

A review of new fundamental principles in exact topology optimization

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Abstract

After reviewing briefly the history of exact topology optimization of structures, a number of fundamental principles for deriving new optimal structural layouts will be presented. These also throw some light on general properties of optimal topologies.

Keywords: topology optimization, trusses, optimality criteria, non-uniqueness, symmetry

1. Introduction

In this lecture new fundamental principles in topology optimization will be reviewed, which are suitable for deriving new exact optimal layouts for structures.

2. Early history of structural topology optimization

Optimality criteria and some examples of least-weight truss layouts were given by the ingenious Australian inventor, Michell [9]. This milestone paper was ignored for about fifty years, after which some researchers in the UK, in particular Hemp [3] derived new solutions for Michell's criteria.

The first general theory of structural topology optimization was presented in the seventies (Prager and Rozvany [10], Rozvany [12]) under the term 'optimal layout theory', which was applied to trusses, grillages, beam grids and perforated plates.

For trusses with a stress constraint and a single load condition, the specific cost function is

$$A = k|F|, \quad k = 1/\sigma_p, \quad (1)$$

where A is the member cross-sectional area, F is the member force, k is a constant, and σ_p is the constant permissible stress.

The known optimality conditions (Michell [9], Prager and Rozvany [10]) for the above problem are

$$\bar{\varepsilon} = k \operatorname{sgn} F \quad (\text{for } F \neq 0), \quad |\bar{\varepsilon}| \leq k \quad (\text{for } F = 0) \quad (2)$$

For the considered class of problems, the adjoint strains $\bar{\varepsilon}$ are given by the subgradient of the specific cost function with respect to the member force F . The adjoint strain field must be kinematically admissible (satisfying kinematic support and continuity conditions).

The optimal adjoint strain fields for plane Michell trusses may consist of the following types of regions (e.g. Prager and Rozvany [10]):

T-region with a tensile and a compression member at right angles, $\bar{\varepsilon}_1 = -\bar{\varepsilon}_2 = k$,

S-region with members having forces of the same sign in any direction, $\bar{\varepsilon}_1 = \bar{\varepsilon}_2$, $|\varepsilon_i| = k$ ($i = 1, 2$),

R-regions with only one member at any point, $|\bar{\varepsilon}_1| = k$, $|\bar{\varepsilon}_2| \leq k$,

or

O-region with no members $|\bar{\varepsilon}_1| \leq k$, $|\bar{\varepsilon}_2| \leq k$,

where subscripts 1 and 2 indicate to principal strains. The same types of optimal regions can be used for least-weight grillages.

3. Developments in the nineties and later

The optimal theory has also been extended to line supports (Rozvany and Gollub [18], Rozvany et al. [20]), multiple load conditions and displacement constraints (Rozvany [13], Rozvany et al. [23]), and layouts with pre-existing members (Rozvany et al. [22]).

The author (Rozvany [14]) (i) pointed out an error in Michell's [9] optimality criteria for unequal permissible stresses in tension and compression, (ii) found the error in Michell's proof, (iii) defined the problem class for which the original Michell criteria are valid, and (iv) presented a simple example showing that the new criteria give a lower structural weight. Earlier Hemp [3] stated the correct optimality criteria, but did not apply these to problems with unequal stresses. A complete theory for the amended optimality criteria with many examples was developed later by Graczykowski and Lewiński [1,2].

The author (Rozvany [15]) has also shown that Hemp's [3] orthogonality conditions do not apply along boundaries between two R-regions.

4. Powerful new general principles for deriving new optimal topologies

Although exact optimal grillage topologies are known for almost all possible load and support conditions (e. g. Prager and Rozvany [11]), optimal truss topologies are only available for a relatively few cases, although Lewiński et al. [7], and Lewiński and Rozvany [4-6] derived new solutions for popular benchmark problems and Sokol and Lewiński [24] solved some important problems. For this reason, principles for facilitating the derivation of new topologies are important.

4.1. Non-uniqueness, symmetry and skew-symmetry principles

A number of the above principles are outlined in a yet unpublished article (Rozvany [16]). These will be discussed in greater detail in the lecture.

4.2. Domain augmentation and reduction principles

These are also very powerful in deriving new topologies and will be examined in the presentation, see also Rozvany [17].

4.3. The problem of subdomains without members, application of R- and O-regions

It was pointed out by Melchers [8], that no adjoint field is known for subdomains without members. It has been found recently that these subdomains can be filled in optimally by using a combination of R-, O- and T-regions.

4.4. Application of nonorthogonal layouts

As mentioned, it has been shown that non-orthogonal layouts can be optimal under certain circumstances. This will be demonstrated further with important applications.

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