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Connectivity of Sensor Networks with Power Control

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***Abstract*— We consider a sensor network with an average of n nodes randomly placed over a region of unit area. We assume that each node is equipped with a wireless transceiver, and are interested in the minimum transmit power required for maintaining connectivity of the network when power control is employed (i.e., each node can choose a power level for transmission independent of any other node). We show that the average power gain per node (the ratio of the transmit power required without and with power control) increases with the number of nodes n as $(\log n)^{\alpha/2}$, where α is the path loss exponent.**

I. Introduction

With the availability of cheap wireless technology and the emergence of micro-sensors based on MEMS technology [3], [8], sensor networks are anticipated to be widely deployed in the near future. Such networks have many potential applications, both in the military domain (eg. robust communication infrastructure or sensing and physical intrusion detection), as well as commercial applications such as air or water quality sensing and control. Such sensor networks are usually characterized by the absence of any large-scale established infrastructure and nodes cooperate by relaying packets to ensure that the packets reach their respective destinations.

A major constraint in such networks is power. As the nodes are usually battery-powered, it is crucial that relaying and communication strategies be developed to minimize power utilization. Further, it is also beneficial for each node to use as small a transmit power as possible for interference reduction [5]. Such a reduction can potentially increase capacity of sensor networks [2]. However, many applications require that the transmit power used at the nodes be large enough to ensure that the sensor network is *strongly connected,* i.e., every node in the network should be able to communicate with any other node in the network (possibly using other nodes as relays).

In this paper, we consider a network of nodes placed according a spatial Poisson process with with intensity n, and consider the restriction of this process to a circular region of unit area. Thus, such a network has an average of n nodes, and for large n, this network approximates an uniformly distributed placement of n nodes over a unit area (see [1, Section 3]). We assume that each node employs *power control,* i.e., a node can choose a power level independent of any other node. We show that the average power gain per node by using power control (i.e., the ratio of the transmit power required without and with power control) increases with n as $(\log n)^{\alpha/2}$, where α is the path loss exponent.

II. Related Work

The basic idea behind packet forwarding schemes that attempt to minimize the power expended by nodes to forward packets is that multiple short hops are better than long hops. The power required to communicate with a node increases as d^{α}, where d is the distance of separation between the two nodes and α is the path loss exponent. Neglecting the power expended by a node to receive data, data transmission over two hops of length D always uses less power than a single transmission over a distance of $2D$ as long as $\alpha \geq 2$. Thus, it is beneficial to force the nodes to use shorter hops in order to save power.

In [1] and [7], the authors have proposed two different schemes to ensure the connectivity of sensor networks while attempting to minimize the power expended by the nodes at the transceiver. The major difference between the schemes is that [1] assumes that all nodes employ a common power level, whereas, in [7] it is assumed that each node is free to regulate its power level.

The authors in [1] consider a network of n nodes, uniformly distributed on a unit circle, i.e., a circle of unit area. All nodes employ a common power level, i.e., *no power control mechanism is used.* When a common transmit power level is used by all nodes, the authors in [1] have shown that as long as the transmission radius of each node is of order $\sqrt{\log(n)/n}$, the network will remain strongly connected. More precisely, a common *critical* transmit power (and thus, a common critical transmission radius) is chosen, which ensures that each node can communicate with all nodes within this critical radius. The critical radius which guarantees asymptotic connectivity under a fixed power scheme is shown in [1] to satisfy the relationship $\pi r^2(n) = \frac{\log n + c(n)}{n}$, where $c(n) \to \infty$ as $n \to \infty$. This scheme ensures that the network is strongly connected. However, the disadvantage of such a scheme is that some nodes use a much higher transmit power than necessary to maintain strong connectivity. A practical protocol based on the common transmission range idea is developed in [4].

Other related work includes that in [7] where power control is employed at each node. Each node determines a collection of *neighbor nodes* to which it can directly transmit without any relay nodes. To each of these neighbors, the node transmits at a power level that depends on the particular neighbor's position. In other words, the node has to switch power levels when transmitting to different neighbors. The authors in [7] show that the network is strongly connected under their proposed scheme. Nevertheless, the disadvantage of the scheme is that a node has to switch power levels when communicating with different neighbors, thus resulting in increased complexity.

In this paper, we first quantify the gain that any power control can yield. We consider the ratio of the average transmit power per node with and without power control. Let us denote $\bar{P}_{com}(n)$ to be the expected transmission power per node when all nodes employ a common power level, and $\bar{P}_{\mathcal{A}}(n)$ be the expected transmit power per node with power control using some power control policy denoted by $\mathcal{A}$ (thus, the total expected power expended in the network is $nP_{com}(n)$ and $nP_{\mathcal{A}}(n)$ for the two schemes). We show that there exists a positive constant $k_2 < \infty$ such that for the node density n large enough, we have

$$\frac{\bar{P}_{com}(n)}{\bar{P}_{\mathcal{A}}(n)} \leq k_2(\log n)^{\alpha/2}, \qquad (1)$$

where $\alpha \geq 2$ is the path loss exponent. Thus, this result implies that in a large-scale network, the gain with power control can grow *at most logarithmly fast.*

Next, we consider a power control scheme where each node can choose a different transmit power. However, *each node uses a single transmit power to communicate with all its "neighbors."* We show that employing such a "fixed" power control scheme is asymptotically optimal in the the sense sense that there exists a positive constant $k_1 > 0$, such that for n large enough, we have

$$k_1(\log n)^{\alpha/2} \leq \frac{\bar{P}_{com}(n)}{\bar{P}_{pow}(n)}, \qquad (2)$$

where $\bar{P}_{pow}(n)$ is the expected transmit power per node with a "fixed" power control scheme (details in Section IV).

III. Problem Setup

We consider the two-dimensional plane, with nodes placed according to a two-dimensional stationary Poisson Process Φ with intensity n, and restrict this process to a unit circle about origin in the 2-D plane.

Let $\alpha \geq 2$ be the path loss exponent. We say that a node can communicate with another node at a distance d from it if the transmission power chosen by the node is at least βd^{α}, where $\beta > 0$ is a constant corresponding to a required receive threshold. If a node a can communicate with a node b, we say that b is a neighbor of a. The network of nodes is said to be strongly connected if there exists a path (possibly using multiple relay nodes) between every pair of nodes.

IV. Results

Let $\bar{P}_{\mathcal{A}}(n)$ be the average transmission power per node under a power control policy $\mathcal{A}$ and with the nodes placed according to a two-dimensional spatial Poisson process with intensity n.

Theorem 4.1: A necessary condition for strong connectivity is the following. There exists a positive constant $0 < k_2 < \infty$ such that

$$\bar{P}_{\mathcal{A}}(n) \geq k_2 \left(\frac{1}{n\pi}\right)^{\frac{\alpha}{2}}.$$

Proof: For the network to be strongly connected, each node must be able to communicate with at least one other node. Using the fact that the node placement is a two-dimensional Poisson process[1] of intensity n, we calculate the palm distribution of the length of a hop corresponding to the closest node. Let us denote R to be the random variable representing the hop length. First, we have

$$Pr(R > r) = Pr(\Phi(B(0,r)) = 0),$$

where $B(0,r)$ is a ball or radius r about origin. From Slivnyak's theorem [9], we have

$$Pr(R > r) = \exp(-n\pi r^2).$$

Thus, we have

$$\begin{aligned} f_R(r) &= 2n\pi r \exp(-n\pi r^2) \\ E(R^{\alpha}) &= 2n\pi \int_0^{\infty} r^{\alpha+1} \exp(-n\pi r^2) dr \\ &= K\left(\frac{1}{n\pi}\right)^{\frac{\alpha}{2}} \end{aligned}$$

where

$$K = \Gamma\left(\frac{\alpha+2}{2}\right)$$

and $\Gamma(\cdot)$ is the Gamma function. As the above argument provides a lower bound on the average transmission power required for strong connectivity, we have

$$\begin{aligned} \bar{P}_{\mathcal{A}}(n) &\geq \beta . E(R^{\alpha}) \\ &= \beta K \left(\frac{1}{n\pi}\right)^{\frac{\alpha}{2}} \end{aligned}$$

and the result follows. ∎

Next, we consider a sufficient condition for strong connectivity with power control. At each node, let us partition the neighborhood into L sectors, with each sector of angle $L/360$ degrees. This is illustrated in Figure 1 where we consider an 8-partition, with each sector of 45 degrees.

Definition 4.1: Suppose that each node in the network transmits at a fixed power (possibly different for different nodes) such that there is at least one node (if at all possible)

[1] It can be shown that edge effects at the boundary of the unit circle are negligible.

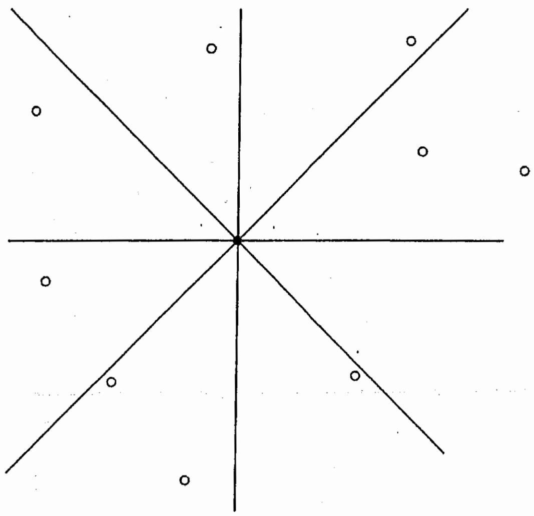

Fig. 1. An 8-partition about the center node.

within its transmission range in each sector of its L-partition. Such a network is said to be L-connected.

Thus, for a L-connected network, each node can reach *at least* L neighboring nodes (unless one or more of the sectors has no nodes). Note that each node could (and in general, will) choose a different transmission power.

Lemma 4.1: A L-connected network is strongly connected for any $L \geq 8$.

Sketch of Proof: The proof follows from the fact that given any source and destination node, with an 8-connected network, it is always possible to get closer to the destination at each hop. As the number of nodes are finite, the result follows. The details are available in [6]. ■

Let $\bar{P}_{pow}(n)$ be the average transmission power per node under a power control policy which maintains 8-connectivity. As before, assume that the nodes placed according to a two-dimensional spatial Poisson process with intensity n restricted to an unit circle.

Theorem 4.2: A sufficient condition for strong connectivity is the following. There exists a positive constant $0 < k_1 < \infty$ such that

$$\bar{P}_{pow}(n) \quad \leq \quad k_1 \left(\frac{1}{n\pi}\right)^{\frac{\alpha}{2}}.$$

Proof: We constrain all nodes to pick a transmit power that ensures 8-connectivity. Given this constraint, we find the expected power at which a node has to transmit. Let R be a random variable representing the range of a node's transmission. Using Slivnyak's Theorem [9], we compute the palm distribution of the transmission range. We have

$$Pr(R \leq r) = [1 - \exp(\tfrac{-n\pi r^2}{8})]^8$$

Differentiating, the density function of R is

$$f_R(r) = 8\left[1 - \exp\left(\frac{-n\pi r^2}{8}\right)\right]^7 \left[\frac{n\pi r}{4}\exp\left(\frac{-n\pi r^2}{8}\right)\right].$$

Expanding the first term using the binomial theorem and simplifying, we have

$$E(R^\alpha) \quad = \quad \frac{K}{(n\pi)^{\frac{\alpha}{2}}},$$

for some $K > 0$. Thus, the expected transmission power for a node is given by $\frac{\beta K}{(n\pi)^{\frac{\alpha}{2}}}$, and the result follows. ■

Let us denote $\bar{P}_{com}(n)$ to be the expected transmission power per node when all nodes employ a common power level. As before, the network considered is that of a collection of nodes placed according to a two-dimensional Poisson process with intensity n and restricted to an unit circle. It has been shown in [1, Section 3] that the conditions for connectivity (when employing a common power) are asymptotically the same for such a restricted Poisson process and placing n nodes in a unit circle with uniform distribution. Further, it follows from [1] that for large enough n the average transmission power is given by

$$\bar{P}_{com}(n) \quad = \quad \beta\left(\frac{\log(n)}{n}\right)^{\alpha/2}$$

Combining this with Theorems 4.1 and 4.2, the main results in (1 and (2) follow.

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REFERENCES

[1] P. Gupta and P. R. Kumar. Critical power for asymptotic connectivity in wireless networks. In *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W.H. Fleming. Edited by W.M. McEneany, G. Yin, and Q. Zhang*, pages 547–566, Boston, 1998. Birkhauser.

[2] P. Gupta and P. R. Kumar. The capacity of wireless networks. *IEEE Transactions on Information Theory*, IT-46(2):388–404, March 2000.

[3] J. M. Kahn, R. H. Katz, and K. S. J. Pister. Mobile networking for smart dust. In *Proceedings of ACM Mobicom*, Seattle, WA, August 1999.

[4] R. S. S. Narayanaswamy, V. Kawadia, and P. R. Kumar. Power control in ad-hoc networks: Theory, architecture, algorithm and implementation of the COMPOW protocol. In *Proceedings of European Wireless 2002. Next Generation Wireless Networks: Technologies, Protocols, Services and Applications*, pages 156–162, Florence, Italy, 2002.

[5] T. S. Rappaport. *Wireless Communications: Principles and Practice.* Prentice Hall, Upper Saddle River, NJ, 2002.

[6] B. Rengarajan, J. Chen, S. Shakkottai, and T. Rappaport. Connectivity of sensor networks with power control. WNCG Technical Report, 2003.

[7] V. Rodoplu and T. H.-Y. Meng. Minimum energy mobile wireless networks. *IEEE Journal on Selected Areas in Communications*, 17(8):1333–1344, August 1999.

[8] K. Sohrabi, J. Gao, V. Ailawadhi, and G.J. Pottie. Protocols for self-organization of a wireless sensor network. *IEEE Personal Communications*, 7(5):16–27, October 2000.

[9] D. Stoyan, W. S. Kendall, and J. Meche. *Stochastic Geometry and its Applications.* John Wiley and Sons, 1995.