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Minimization of Nonlinear Functions by Certain Numerical Algorithms

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***Abstract:* In this paper, we propose few new algorithms, for minimization of nonlinear functions. Then comparative study among the new algorithms and Newton's algorithm is established by means of various examples.**

Index terms: **Nonlinear Functions; Newton's Method; Ostrowski's Method; Halley' method; Newton Secant Method; Third-Order Convergence.**

I. INTRODUCTION

In recent times, many problems in business situations and engineering designs have been modeled as an optimization problem for taking optimal decisions. Optimization problems with or without constraints arise in various fields such as science, engineering, economics, management sciences, etc., where numerical information is processed In fact, numerical optimization techniques have made deep in to almost all branches of engineering and mathematics. Several methods [3], [10], [11], [12], [13], [14], [16], [21] are available for solving unconstrained minimization problems. These methods can be classified in to two categories as non gradient and gradient methods. The non gradient methods require only the objective function values but not the derivatives of the function in finding minimum. The gradient methods require, in addition to the function values, the first and in some cases the second derivatives of the objective function. Since more information about the function being minimized is used through the use of derivatives, gradient methods are generally more efficient than non gradient methods. All the unconstrained minimization methods are iterative in nature and hence they start from an initial trial solution and proceed towards the minimum point in a sequential manner.

To solve unconstrained nonlinear minimization problems arising in the diversified field of engineering and technology, we have several methods to get solutions. For instance, multi-step nonlinear conjugate gradient methods [6], ABS-MPVT algorithm [19] are used for solving unconstrained optimization problems. A proximal bundle method with inexact data [23] is used for minimizing unconstrained non smooth convex function. A new algorithm [8] is used for solving unconstrained optimization problem with the form of sum of squares minimization.

Vinay Kanwar et al. [25] introduced new algorithms called, external touch technique and orthogonal intersection technique for solving the non linear equations. A.M.Ostrowski's [18] introduced fourth order convergence iteration scheme for solving non linear equations. Chun and Ham [5] proposed some sixth order variants of Ostrowski's root finding methods. Grau et.al[7] proposed an improvement to Ostrowski's root finding method. Miquel Grau-Sanchez [15] proposed improvements of the efficiency of some three step iterative like Newton's methods. Several algorithms are available to solve [1], [2], [4], [17], [20], [22] nonlinear equations. Jovana Dzunic, Miodrag S. Petkovic[9] introduced derivative free method for solving nonlinear equations of Steffensen's type and compared with standard algorithms. In this paper, we introduce few new algorithms for minimization of nonlinear functions and comparative study is established among the new algorithms with Newton's algorithm by means of examples.

II. NEW ALGORITHMS

In this section, we introduce three new numerical algorithms for minimizing nonlinear real valued and thrice differentiable real functions.

Consider the nonlinear optimization problem: Minimize $\{f(x),\ x\in R,\ f:R\to R\}$ where f is a nonlinear thrice differentiable function.

Consider the function $G(x)=x-(g(x)/g'(x))$ where $g(x)=f'(x)$. Here $f(x)$ is the function to be minimized. $G'(x)$ is defined around the critical point x^* of $f(x)$ if $g'(x^*)=f''(x^*)\neq 0$ and is given by $G'(x)=g(x)g''(x)/g'(x)$.

If we assume that $g''(x^*)\neq 0$, we have $G'(x^*)=0$ iff $g(x^*)=0$.

Consider the equation $g(x) = 0$ whose one or more roots are to be found. $y = g(x)$ represents the graph of the function $g(x)$ and assume that an initial estimate x_0 is known for the desired root of the equation $g(x) = 0$. Here we consider iterative techniques to find the simple root of a non linear equation $g(x) = 0$ where $g : D \subset R \to R$ for an open interval D is a scalar function.

Let α be a simple real zero of a real function and let x_0 be an initial approximation to α. Consider the iterative function of J.F Traub [24] for g, we have

$$\phi(x) = \phi(x, y) = x - \gamma\, g(x)^2 / (g(x + \gamma\, g(x)) - g(x)) \qquad \text{\_\_\_\_\_\_\_\_(2.1)}$$

where $\gamma \neq 0$ is a real constant. Introducing $v(x) = g(x)/g'(x)$ and expanding the denominator in (2.1) in a geometrical series, we will get the following relation

$$\phi(x) - \alpha = (1 + \gamma\, g'(x))\, c_2(x)\, v(x)^2 + O(v(x)^3) \qquad \text{\_\_\_\_\_\_\_\_(2.2)}$$

In particular, choosing $x = \alpha$ it follows that $\phi(\alpha) = \alpha$ and $\phi'(\alpha) = 0$ which means that (2.1) defines at least a second-order iteration according to the Schröder–Traub theorem [24].

Taking $\gamma = -1/g'(x)$ and $g(x) = O(x - \alpha)$, we conclude from (2.2) that

$$\phi(x) - \alpha = O((x - \alpha)^3) \qquad \text{\_\_\_\_\_\_\_\_ (2.3)}$$

New method - 1

For this particular choice of γ, from (2.2) we introduce new method which is based on Newton secant iterative method with third order convergence is given by

$$x_{k+1} = x_k - \frac{(g(x_k)/g'(x_k))\, g(x_k)}{[g(x_k) - g(x_k - (g(x_k)/g'(x_k))]} \qquad \text{\_\_\_\_\_\_\_\_(2.4)}$$

Since $g(x) = f'(x)$, the equation (2.4) becomes

New Algorithm –I

$$x_{k+1} = x_k - \frac{(f'(x_k)/f''(x_k))\, f'(x_k)}{[f'(x_k) - f'(x_k - (f'(x_k)/f''(x_k))]} \qquad \text{\_\_\_\_\_\_\_\_(2.5)}$$

New method – 2

We introduce new method – (2) which is based on Halley's method with third order convergence [9], [22] is given by

$$x_{k+1} = x_k - \frac{g(x_k)}{g'(x_k)} \left[1 - \frac{g(x_k) g''(x_k)}{2(g'(x_k))^2} \right]^{-1} \qquad \text{\_\_\_\_\_\_\_\_(2.6)}$$

Since $g(x) = f'(x)$, the equation (2.6) becomes

New Algorithm –II

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)} \left[1 - \frac{f'(x_k) f'''(x_k)}{2(f''(x_k))^2} \right]^{-1} \qquad \text{\_\_\_\_\_\_\_\_(2.7)}$$

New method – 3

We introduce new method – (3) which is a variant of Ostrowski's method [9], [22] with third order convergence is given by

$$x_{k+1} = x_k - \frac{g(x_k)}{[(g'(x_k))^2 - g(x_k)\, g''(x_k)]^{1/2}} \qquad \text{\_\_\_\_\_\_\_\_(2.8)}$$

Since $g(x) = f'(x)$, the equations (2.8) becomes

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New Algorithm –III

$$x_{k+1} = x_k - \frac{f'(x_k)}{\left[(f''(x_k))^2 - f'(x_k)\, f'''(x_k)\right]^{1/2}} \qquad \text{————(2.9)}$$

III. CONVERGENCE ANALYSIS

The convergence analysis of new algorithm-I, new algorithm-II and new algorithm-III are of cubically convergence since the new algorithm-I is just a modification of Newton secant iterative method which is a third order of convergence, the new algorithm-II is just a modification of Halley's method which is a third order of convergence and the new algorithm-III is just a modification of a variant of Ostrowski's method which is also a third order of convergence.

IV. NUMERICAL ILLUSTRATIONS

Example 4.1: Consider the function $f(x) = x^3 - 2x - 5$. The minimized value of the function is 0.816497. The following table depicts the number of iterations needed to converge to the minimized value for all the new algorithms with three initial values $x_0 = 1$, $x_0 = 2$ and $x_0 = 3$.

Table – I: shows a comparison between the New iterative Algorithms and Newton's Algorithms

Sl. No	Methods	For initial value x_0 =1.000000	For initial value x_0 =2.000000	For initial value x_0 =3.000000
1	Newton's Algorithm	3	5	5
2	New Algorithm-I	2	3	3
3	New Algorithm-II	2	3	3
4	New Algorithm-III	2	3	3

Example 4.2: Consider the function $f(x) = xe^x - 1$. The minimized value of the function is -1. The following table depicts the number of iterations needed to converge to the minimized value for all the new algorithms with three initial values $x_0 = 1$, $x_0 = 2$ and $x_0 = 3$.

Table – II: shows a comparison between the New iterative Algorithms and Newton's Algorithms

Sl. No	Methods	For initial value x_0 =1.000000	For initial value x_0 =2.000000	For initial value x_0 =3.000000
1	Newton's Algorithm	7	8	10
2	New Algorithm-I	-	5	7
3	New Algorithm-II	4	5	5
4	New Algorithm-III	1	1	1

Example 4.3: Consider the function $f(x) = x^5 + x^4 + 4x^2 - 15$. The minimized value of the function is 0.0000. The following table depicts the number of iterations needed to converge to the minimized value for all the new algorithms with three initial values $x_0 = 1$, $x_0 = 2$ and $x_0 = 3$.

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Table – III: shows a comparison between the New iterative Algorithms and Newton's Algorithms

Sl. No	Methods	For initial value x_0 =1.000000	For initial value x_0 =2.000000	For initial value x_0 =3.000000
1	Newton's Algorithm	5	6	8
2	New Algorithm-I	3	4	5
3	New Algorithm-II	3	5	5
4	New Algorithm-III	3	4	4

Example 4.4: Consider the function $f(x) = x^4 - x - 10$. The minimized value of the function is 0.629961. The following table depicts the number of iterations needed to converge to the minimized value for all the new algorithms with three initial values $x_0 = 1$, $x_0 = 2$ and $x_0 = 3$.

Table – IV: shows a comparison between the New iterative Algorithms and Newton's Algorithms

Sl. No	Methods	For initial value x_0 =1.000000	For initial value x_0 =2.000000	For initial value x_0 =3.000000
1	Newton's Algorithm	4	6	7
2	New Algorithm-I	3	4	5
3	New Algorithm-II	3	4	5
4	New Algorithm-III	3	3	4

Example 4.5: Consider the function $f(x) = e^x - 3x^2$. The minimized value of the function is 0.20448. The following table depicts the number of iterations needed to converge to the minimized value for all the new algorithms with three initial values $x_0 = -1$, $x_0 = 0$, and $x_0 = 1$.

Table – V: shows a comparison between the New iterative Algorithms and Newton's Algorithms

Sl. No	Methods	For initial value x_0 =−1.000000	For initial value x_0 =0.000000	For initial value x_0 =1.000000
1	Newton's Algorithm	3	3	4
2	New Algorithm-I	2	2	3
3	New Algorithm-II	3	2	3
4	New Algorithm-III	-	-	-

V. CONCLUSION

In this paper, we have introduced three new numerical algorithms for minimizing nonlinear unconstrained optimization problems and compared with Newton's method. It is clear from numerical results that the rate of convergence of New algorithms-I, New algorithm-II and New algorithm-III are better than Newton's method in almost all cases except few cases. In real life problems, the variables can not be chosen arbitrarily rather they have to satisfy certain specified conditions called constraints. Such problems are known as constrained

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optimization problems. In near future, we have a plan to extend the proposed new algorithms to constrained optimization problems.

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