Topological structure vulnerability assessment of Shanghai Urban Metro Networks

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ABSTRACT

Topological structure vulnerability assessment approach for Urban Metro Networks (UMNS) was proposed in order to decrease the impact caused by incidents. Failure scale of stations and sections random failure and target attacks was evaluated. The results show that UMNS is more vulnerable to target attacks on stations than random failure on stations. But UMNS is less vulnerable to target attacks on sections than random failure on sections. Additionally, UMNS is more vulnerable to station failure than sections. It could be concluded as more resources should be put on big transfer stations in UMNS operation management to avoid large scale impacts. The proposed methodology is not intended to predict the occurrence of events but rather to be used a management tool. Results from the evaluation are valuable elements in planning UMNS. They can be used for network planning, further detailed hazard studies, deciding on the arrangement of emergency resources.

Keywords

Urban metro networks, vulnerability, robustness, target attack, random failure.

INTRODUCTION

UMNS as a networked operation system, once random failure or target attacks happened on one station, passengers would be delayed not only in this station but also in the interrelated stations too. Therefore, an overall and methodological approach for assessment of UMNS vulnerability would provide strategies to find out the critical stations and sections which can finally could reduce the loss in economic. Additionally, the analysis on structural vulnerability of UMNS will be helpful to subway network design.

Vulnerability is an inherit property of system which is determined by the topological structure of the UMNS and can be amplified by the passenger behaviors. UMNS vulnerability refers to the small disturbance in the key parts of the system, can cause great damage and even cause the system to crash. Therefore, this paper only studies the vulnerability of UMNS topology, regardless of the external factors, such as passenger behaviors. The domestic and foreign scholars made a valuable research in this area. The biggest connected component (Zhang, 2012; Wang, 2008; Angeloudis and Fisk, 2006; Han and Liu, 2009), path length, diameter of network, network efficiency (Zhang, 2012; Wang, 2008; Angeloudis and Fisk, 2006; Derrible and Kennedy, 2010; Latora and Marchiori, 2001) based on the average shortest path length (Li and Ma, 2009), and network redundancy indicators are used for UMNS vulnerability evaluation. Previous research found that number of transfer is an important indicator considered by managers in UMNS (Wang, Chan, Yuan, Xia, Skitomore and Li, 2015), however, which cannot be reflected by the existing indicators. Additionally, not only the biggest connected component can be operated after disruption, but also the other side of the degraded station can also be operated by changing direction of train. As the aim of this paper is to propose a methodology which can be used evaluate how the metro network would be affected when station and section damaged, so those probabilistic approaches like event tree analysis or FMEA are not considered here.

Based on the above shortages, writers plan to propose new indicators for vulnerability evaluation of topological structure. Largest number of connected Original and Destination stations (OD pair), network efficiency based on transfer, network efficiency based on path length indicators are proposed for vulnerability evaluation of UMNS topological structure.

METHODOLOGY

In this work, we attempt to use complexity science to perform a qualitative assessment of the topological vulnerability of one of the largest urban metro network systems in China. A system is said to be vulnerable if its functioning can be significantly reduced by intentional or non-intentional means. Thus, in order to assess a systems' vulnerability, we need to evaluate its functioning, to design failure scenarios, and to examine its response to failures.

Important indicator of station and section

The UMNS is assumed to a network: G=G (V, E) with V vertices and E arcs. Since we treat the UMNS as a network, closing a station or section corresponds to removing the segments or reducing the train speed passing through the segments. In this work, only segment removal is considered. From the topology point of view, the amount of damage caused by the removal of one particular segment depends on the position of that segment in the network, on its degree, on its values of centrality properties and so on important indicators. Therefore, three important indicators which are line degree of station, path distance based betweenness, transfer based betweenness, are used here.

Line degree

Line degree is defined as the number of lines passing through a station. The more the lines passing through a station, more important the station is.

Betweenness

Betweenness is used to represent the proportion that the number of shortest path passing through the station or section. Betweenness can be used to describe the importance of stations and sections in information transfer process. Once the station cannot service normally, all the paths passing through it would be affected. In UMNS, length of information transfer is the distance between two stations. The distance is determined by the number of sections between an OD pair, defined as path distance based betweenness and the number of transfer time between an OD pair, defined as transfer number based betweenness.

Path distance based station betweenness $C_B^P(v)$ is formulated as following (Tang and Wang, 2008):

$$C_B^P(v) = \sum_{s \neq t \neq v} \frac{\partial_{st}(v)}{\partial_{st}} \qquad (1)$$

Where ∂_{st} is the number of shortest path which contains minimum sections from station *s* to station *t* and $\partial_{st}(v)$ is the number of shortest path which contain minimum sections passing through station *v* from station *s* to station *t*. Value of path distance based station betweenness for the sample metro network shown in Figure 1 is calculated in Table 1.

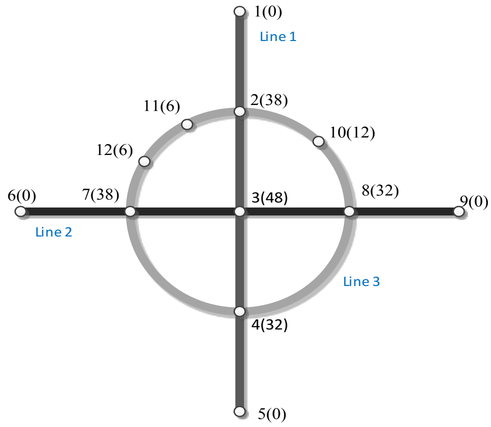


Figure 1. Sample metro network

Where the spot respects to subway station, the line between two spots respects to subway section. In the sample metro network, there are three lines and 12 stations. Value of path distance based station betweenness is calculated in Table 1. Similarly, path distance based section betweenness ($C_B^P(e)$) can also be formulated.

Station number	1	2	3	4	5	6	7	8	9	10	11	12
$C_B^P(v)$	0	38	48	32	0	0	38	32	0	12	6	6

Table 1. Path distance based betweenness of stations $C_B^P(v)$ in sample metro network

Transfer number based betweenness of station is formulated as:

$$C_B{}^T(v) = \sum_{s \neq t \neq v} \frac{\partial_{st}{}^T(v)}{\partial_{st}{}^T} \tag{2}$$

Where $\partial_{st}{}^T$ is the number of shortest path which contains minimum transfer stations from station *s* to station *t* and $\partial_{st}{}^T(v)$ is the number of shortest path which contains minimum transfer stations passing through station *v* from station *s* to station *t*. Similarly, transfer-based section betweenness $C_B{}^T(\mathrm{e})$ can be formulated.

Range of the betweenness of station and section is both [0, 1]. The betweenness of station v equals to 0 when there is no shortest path passing through it. The betweenness of station v equals to 1 when all shortest path passing through it. For example, betweenness of central station in a star network equals to 1 (De Nooy, Mrvar and Batagelj, 2011).

Modeling failure scenarios

Once the importance of every station and section has been calculated, failure scenarios can be designed accordingly. A failure scenario is defined as a set of failed stations and sections, which causes the loss of function of the UMNS. A failure scenario *F* is described by the set $F=\{k_1, k_2, \ldots, k_s\}$, where the components $k_1, k_2, \ldots, k_s$ are the components disabled from the network. There are seven failure scenarios are designed as following:

F_1: randomly remove one or a proportion stations in each time step;

F_2: randomly remove one or a proportion sections in each time step;

F_3: remove the station with the highest line degree among all station in each time step;

F_4: remove the station with highest path distance based betweenness among all stations in each time step;

F_5: remove the section with highest path distance based betweenness among all section in each time step;

F_6: remove the station with highest transfer number based betweenness among all stations in each time step;

F_7: remove the section with highest transfer number based betweenness among all section in each time step;

Network-failure scenario analysis

Once a specific or multiple-station or section failure, the effects of a particular failure over the whole network has to be considered. Therefore, the network function index (N_F) is proposed to measure the connectivity change of the whole network for a given failure scenario F. The comparison between the connectivity indexes for optimum operation conditions and the operation condition for a particular failure scenario is made through the maximum connective OD pair ratio $N_F(W)$, path distance based efficiency $N_F(E^P)$, and transfer based efficiency $N_F(E^T)$.

Maximum connective OD pair ratio $N_F(W)$ is used to understand how connectivity the network decreases as one or more stations or sections fail, formulated as:

$$N_F(W) = \frac{W_F}{W_0} \tag{3}$$

where W_0 is the connectivity of the network under optimum operation condition and W_F is the connectivity of the network under a particular failure scenario *F*. The maximum connective OD pair ratio $N_F(W)$ values will range between 0 and 1. A value closer to 0 corresponds to a dramatic reduction of the connectivity of the network under failure scenario *F*, while a value closer to 1 indicates minor connectivity changes. Thus, we define the decrease of the total connectivity $\Delta N_F(W)$ under the failure scenario *F* as the topological vulnerability indicator L, formulated as:

$$L = \Delta N_F(W) = 1 - (\frac{W_F}{W_0}) \tag{4}$$

The path distance based topological efficiency of the network E^P provides an estimated of the average efficiency with which the network ensures that all nodes are reachable (Latora and Marchiori, 2001), formulated as:

$$E^P = \frac{1}{N(N-1)} \sum_{1 \neq j} \frac{1}{a_{ij}} \tag{5}$$

where a_{ij} is the shortest path which is determined by the number of sections between station *i* and *j*. This definition is extremely general and holds also for non-connected networks where no path connects station *i* and *j*, $a_{ij} \to \infty$. Similarly, we define the decrease of path distance based efficiency $\Delta N_F(E^P)$ under the failure scenario *F* as the second topological vulnerability indicator L and expressed as,

$$L = \Delta N_F(E^P) = 1 - (\frac{E_F^P}{E_0^P}) \tag{6}$$

where E_0^P is the path distance based efficiency of the whole network under optimum condition and E_F^P is the path distance based efficiency of the whole network under a particular failure scenario *F*.

The third indicator used for topological vulnerability evaluation is transfer based efficiency. The distance between station *i* to *j* is considered as transfer time and can be expressed as,

$$E^T = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{\delta_{ij} + 1} \qquad (7)$$

where δ_{ij} is the shortest path between station *i* and *j* quantitated by the transfer time. However, when there is no need for a passenger transfers from one line to another, δ_{ij} equals to 0, the inverse of it is meaningless. Therefore, transfer time pluses one is used as denominator. Similarly, we define the decrease of the transfer based efficiency $\Delta N_F(E^T)$ under the failure scenario *F* as the third topological vulnerability indicator *L* and expressed as,

$$L = \Delta N_F(E^T) = 1 - (\frac{E_F^T}{E_0^T}) \qquad (8)$$

where E_0^T is the transfer based efficiency of the whole network under optimum condition and E_F^T is the transfer based efficiency of the whole network under a particular failure scenario *F*.

DATA SET AND PRETREATMENT

The Shanghai metro network contains 242 metro station, 536 sections, and 14 topological lines as shown in Figure 2. This network is frequently interrupted by the occurrence of passengers' interruption and mechanical failure of train, power system, communication system etc. in stations and sections. Therefore, seven failure scenarios randomly and deliberately happen on station and section are all considered to evaluate the vulnerability of the UMNS.

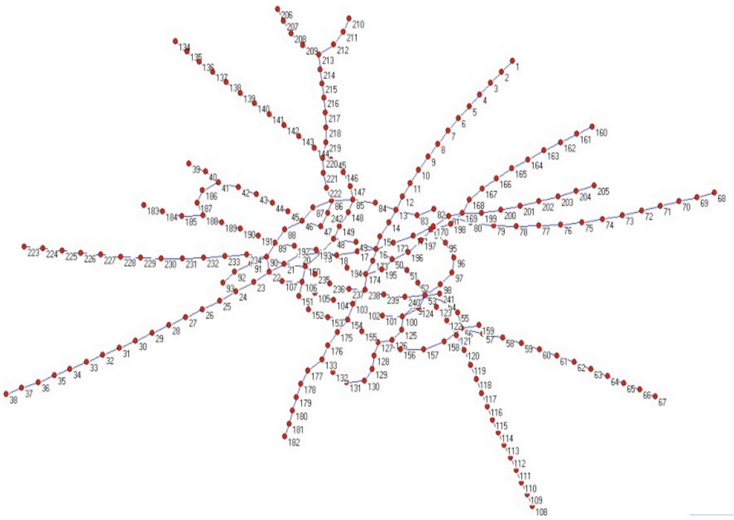


Figure 2. Topological structure of Shanghai metro network

Important indicator of station and section in Shanghai Metro network

In order to select out the more important stations in the network, stations were ordered according to line degree, path distance based betweenness, and transfer based betweenness. The top 10 important stations were listed in Table 2.

Order	Line degree		Path distance based betweenness		Transfer based betweenness	
	L	No.	$C_B^P(v)$	No.	$C_B{}^T(v)$	No.
1	8	53	0.3988	16	0.386892	16
2	6	16	0.307778	53	0.334306	53
3	6	80	0.237908	48	0.236082	48
4	6	90	0.226856	50	0.220944	50
5	4	13	0.200817	81	0.212584	19
6	4	18	0.198765	86	0.20157	22
7	4	19	0.181349	85	0.201438	21
8	4	21	0.171088	49	0.194886	90
9	4	22	0.170843	238	0.194472	238
10	4	24	0.169582	51	0.193493	86

Table 2. Top ten metro station with highest L, $C_B^P(v)$ and $C_B{}^T(v)$ in Shanghai metro network

In order to select out the more important sections in the network, sections were ordered according to path distance based betweenness and transfer based betweenness. The top 10 important sections were listed in Table 3.

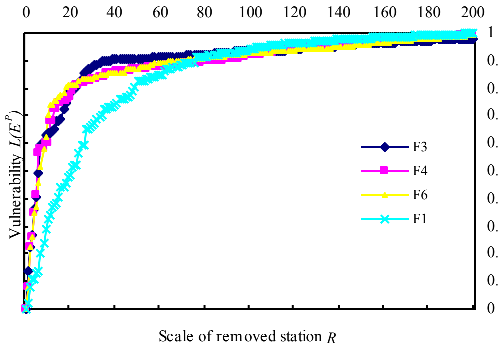
Order	Path distance based betweenness		Transfer based betweenness	
	$C_B^P(e)$	No.	$C_B^T(e)$	No.
1	0.093875	16_50	0.081468	50_51
2	0.093499	50_16	0.079773	51_52
3	0.090053	49_16	0.078757	16_50
4	0.089432	48_49	0.07855	51_50
5	0.089018	50_51	0.078267	16_172
6	0.087512	51_50	0.078211	52_53
7	0.086928	16_172	0.077062	52_51
8	0.086608	51_52	0.07629	50_16
9	0.086194	16_49	0.075499	53_52
10	0.085572	49_48	0.075142	172_171

Table 3. Top ten sections with highest $C_B^P(e)$ and $C_B^T(e)$ in the metro network

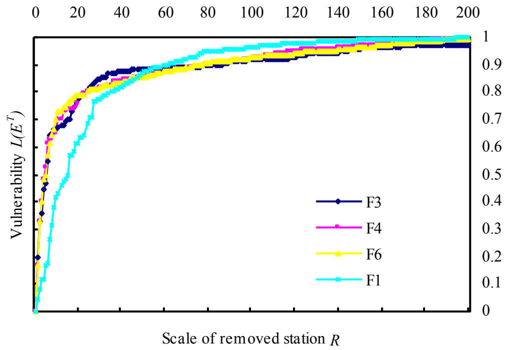
RESULTS AND DISCUSSION

Vulnerability evaluation under station failure scenario

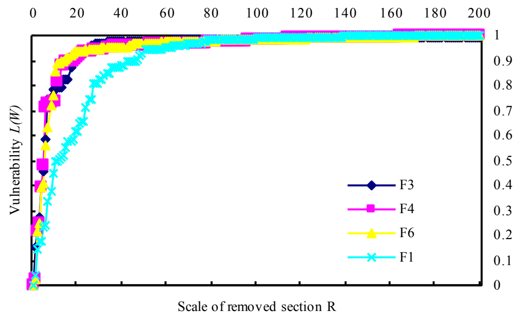
Under F_1, station was randomly removed in each time step; under F_3, station with the highest line degree was removed in each time step; under F_4, station with the highest path distance based betweenness was removed in each time step; under F_6, station with the highest transfer based betweenness was removed in each time step. The relationship between the decrease in function of three topological vulnerability indicators *L* and scale of removed stations *R* is shown in Figure 3.



(a) Relationship between vulnerability $L(E^P)$ and scale of removed station R



(b) Relationship between vulnerability $L(E^T)$ and scale of removed station R



（c)Relationship between vulnerability $L(W)$ and scale of removed station R

Figure 3. Relation curve of failure scale and the number of disturbance of the metro networks under stations random failure and target attacks

In Figure 3, it is obviously to see that decrease in function caused by random failure (failure scenario F_1) is less than targeted attack (failure scenario F_3, F_4, F_6). Since targeted attack strategy focuses on important vertices such as high degree and high betweenness, which play an important part in the overall connectivity of the network. The decrease of path distance based efficiency caused by removing the top 8 most important station equals to removing 27 stations randomly (see Figure 3 (a)). Similar phenomenon can be found in decrease of transfer based efficiency and maximum connective OD pair. Additionally, decrease speed of maximum connective OD pair is faster than the other two vulnerability indicators.

Vulnerability evaluation under section failure scenario

Three section failure scenarios were used to simulate random failure and targeted attack caused by passengers' interruption and mechanical failure from train, power system, communication system etc. in sections. Under F_2, section was randomly removed in each time step; under F_5, section with the highest path distance based betweenness was removed in each time step; under F_7, section with the highest transfer based betweenness was removed in each time step. The relationship between the decrease in function of three topological vulnerability indicators L and scale of removed sections R is shown in Figure 4. Whereby, Figure 4(a) reports relationship between the decreases of path distance based efficiency L and scale of removed sections R under three failure scenarios; Figure 4(b) reports relationship between the decrease of transfer based efficiency L and scale of removed sections R under three failure scenarios; Figure 4(c) reports relationship between the decrease of maximum connective OD pair L and scale of removed sections R under three failure scenarios.

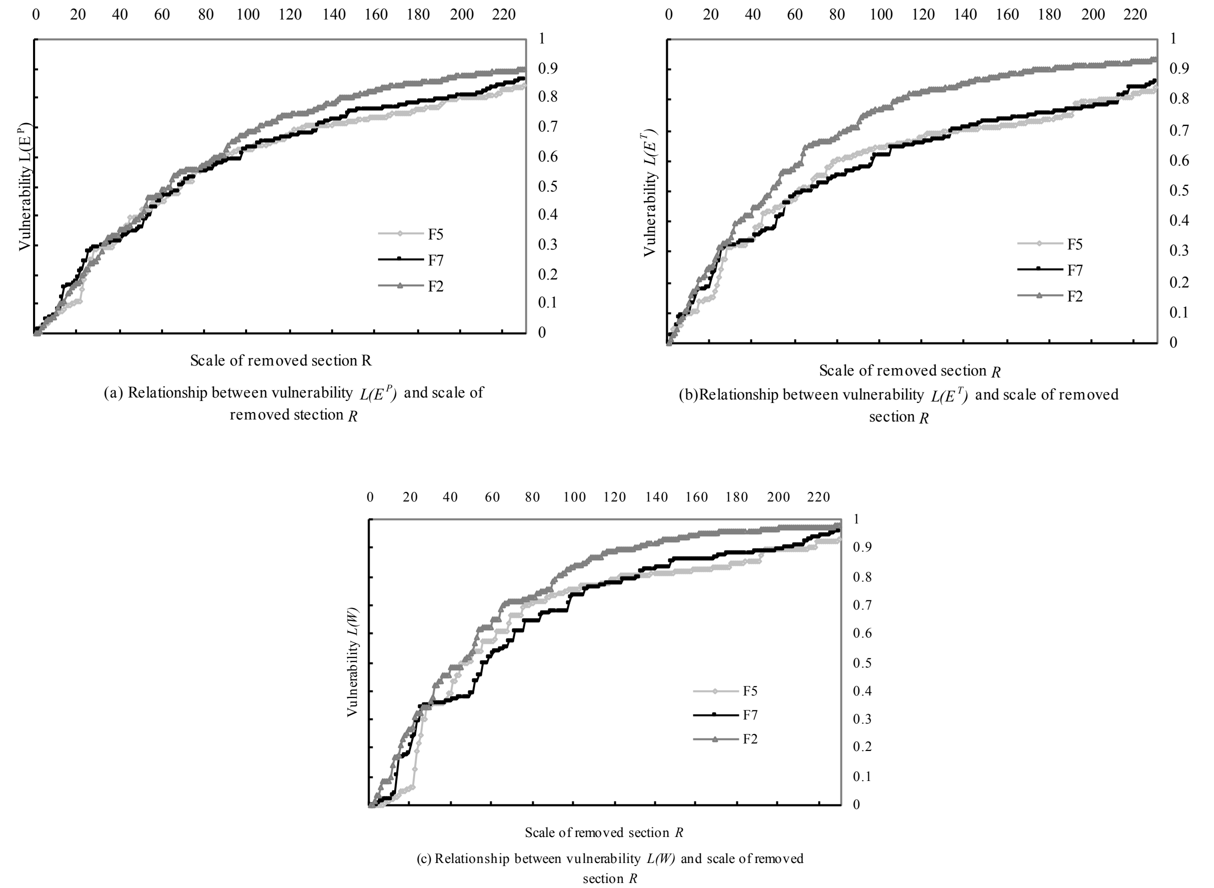


Figure 4. Relation curve of failure scale and the number of disturbance of the metro networks under sections random failure and target attacks

We would expect that UMNS would be more vulnerable to targeted attack than random failure. However, decrease in function caused by randomly section removing almost equals to deliberately removing when the scale of removed section R less than a specific value. With increasing of the scale of the removed section R, the decrease in function caused by randomly section removing will larger than deliberately section removing. Since there are two separate sections which services opposite direction between two neighbor stations, have the same degree of importance. If one section is removed, the neighbor stations may be arrived by other path which the opposite direction section is impossible contained. Therefore, although two opposite sections are removed, the connectivity of two neighbor station would not be affected. But sections on other path when section which connects two neighbor stations is removed would be selected under failure scenario F_2. This is the reason why UMNS is more vulnerable to randomly section removing than targeted section removing.

By comparing Figure 3 and Figure 4, it is obviously to see that UMNS is more vulnerable to targeted station removing than randomly section removing. When 51 stations are removed, path distance based efficiency, transfer based efficiency, and maximum connected OD pair decrease more than 80%, but only 40% function decrease when the same scale sections are removed. The decrease in function caused by removing the station with highest line degree equals to removing 20 sections. The results suggest that more attention should be pay on station maintain in daily operation.

CONCLUSION

The paper proposes a complexity theory based approach for vulnerability analysis of urban metro network system. The model provides information that allows evaluating the response of each component of the network individually and the network as a whole. The methodology is easy to use which only need input damage random or targeted and the damage strength, which avoid probability calculation of risk events happened on stations and sections.

The proposed methodology focuses on important indicator of station and section, vulnerability indicator which should reflect the specific attributes of urban metro networks. The results of simulation show that UMNS is more vulnerable to targeted station attack than randomly station removing; decrease in function caused by randomly section removing almost equals to deliberately removing when the scale of removed section R less than a specific value; UMNS is more vulnerable to targeted station removing than randomly section removing.

The proposed methodology is not intended to predict the occurrence of events but rather to be used a management tool. Results from the evaluation are valuable elements in planning UMNS. They can be used for network planning, further detailed hazard studies, deciding on the arrangement of emergency resources.

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