**Proceedings of the ASME 2011 International Mechanical Engineering Congress & Exposition**
**IMECE2011**
**November 11-17, 2011, Denver, Colorado, USA**

# IMECE2011-63112

# CLOSED FORM SOLUTIONS FOR THE PROBLEM OF STATICAL BEHAVIOR OF NANO/MICROMIRRORS UNDER THE EFFECT OF CAPILLARY FORCE AND VAN DER WAALS FORCE

**Ali Darvishian**
M.Sc. Student, School of Mechanical Engineering,
Sharif University of Technology, Tehran, Iran.
alidarvishian@gmail.com

**Hamid Moeenfard**
PhD Candidate, Department of mechanical engineering, Sharif University of Technology, Tehran, Iran.
hamid_moeenfard@gmail.com

**Hasan Zohoor**
Center of Excellence in Design, Robotics and Automation, Department of Mechanical Engineering, Sharif University of Technology, Tehran, Iran.
zohoor@sharif.edu

**Mohammad Taghi Ahmadian**
Center of Excellence in Design, Robotics and Automation, Department of Mechanical Engineering, Sharif University of Technology, Tehran, Iran.
ahmadian@mech.sharif.edu

**ABSTRACT**
The current paper deals with the problem of static instability of Micro/Nano mirrors under the combined effect of capillary force and van der Waals force. First the governing equations of the statical behavior of Micro/Nano mirrors under the combined effect of capillary force and casimir force is obtained using the newtons first law of motion. The dependence of the critical tilting angle on the physical and geometrical parameters of the nano/micromirror and its supporting torsional beams is investigated. It is found that existence of vdW torque can considerably reduce the stability limits of the nano/micromirror. It is also found that rotation angle of the mirror due to capillary force highly depends on the vdW toque applied to the mirror. Finally analytical tool Homotopy Perturbation Mehtod (HPM) is utilized for prediction of the nano/micromirror behaviour under combined capillary and vdW force. It is observed that a sixth order perturbation approximation accurately predicts the rotation angle and stability limits of the mirror. Results of this paper can be used for successful fabrication of nano/micromirrors using wet etching process where capillary force plays a major role in the system.

**Keywords:** Nano/micromirror, capillary force, vdW force, HPM.

## 1) Introduction

The technology of MEMS devices has experienced a lot of progress recently. Their low manufacturing cost, batch production, light weight, small size, durability, low energy consumption and compatibility with integrated circuits, makes them extremely attractive [1, 2]. Successful MEMS devices rely not only on well developed fabrication technologies, but also on the knowledge of device behavior, based on which a favorable structure of the device can be forged [3]. The important role of MEMS devices in optical systems initiate the development of a new class of MEMS called MicroOptoElectroMechanical Systems (MOEMS). MOEMS mainly include micromirrors and torsional micro-actuators. These devices has found variety of applications such as digital micromirror devices (DMD)



Copyright © 2011 by ASME

Downloaded From: https://proceedings.asmedigitalcollection.asme.org on 07/02/2019 Terms of Use: http://www.asme.org/about-asme/terms-of-use

[4], optical switches [5], micro scanning mirrors [6], optical cross connects [7, 8], and etc.

Existence of a liquid bridge between two objects results in forming capillary force [9]. The existence of capillary force even in low relative humidity is observed experimentally [10]. Parallel plate MEMS actuators are conventionally fabricated by forming a layer of a plate or beam material on the top of a sacrificial layer of another material and wet etching the sacrificial layer. In this process, capillary force can be easily formed and in the case of poor design, the structure will collapse and adhere to the substrate. So investigating the effect of capillary force on micromirrors is extremely important in their design and fabrication.

Many researchers investigated the effect of capillary force on MEMS devices. Mastrangelo and Hsu [11, 12] studied the stability and adhesion of thin micromechanical structures under capillary force, theoretically and experimentally. Moeenfard et al [13] studied the effect of capillary force on the static pull-in instability of fully clamped micromirrors. The effects of capillary force on the static and dynamic behaviours of atomic force microscopes (AFM) are widely assessed [14-16]. The instability of torsional MEMS/NEMS micro-actuators under capillary force was investigated by Guo et al [17]. Intermolecular surface forces which are mainly include casimir and vdW force, play an important role in the stability of micromirrors. VdW force is a short range force in nature, but it can lead to long range influences more than 0.1 μm [18].This force is the interaction force between neutral atoms, and it varies from covalent and ionic bondings in that it is caused by correlations in the fluctuating polarizations of nearby particle [19]. Casimir effect can be simply understood as the long range analog of the Van der Waals force, resulting from the propagation of retarded electromagnetic wave [20].

When the size of a structure is sufficiently small, vdW force plays a key role on the stability of the structure and in the case of poor design, can lead to the collapse of the structure. Influence of vdW and casimir forces in pull-in phenomena and dynamic response of a capacitive nano-beam switch has been studid by Tahami et al [21]. Batra et al [22] studied effects of van der Waals force and thermal stresses on pull-in instability of clamped rectangular microplates. Modelling and simulation of nano electrostatic switches under the effect of vdW and casimir forces have been studied by Mojahedi et al [23]**.** Ramezani et al [24] investigated influence of van der Waals force on the pull-in parameters of cantilever type nanoscale electrostatic actuators. Guo and Zhao [25] studied dynamic stability of electrostatic torsional actuators with van der Waals effects. They [26] also discussed the effect of vdW force on the pull-in of electrostatic torsional actuators. But statical behaviour and pull-in of nano/micromirrors under combined effect of vdW and capillary forces has not been investigated. So in this paper, the combined effect of vdW and capillary forces on the tilting angle and stability of torsional nano/micromirror is studied. In this study, HPM is used as an perturbational based analytical tool. Perturbation methods have been used to analytically solve the nonlinear problems in MEMS. Younis and Nayfeh [27] investigate the response of a resonant microbeam to an electric actuation using the multiple-scale perturbation method. Abdel-Rahman and Nayfeh [28] used the same method to model secondary resonances in electrically actuated microbeams. Since perturbation methods are based upon the assumption that there is a small parameter in the equations, they have some limitations in problems without involvement of small parameters. In order to overcome this limitation a new perturbational based method, namely Homotopy Perturbation Method (HPM) was developed by He et al [29]. His new method takes full advantages of the traditional perturbation methods and homotopy techniques. HPM has also been used for solving the nonlinear problems encountered in N/MEMS. For example, Moeenfard et al [30] used HPM to model the nonlinear vibrational behavior of Timoshenko micobeams. Mojahedi et al [31] applied the HPM method to simulate the static response of nano-switches to electrostatic actuation and intermolecular surface forces. But so far no analytic solution has been presented to model the behavior of nano/micromirrors under vdW and capillary forces.

In the current paper, the equation governing the statical behavior of rectangular nano/micromirrors under vdW and capillary forces is obtained using Newton's first law of motion then pull-in parameters of nano/micromirrors under effect of vdW and capillary forces is investigated. At the end, tilting angle of a nano/micromirror under vdW and capillary forces is calculated both numerically and analytically using HPM.

**2) Theoretical model**

The schematic view of a rectangular nano/micromirror with a layer of liquid between mirror and substrate is shown in figure (1). The mirror considered to be a rotational rigid body with $\theta$ being the tilting angle of the mirror.

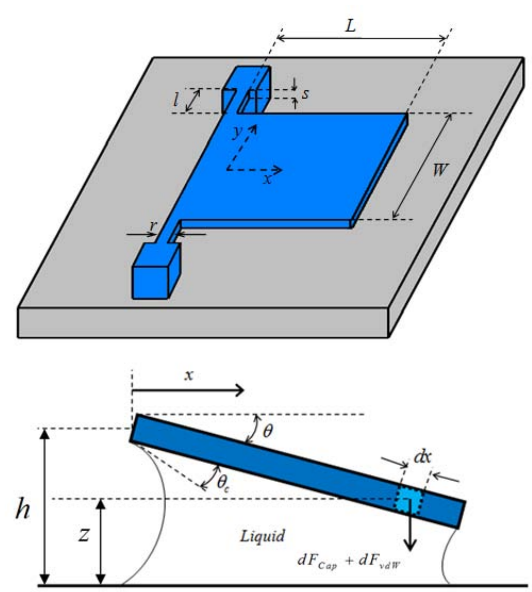


**Fig. 1:** Schematic view of a nano/micromirror under the combined effect of capillary and vdW force.


Copyright © 2011 by ASME

Downloaded From: https://proceedings.asmedigitalcollection.asme.org on 07/02/2019 Terms of Use: http://www.asme.org/about-asme/terms-of-use

The capillary pressure, $P_{cap}$ underneath the mirror is [13]:

$$P_{cap} = \frac{2\gamma \cos\theta_c}{z} \quad (1)$$

Where $z$ is the vertical distance between the mirror and the substrate at the point with a capillary pressure of $P_{Cap}$, $\gamma$ is surface energy of liquid and $\theta_c$ is contact angle between liquid and solid surface. So the capillary torque applied to the mirror can be derived as:

$$M_{cap} = \int_0^L \frac{2\gamma \cos\theta_c}{h - x\sin\theta} xWdx \quad (2)$$

Where $h$ is the initial distance between the mirror and the substrate, $L$ is length, and $W$ is width of mirror respectively as illustrated in figure (1).

Furthermore the differential vdW force applied to a differential surface element of the mirror shown in figure (1) is [18]:

$$dF_{vdW} = \frac{A}{6\pi z^3} Wdx \quad (3)$$

Where $A$ is the Hamaker constant. So the torque applied to the mirror due to vdW force is calculated as:

$$M_{vdW} = \int_0^L \frac{A}{6\pi (h - x\sin\theta)^3} Wxdx \quad (4)$$

Since $\frac{h}{L} << 1$, the tilting angle is small, and $\sin\theta$ can be closely approximated by $\theta$. Therefore equations (2) and (4) can be restated as:

$$M_{cap} = \int_0^L \frac{2\gamma \cos\theta_c}{h - x\theta} xWdx \quad (5)$$

$$M_{vdW} = \int_0^L \frac{A}{6\pi (h - x\theta)^3} Wxdx \quad (6)$$

Due to torsional stiffness of the supporting beams, rotation angle of the mirror would produce an opposing mechanical torque which can be calculated as follows.

$$M_{Mech} = K\theta \quad (7)$$

Where

$$K = \frac{2GI_p}{l} \quad (8)$$

In equation (8), $G$ is the shear modulus of the beam's material, $l$ is length of each torsion beam and $I_p$ is the polar momentum of inertia of the beam's cross section which is calculated using equation (9) [32].

$$I_p = \frac{1}{3} rs^3 - \frac{64}{\pi^5} s^4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \tanh \frac{(2n-1)\pi r}{2s} \quad (9)$$

Where $r$ and $s$ are the width and length of the torsion beam's cross section respectively as illustrated in figure (1).

For simplification purpose, the following dimensionless variable is introduced

$$\Theta = \frac{\theta}{\theta_0} \quad (10)$$

Where $\theta_0 \approx \sin\theta_0 = \frac{h}{L}$ is the maximum physically possible rotation angle of the mirror. By using normalized tilting angle introduced in equation (10), equations (5), (6) and (7) can be more simplified as equations (11), (12) and (13) respectively.

$$M_{cap} = \frac{-2\gamma \cos\theta_c wL^2}{h\Theta} \left[ 1 + \frac{1}{\Theta} \mathrm{Ln}(1 - \Theta) \right] \quad (11)$$

$$M_{vdw} = \frac{AwL^2}{6\pi h^3 \Theta^2} \left[ \frac{1}{2} + \frac{2\Theta - 1}{2(\Theta - 1)^2} \right] \quad (12)$$

$$M_{Mech} = \frac{2GhI_p}{lL} \Theta \quad (13)$$

At equilibrium point, the resultant of the capillary and the vdW torques, reaches a balance with the elastic restoring torque of torsion beams, and so equation (14) must be satisfied:

$$M_{cap} + M_{vdW} - M_{els} = 0 \quad (14)$$

By substituting equations (11), (12) and (13) into equation (14), this equation is simplified as equation (15):

$$\frac{\eta}{\Theta} \left[ 1 + \frac{1}{\Theta} \mathrm{Ln}(1 - \Theta) \right] - \frac{\mu}{\Theta^2} \left[ \frac{1}{2} + \frac{2\Theta - 1}{2(\Theta - 1)^2} \right] + \Theta = 0 \quad (15)$$

where

$$\eta = \frac{2\gamma \cos\theta_c wL^3}{Kh^2} \quad (16)$$

$$\mu = \frac{AwL^3}{6\pi Kh^4} \quad (17)$$

$\eta$ and $\mu$ are defined as instability numbers of the nano/micromirror due to capillary and vdW torque respectively. The dimensionless elastic restoring torque of beam is

$$f(\Theta) = \Theta \quad (18)$$

and the dimensionless resultant of the capillary and the vdW torques is

$$g(\eta, \mu, \Theta) = \frac{\mu}{\Theta^2} \left[ \frac{1}{2} + \frac{2\Theta - 1}{2(\Theta - 1)^2} \right] - \frac{\eta}{\Theta} \left[ 1 + \frac{1}{\Theta} \mathrm{Ln}(1 - \Theta) \right] \quad (19)$$

The variation of these two dimensionless torques with normalized tilting angle while $\mu$ is constant is shown in figure (2).

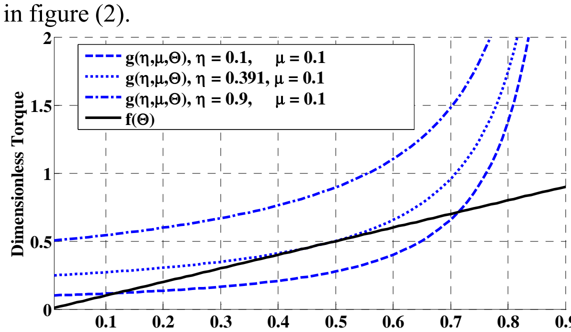


**Fig. 2:** Dimensionless elastic restoring torque and attractive torque (at different values of $\eta$) applied to the mirror.

It can be seen that for small values of $\eta$, there exist two equilibrium points where the smaller one is the stable



Copyright © 2011 by ASME

Downloaded From: https://proceedings.asmedigitalcollection.asme.org on 07/02/2019 Terms of Use: http://www.asme.org/about-asme/terms-of-use

and the larger one is the unstable equilibrium point. There is a certain value of $\eta$ for which equation (15) has just one root. This value of $\eta$ is the $\eta$ at the pull-in state and is denoted by $\eta_p$. For $\eta$ larger than the value of $\eta_p$ there would be no equilibrium point and so the mirror would be unstable.

The same discussion exist for pull-in due to the van der Waals force. In fact when $\eta$ is constant, increasing the value of $\mu$ can lead to pull-in of the mirror.

At constant value of $\mu$, pull-in occurs when $\eta$ reaches its maximum value, so at pull-in, the following equation is satisfied.

$$\frac{\partial \eta(\mu,\Theta)}{\partial \Theta} = 0 \tag{20}$$

where $\eta$ can be calculated using equation (15). Similarly at constant value of $\eta$, pull-in occurs when $\mu$ reaches it maximum value and as a result at pull-in, equation (21) would be satisfied

$$\frac{\partial \mu(\eta,\Theta)}{\partial \Theta} = 0 \tag{21}$$

Where $\mu$ is easily calculated using equation (15). Equations (20) and (21) can be solved for finding the values of $\mu$ and $\eta$ at pull-in respectively. The results are as follows.

$$\mu_p = \frac{2\Theta(1-\Theta)^2\left(-2\Theta^2+3\Theta+3(1-\Theta)\ln(1-\Theta)\right)}{\Theta^2+2\Theta+2\ln(1-\Theta)} \tag{22}$$

$$\eta_p = \frac{\Theta^3(1-3\Theta)}{\Theta^2+2\Theta+2\ln(1-\Theta)} \tag{23}$$

Where $\mu_p$ and $\eta_p$ are the values of $\mu$ and $\eta$ at pull-in respectively. Figures (3) and (4) show the values of pull-in angle versus $\mu_p$ and $\eta_p$ respectively. It is observed that with increasing $\mu_p$ the pull-in angle of the mirror is reduced while this angle is increased with increasing $\eta_p$.

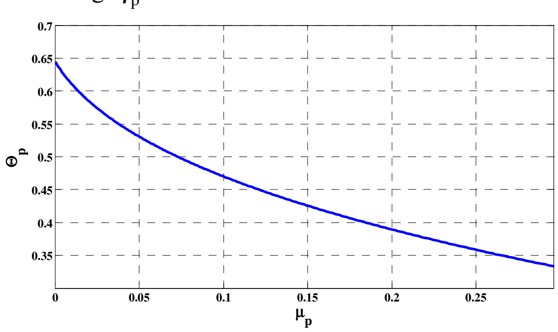


**Fig. 3:** The values of pull-in angle versus $\mu_P$ .

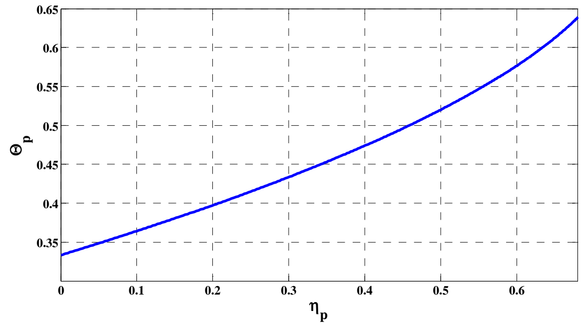


**Fig. 4:** The values of pull-in angle versus $\eta_P$ .

I By eliminating $\Theta$ between equations (22) and (23), $\eta_P$ can be obtained versus $\mu_p$ as plotted in figure (5).

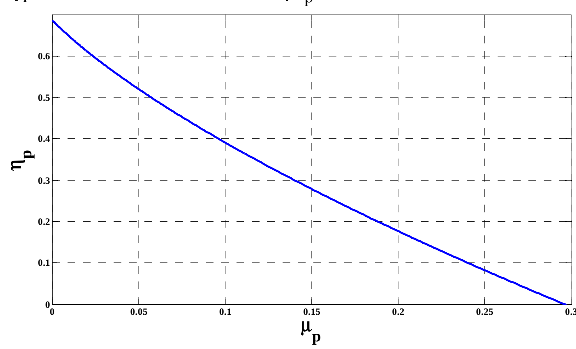


**Fig. 5:** The values of $\eta_P$ versus $\mu_P$ .

It is observed that with increasing $\mu_P$ pull-in occurs at lower values of $\eta_p$. In fact this figure shows that vdW force can significantly reduce the maximum allowable value for $\eta$ and as a result, the stability limits of the nano/micromirror is reduced. In addition it can be concluded that even in the absence of capillary force, vdW force can be lead to the occurrence of pull-in. equations (24) and (25) can be used for successful and stable design of nano/micromirrors fabricated using wet etching process where capillary force plays a major role. In fact in a safe design, the following inequalities must be satisfied:

$$\eta = \frac{2\gamma\cos\theta_c wL^3}{Kh^2} < \eta_p \tag{24}$$

$$\mu = \frac{AwL^3}{6\pi Kh^4} < \mu_p \tag{25}$$

In order to investigate the mirror behavior under combined capillary and vdW loading, the dimensionless rotation angle is plotted versus $\eta$ in figure (6).



Copyright © 2011 by ASME

Downloaded From: https://proceedings.asmedigitalcollection.asme.org on 07/02/2019 Terms of Use: http://www.asme.org/about-asme/terms-of-use

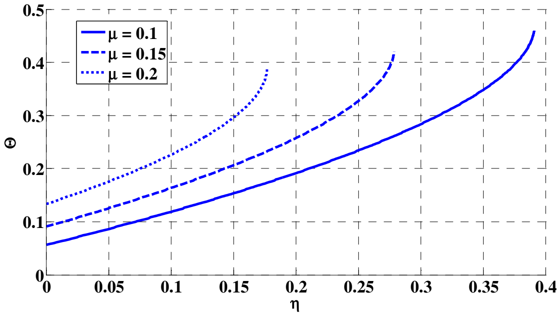

**Fig. 6:** Stable equilibrium angle versus $\eta$ at different values of $\mu$.

L It is observed that by increasing the value of $\eta$ the rotation angle of the nano/micromirror is increased, but the maximum value of $\eta$ at pull-in, highly depends on the value of $\mu$ and it is verified that by increasing $\mu$, the maximum allowable value for $\eta$ is reduced. Furthermore it is concluded that at a constant $\eta$, larger values of $\mu$ would lead to larger values for stable equilibrium angle.

**3) Analytical solution of equilibrium equations**

In this section, it is tried to find the value of the rotation angle of the nano/micromirror analytically in terms of $\eta$ and $\mu$. For this purpose, the analytical tool, HPM is utilized.

The linear part of equation (15) can be found by using Taylor series expansion of the equilibrium equation as follows.

$$L(\Theta,\eta,\mu) = -\frac{\eta+\mu}{2} + \left(1-\mu-\frac{\eta}{3}\right)\Theta \qquad (26)$$

Where $L(\Theta,\eta,\mu)$ is the linear part of equation (15). Obviously the nonlinear part of equilibrium equation is obtained by subtracting $L(\Theta,\eta,\mu)$ from equation (26).

$$N(\Theta,\eta,\mu) = \frac{\eta}{\Theta}\left(1+\frac{1}{\Theta}\ln(1-\Theta)\right) - \frac{\mu}{\Theta^2}\left(\frac{1}{2}+\frac{2\Theta-1}{2(\Theta-1)^2}\right) + \frac{\eta+\mu}{2} + \left(\mu+\frac{\eta}{3}\right)\Theta \qquad (27)$$

Now, the homotopy form is constructed as follows.

$$\Im(\Theta,\eta,\mu,P) = L(\Theta,\eta,\mu) + P.N(\Theta,\eta,\mu) = 0 \qquad (28)$$

In equation (28), $\Im(\Theta,\eta,\mu,P)$ is the homotopy form and $P$ is an embedding parameter which serves as perturbation parameter. When $P=1$, the homotopy form would be the same as the equilibrium equation and when $P=0$, homotopy form would be the linear part of equilibrium equation. The value of the dimensionless rotation angle $\Theta$ can also be expanded in terms of the embedded parameter $P$.

$$\Theta = \Theta_0 + P\Theta_1 + P^2\Theta_2 + P^3\Theta_3 + \ldots \qquad (29)$$

Substituting equation (29) into homotopy form yields:

$$\Im(\Theta,\eta,\mu,P) = L\left(\Theta_0 + P\Theta_1 + P^2\Theta_2 + \ldots,\eta,\mu\right) + P.N\left(\Theta_0 + P\Theta_1 + P^2\Theta_2 + \ldots,\eta,\mu\right) = 0 \qquad (30)$$

The Taylor series expansion of right hand side of equation (30) in terms of $P$ would be as

$$\begin{aligned}\Im(\Theta,\mu,P) &= L(\Theta_0,\eta,\mu) \\ &+ \left(\Theta_1\frac{\partial L(\Theta_0,\eta,\mu)}{\partial\Theta_0} + N(\Theta_0,\eta,\mu)\right)P \\ &+ \left(\Theta_2\frac{\partial L(\Theta_0,\eta,\mu)}{\partial\Theta_0} + \Theta_1\frac{\partial N(\Theta_0,\eta,\mu)}{\partial\Theta_0}\right)P^2 \\ &+ \left(\Theta_3\frac{\partial L(\Theta_0,\eta,\mu)}{\partial\Theta_0} + \Theta_2\frac{\partial N(\Theta_0,\eta,\mu)}{\partial\Theta_0} + \frac{1}{2}\Theta_1^2\frac{\partial^2 N(\Theta_0,\eta,\mu)}{\partial\Theta_0^2}\right)P^3 + \ldots = 0\end{aligned} \qquad (31)$$

Since the homotopy form must be unified with zero, the coefficients of all powers of $P$ must be zero. This, leads to the following equations.

$$L(\Theta_0,\eta,\mu) = 0 \qquad (32)$$

$$\Theta_1\frac{\partial L(\Theta_0,\eta,\mu)}{\partial\Theta_0} + N(\Theta_0,\eta,\mu) = 0 \qquad (33)$$

$$\Theta_2\frac{\partial L(\Theta_0,\eta,\mu)}{\partial\Theta_0} + \Theta_1\frac{\partial N(\Theta_0,\eta,\mu)}{\partial\Theta_0} = 0 \qquad (34)$$

$$\Theta_3\frac{\partial L(\Theta_0,\eta,\mu)}{\partial\Theta_0} + \Theta_2\frac{\partial N(\Theta_0,\eta,\mu)}{\partial\Theta_0} + \frac{1}{2}\Theta_1^2\frac{\partial^2 N(\Theta_0,\eta,\mu)}{\partial\Theta_0^2} = 0 \qquad (35)$$

With solving equations (32) to (35), the parameters $\Theta_i$ $0 \le i \le 3$ are found as follows.

$$\Theta_0 = \frac{3\mu}{6-8\mu} \qquad (36)$$

$$\Theta_1 = -N(\Theta_0,\eta,\mu)\Big/\left(\frac{\partial L(\Theta_0,\eta,\mu)}{\partial\Theta_0}\right) \qquad (37)$$

$$\Theta_2 = -\Theta_1\left(\frac{\partial N(\Theta_0,\eta,\mu)}{\partial\Theta_0}\right)\Big/\left(\frac{\partial L(\Theta_0,\eta,\mu)}{\partial\Theta_0}\right) \qquad (38)$$

$$\Theta_3 = -\left(\Theta_2\frac{\partial N(\Theta_0,\eta,\mu)}{\partial\Theta_0} + \frac{1}{2}\Theta_1^2\frac{\partial^2 N(\Theta_0,\eta,\mu)}{\partial\Theta_0^2}\right)\Big/\left(\frac{\partial L(\Theta_0,\eta,\mu)}{\partial\Theta_0}\right) \qquad (39)$$

The value of $\Theta$ is found by substituting $\Theta_i$ $0 \le i \le 3$ and $P=1$ in equation (29). In figure (7) the results of the numerical simulations is compared with those of analytical HPM results. It is observed that HPM closely approximates the rotation angle of the mirror. Obviously increasing the order of perturbation approximation would lead to more precise results, but increasing the order of the perturbation approximation more than 6 will not improve the accuracy of the obtained response significantly. As a result, a sixth order perturbation approximation used in HPM can precisely predict the nano/micromirror behaviour under the combined effects of capillary and vdW force.


Copyright © 2011 by ASME

Downloaded From: https://proceedings.asmedigitalcollection.asme.org on 07/02/2019 Terms of Use: http://www.asme.org/about-asme/terms-of-use

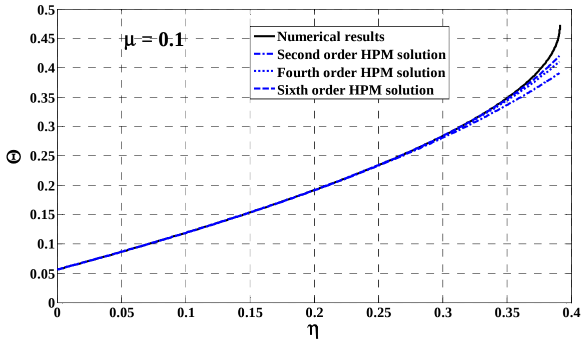


**Fig. 7:** Estimation of nano/micromirror's rotation angle using HPM.

## Conclusion

The dimensionless equilibrium equation of nano/micromirror under capillary force was found considering vdW force. The dependence of the critical tilting angle on the instability numbers defined in the paper was investigated. Results show that neglecting vdW effect on the static equilibrium of nano/micromirrors under capillary force may lead to considerable error in predicting stability limits of the mirror and can lead to an unstable design.

It was observed that rotation angle of the mirror due to capillary force highly depends on the vdW torque applied to the mirror. HPM was utilized to analytically predict the rotation angle and stability limits of the nano/micromirrors. It was found that a sixth order perturbation approximation can accurately estimate the rotation angle of the mirror due to capillary and vdW loading. Presented results in this paper can be used for stable design and fabrication of nano/micromirrors using wet etching process where the gap between the mirror and the underneath substrate is less than 100 nm and as a result, both capillary and vdW torque have significant effects on the system.

## References

[1] N. Maluf, K. Williams, 1999, "An Introduction to Microelectromechanical Systems Engineering, Second Edition, MicroelectromechanicalSystems (MEMS) Series, Boston, London, Artech House Inc.

[2] M.I Younis, 2004, "Modeling and Simulation of Microelectromechanical Systems in Multi-Physics Fields", PhD thesis, Virginia Polytechnic Institute and State University.

[3] P.C.P Chao, C.W. Chiu, C.Y. Tsai, 2006, "A novel method to predict the pull-in voltage in a closed form for micro-plates actuated by a distributed electrostatic force", J. Micromech.Microeng. 16, pp.986–998.

[4] L.J.Hornbeck, 1991 "Spatial light modulator and method", US Patent 5,061,049.

[5] J.E. Ford, V.A. Aksyuk, D.J. Bishop, J.A. Walker, 1999, "Wavelenghth add-drop swiching using tilting micromirrors", J. Lightwave technol. 17, 904-911.

[6] D.L. Dickensheets, and G.S. Kino, 1998, "Silicon-micromachined scanning confocal optical microscope", Journal of Microelectromechanical Systems, vol. 7, no. 1 , pp. 38-47.

[7] P.M. Zavracky, S. Majumber, E. McGruer, 1997, "Micromechanical switches fabricated using nickel surface micromachining", J. Microelectromech. Syst. 6 , pp. 3–9.."

[8] H. Toshiyoshi, H. Fujita, 1996, "Electrostatic micro torsion mirrors for an optical switch matrix", J. Microelectromech. Syst. 5, 231–237.

[9] Z. Wei, and Y.P. Zhao, 2007, "Growth of liquid bridge in AFM", Journal of Physics D: Applied Physics, vol. 40, no. 14, pp. 4368-4375.

[10] P. J. van Zwol, G. Palasantzas, J. Th. M. De Hosson, 2008, "Influence of roughness on capillary forces between hydrophilic surfaces", Physical Review E 78, 03160.

[11] C. H. Mastrangelo, C. H. Hsu, 1993, "Mechanical Stability and Adhesion of Microstructures Under Capillary Forces-Part I: Basic Theory", Journal of Microelectromechanical Systems 2 (1), pp. 33-43.

[12] C. H. Mastrangelo, C. H. Hsu, 1993, "Mechanical Stability and Adhesion of Microstructures Under Capillary Forces-Part 2: Experiments", Journal of Microelectromechanical Systems 2 (1), pp. 44-55.

[13] H. Moeenfard, M.H.Kahrobaiyan, M.T. Ahmadian, 2010, "Aplication of the extended Kantorovich method to the static deflection of microplates under capillary force", ASME International Mechanical Engineering Congress and Exposition, IMECE2009-12186.

[14] L. Zitzler, S. Herminghaus, F. Mugele, 2002, "Capillary forces in tapping mode atomic force microscopy", PHYSICAL REVIEW B 66, 155436.

[15] Xide Li, Yun Peng, 2006; "Investigation of capillary adhesion between the microcantilever and the substrate with electronic speckle pattern interferometry", APPLIED PHYSICS LETTERS 89, 234104.

[16] J. Jang, G.C. Schatz, M.A Ratner, 2004, "Capillary force in atomic force microscopy",Journal of Chemical Physics 120 (3), pp. 1157-1160.

[17] J.G. Guo, L.J. Zhou, and Y.P. Zhao, 2009, "Instability analysis of torsional MEMS/NEMS actuators under capillary force", Journal of Colloid and Interface Science, vol. 331, no. 2, pp. 458-462.

[18] H. Liu, S. Gao, S. Niu, L. Jin, 2010, "Analysis on the adhesion of microcomb structure in MEMS", International Journal of Applied Electromagnetics and Mechanics 33 , pp. 979–984.

[19] G. Xie, J. Ding, S. Liu, W. Xued, J. Luob, 2009, "Interfacial properties for real rough MEMS/NEMS surfaces by incorporating the electrostatic and Casimir effects–a theoretical study",Surf. Interface Anal. 41, pp.338–346.

[20] A. Gusso, G.J. Delben, 2007, "Influence of the Casimir force on the pull-in parameters of silicon based electrostatic torsional actuators", Sensors and Actuators A 135 , pp. 792–800.

[21] F.V. Tahami, H. Mobki, A.A.K.Janbahan, G. Rezazadeh, 2009, "Pull-in Phenomena and Dynamic Response of a Capacitive Nano-beam Switch", Sensors & Transducers Journal, Vol. 110, Issue 11, pp. 26-37.


Copyright © 2011 by ASME

Downloaded From: https://proceedings.asmedigitalcollection.asme.org on 07/02/2019 Terms of Use: http://www.asme.org/about-asme/terms-of-use

[22] R. C. Batra, M. Porfiri, D. Spinello, 2008, "Effects of van der Waals Force and Thermal Stresses on Pull-in Instability of Clamped Rectangular Microplates",Sensors 8, pp.1048-1069.
[23] M. Mojahedi, H. Moeenfard, M. T. Ahmadian, 2009, "A new efficient approach for modeling and simulation of nano- switches under combined effects of intermolecular surface forces and electrostatic actuation", International Journal of Applied Mechanics Vol. 1, No. 2, pp. 349–365.
[24] A. Ramezani, A. Alasty , J. Akbari, 2007, "Closed-form solutions of the pull-in instability in nano-cantilevers under electrostatic and intermolecular surface forces", International Journal of Solids and Structures 44, pp. 4925–4941.
[25] J.G. Guo, Y.P. Zhao, 2006, "Dynamic stability of electrostatic torsional actuators with van der Waals effect", International Journal of Solids and Structures 43, pp.675–685.
[26] J.G. Guo, Y.P. Zhao, 2004, "Influence of van der Waals and Casimir Forces on Electrostatic Torsional Actuators", JOURNAL OF MICROELECTROMECHANICAL SYSTEMS, VOL. 13, NO. 6.
[27] M.I. Younis, A.H. Nayfeh, 2003, "A study of the nonlinear response of a resonant microbeam to an electric actuation", Nonlinear Dynamics, vol. 31, no. 1, pp. 91-117.
[28] E.M. Abdel-Rahman, A.H. Nayfeh, 2003, "Secondary resonances of electrically actuated resonant microsensors", Vol. 13, pp. 491–501.
[29] J. H. He, 2000, "A coupling method of a homotopy technique and a perturbation technique for non-linear problems", Int. J. Non-Linear Mechanics, 35, pp. 37–43.
[30] H.Moeenfard, M.Mojahedi, M.T.Ahmadian, "A homotopy perturbation analysis of nonlinear free vibration of timoshenko microbeams", Journal of mechanical science and technology 25 (3), 279-285, DOI 10.1007/s12206-010-0130-8, article in press.
[31] M. Mojahedi, H. Moeenfard, and M.T. Ahmadian, 2010, "Analytical solutions for the static instability of nano-switches under the effect of casimir force and electrostatic actuation", ASME International Mechanical Engineering Congress and Exposition, Proceedings, pp. 63-69.
[32] J.M. Huang, A.Q. Liu, Z.L. Deng, Q.X. Zhang, J. Ahn, and A. Asundi, 2004, "An approach to the coupling effect between torsion and bending for electrostatic torsional micromirrors", Sensors and Actuators, A: Physical, vol. 115, no. 1, pp. 159-167.


Copyright © 2011 by ASME

Downloaded From: https://proceedings.asmedigitalcollection.asme.org on 07/02/2019 Terms of Use: http://www.asme.org/about-asme/terms-of-use