this paper, we propose MIMO-relay deployment algorithms which exploit MIMO features to flexibly select among various possible transmission strategies based on network conditions to effectively bridge weak links and provide performance provisioning. We first present constraints that capture

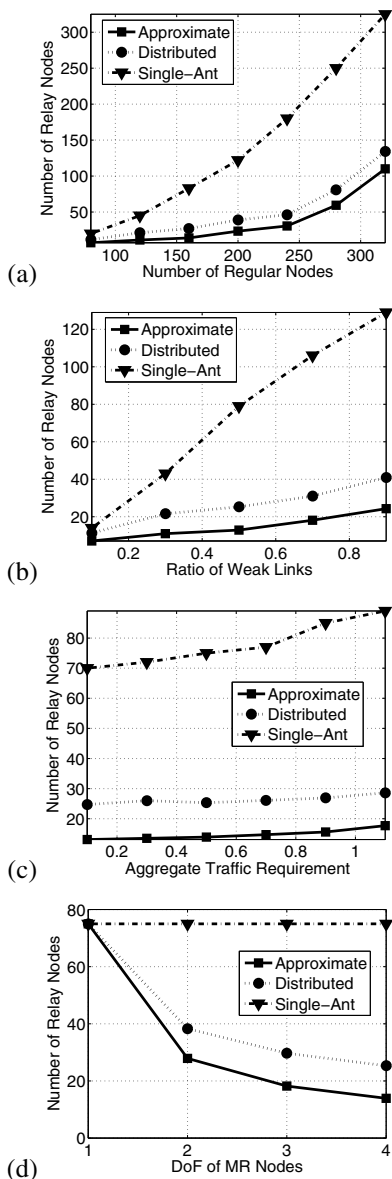


Fig. 6. Performance of the algorithms: (a) impact of the node density, (b) impact of the ratio of weak links, (c) impact of the traffic requirement, (d) impact of the DoF value.

the characteristics of transmissions between MIMO relays and regular nodes, and mathematically formulate the MIMO relay deployment problem with the objective of minimizing the number of relays while satisfying the transmission requirement of each weak link. We then propose a polynomial-time approximation scheme (PTAS) to provide a performance upper bound in the centralized scenario, as well as a distributed algorithm that can facilitate practical in-field implementation. The performance of our algorithms is evaluated through simulations with varied node density, percentage of weak links, aggregate traffic requirement and DoF value of MR nodes. The results demonstrate that MIMO relays can more effectively assist weak links, especially for relieving traffic bottlenecks, and the proposed heuristic algorithm achieves very close performance compared with the upper bound provided by the PTAS.

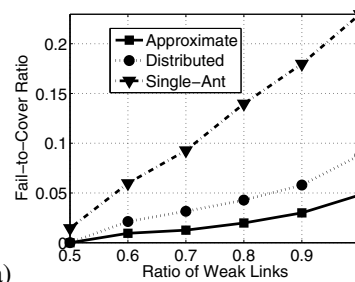


Fig. 7. The Fail-to-Cover Ratio for different traffic requirements.

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