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RESPONSE SURFACE METHOD FOR UPDATING DYNAMIC FINITE ELEMENT MODELS

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ABSTRACT

A finite element model of a structure can be updated as certain criteria based on experimental data are satisfied. The updated FE model is considered a better model for future studies in dynamic response prediction, structural modification, and damage identification. A finite element model updating technique incorporating the concept of response surface approximation (RSA) requires no sensitivity calculations and is much easier to implement with a general-purpose finite element code. The proposed updating method was incorporated with MSC.Nastran to solve the updating problem for an H-shaped frame structure. The updated results show that the predicted and experimental modes are correlated well with high MAC values and with a maximum frequency difference of 1.5%. Moreover, the updated parameters provide a physical insight to the modeling of bolted and welded joints of the H-frame structure.

INTRODUCTION

Finite element model updating [1] is a study in how to combine the strength of both the finite element (analytical) and experimental analyses for studying the dynamic behavior of a structure. Although the finite element (FE) method produces more natural frequencies, mode shapes, and spatial information to characterize the structure, the accuracy of these results usually requires further experimental confirmation. The experimental modal testing, on the other hand, yields fewer modes and less spatial resolution, but generally inspires more confidence in its results. A finite element model of a structure can be updated as certain criteria based on experimental data are satisfied. The updated FE model is considered a better model for future studies in dynamic response prediction, structural modification, and damage identification. In the early years of the development of finite element model updating, most authors studied cases for simulated structures only. In recent years, more and more model updating cases for real structures have been reported, e.g., [2-5].

Model updating techniques have been generally categorized as either direct or sensitivity-based methods [1,2]. The sensitivity methods guided by eigen-sensitivities can produce updated FE models with physical meaning whereas the direct methods may not. Yet, the need for calculations of sensitivities may post a problem for the sensitivity methods. There are three ways to perform a sensitivity calculation, through analytical, semi-analytical, or finite difference methods. Among them, the most popular one is the finite difference scheme, which is also the logical choice for sensitivity calculations in those cases when an analytical expression of the sensitivity is either not available or too complex to implement. However, the finite difference method can be computationally costly and susceptible to truncation and condition errors that are associated with the various step sizes used [6]. Furthermore, it is still not a simple task to incorporate a model updating method and the finite difference scheme in a general-purpose FE software package.

In recent years, the response surface methodology (RSM) [7] has been employed by many authors to solve design optimization problems, especially in the area of multidisciplinary design optimization (MDO), e.g., [8-10]. The RSM is a collection of procedures including design of experiments (DOE), model selection and fitting, and optimization on the fitted model. A (RSA), usually in the form of a simple polynomial function, can be built from DOE and model fitting. Venter and Haftka [11] employed response surface approximations for filtering numerical noise in FE analyses, which is associated with the dependence of discretization error on shape design variables. Once a polynomial RSA is created, the optimization on the function can be easily accomplished by most optimization packages.

A finite element model updating technique incorporating the concept of RSA does not fall into either of the two categories mentioned earlier. The method not only requires no sensitivity calculations and therefore eliminates the possible problems for methods needing sensitivity calculations, but also is much easier to implement with a general-purpose FE code.

The purpose of this paper is to present such an approach to updating dynamic FE models, and to integrate the process with a commercial FE package, i.e., MSC.Nastran, to solve model updating problems for real structures.

RESPONSE SURFACE APPROXIMATIONS

Response surface approximations (RSAs) play a crucial role in RSM. A response surface is a functional expression for a relationship between a response and a set of dependent variables. The response surface approximations are usually applied for two purposes: (1) to build an empirical model from experimental data; and (2) to approximate a complicated or noise-polluted function with a combination of much simpler functions. The second purpose is served in this research. A complex function $\boldsymbol{y}$ can be approximated by a response surface approximation $\boldsymbol{g}$ with $\boldsymbol{n}$ independent variables (or design variables) $\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_n$ as

$$\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_n) + \varepsilon \tag{1}$$

where ε is the difference between the approximated and exact value of $\boldsymbol{y}$. The approximating function $\boldsymbol{g}$ usually takes on the form of a polynomial whose coefficients can be determined by the least squares method based on a chosen set of evaluated values of $\boldsymbol{y}$. Moreover, in order to attain a better numerical condition, the dependent variables are in general normalized and non-dimensionalized. For a second order polynomial approximation, $\boldsymbol{g}$ has the form

$$g = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \beta_{ii} x_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \beta_{ij} x_i x_j \tag{2}$$

where the βs are the coefficients to be determined and there are $(\boldsymbol{n}+1)(\boldsymbol{n}+2)/2$ such coefficients; the $\boldsymbol{x}$s denote the normalized independent variables.

A successful application of RSA is greatly dictated by a proper choice of sampling points in design space, i.e., design of experiments. A face-centered central composite design (FCCD) [7] with its design variables confined within certain upper and lower bounds belongs to a family of central composite designs (CCDs), which are the most popular second-order designs. An FCCD consists of $2^{\boldsymbol{n}}$ factorial points, $2\boldsymbol{n}$ face-centered configurations, and one center point, for a total of $2^{\boldsymbol{n}}+2\boldsymbol{n}+1$ design points with $\boldsymbol{n}$ being the number of design variables. Figure 1 shows the design for the case of 3 design variables and there are 15 dots corresponding to as many design configurations.

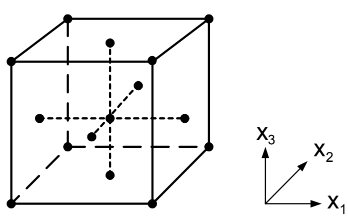

Figure 1: Face-centered central composite design with *k*=3

THE MODEL UPDATING PROCEDURE

An error vector is defined as a vector containing the relative differences between the experimental and FE natural frequencies. Thus, a model updating problem can be defined as to minimize the length of the error vector as follows:

$$Minimize \quad \|e\| = \left[\sum_{i=1}^{m} e_i^2\right]^{\frac{1}{2}} \tag{3}$$

Subject to

$$x_j^{l0} \le x_j \le x_j^{u0} \qquad j = 1,\ldots,n$$

and

$$e_i = \frac{f_i^a - f_i^e}{f_i^e} \qquad i = 1,\ldots,m \tag{4}$$

where $\boldsymbol{f}_i$ is the natural frequency for the matched mode $\boldsymbol{i}$; the superscripts $\boldsymbol{a}$ and $\boldsymbol{e}$ represent FE analysis and experimental results, respectively; $\boldsymbol{x}_j$ is the $\boldsymbol{j}$th normalized design variable (or independent variables); x_i^{u0} and x_i^{l0} denote the upper and lower bounds, respectively, for the normalized design variable; and $\boldsymbol{m}$ is the number of modes included in the updating process. To form the relative differences, which constitute the elements of the error vector in Eqs. (3) and (4), the test and the analysis natural frequencies have to be properly checked and correctly paired. The modal assurance criteria (MAC), which compare the similarity between two vectors, can be used to ensure that FE analysis and experimental modes are matched correctly, and are given as [12]

$$MAC_{ij} = \frac{\left|\{\psi_i^e\}^H\{\psi_j^a\}\right|^2}{\left|\{\psi_i^e\}^H\{\psi_i^e\}\right|\left|\{\psi_j^a\}^H\{\psi_j^a\}\right|} \tag{5}$$

where ψ_i^e and ψ_j^a are the $\boldsymbol{i}$th experimental and the $\boldsymbol{j}$th FE mode shape vectors, respectively. The superscript $\boldsymbol{H}$ denotes the hermitian operator. According to Eq. (5), a ***MAC*** value can varies from 0 and 1 with the value of 1 representing the pairing of two identical modes and a smaller value a less similarity between two modes. In general, a MAC value of at least 0.8 to 0.9 is usually required for a correct match.

The relative differences, i.e., $\boldsymbol{e}_i$, $\boldsymbol{i}$=1, ..., $\boldsymbol{m}$ in Eq. (4), are complex and implicit functions of the design variables. To approximate these functions by RSAs, repeated FE analyses are performed on all design configurations, after which explicit functional relations, also known as response surfaces, of $\boldsymbol{e}_i$, $\boldsymbol{i}$=1, ..., $\boldsymbol{m}$ with respect to the design variables are created by least squares curve fitting on the relative differences to polynomial models as

$$e_i = \beta_0^i + \sum_{j=1}^{n} \beta_j^i x_j + \sum_{j=1}^{n} \beta_{jj}^i x_j^2 + \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \beta_{jk}^i x_j x_k \quad i = 1, ..., m \tag{6}$$

As for the length of the error vector, i.e., $\|e\|$ in Eq. (3), there are two ways to build RSAs. One is to follow the above procedure with $\|e\|$ replacing $\boldsymbol{e}_i$ as

$$\|e\| = \beta_0^l + \sum_{i=1}^{n} \beta_i^l x_i + \sum_{i=1}^{n} \beta_{ii}^l x_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \beta_{ij}^l x_i x_j \tag{7}$$

The minimization of the quadratic function in the above equation subjected to side constraints can be easily carried out by common optimization routines. The other way to build an RSA for $\|e\|$ is to employ all $\boldsymbol{m}$ sets of RSAs for $\boldsymbol{e}_i$, $\boldsymbol{i}$=1, ..., $\boldsymbol{m}$ to form the error vector, whose length can be calculated and then minimized. The composite response surface obtained in the latter approach is no longer a simple polynomial function and its global optimum is not guaranteed by common optimization routines. The first approach was adopted in the application example presented in the next section.

Due to the fact that good RSA results are usually valid only within certain distance around the center design points, move limits [6] should be imposed on the updating parameters to ensure a better curve fitting result for RSAs. This is particularly true for approximating highly nonlinear functions like the eigenfunctions (or the frequency functions) using quadratic polynomials in our study.

As an example to further clarify the mechanism for approximating $\|e\|$ using RSAs, assume that we have two updating parameters and their initial values are $\boldsymbol{p}_1$=200 GPa and $\boldsymbol{p}_2$=3000 kg/m^3, and at the current state the parameters have been updated to $\boldsymbol{p}_1$=160 GPa and $\boldsymbol{p}_2$=3600 kg/m^3. Then the normalized parameters are $\boldsymbol{x}_1$=0.8 and $\boldsymbol{x}_2$=1.2. If the move limits at the current state are $\pm$0.2 for $\boldsymbol{x}_1$ and $\pm$0.3 for $\boldsymbol{x}_2$, FCCD gives the following nine ($\boldsymbol{n}$=2, $2^n+2\boldsymbol{n}+1=9$) design configurations ($\boldsymbol{x}_1$,$\boldsymbol{x}_2$): (0.8,1.2), (0.8,1.56), (0.8,0.84), (0.96,1.2), (0.96,1.56), (0.96,0.84), (0.64,1.2), (0.64,1.56), and (0.64,0.84). The normalized updating parameters of these configurations are transformed back to their original dimensions and then FE analyses are executed nine times using as many sets of parameters to produce frequencies and mode shapes. The calculated frequencies together with the test data yield nine different observations of $\|e\|$. Finally, Eq. (7) is employed repeatedly to create nine linear equations with six (($\boldsymbol{n}$+1)($\boldsymbol{n}$+2)/2=6) unknown βs and then the response surface for $\|e\|$ is constructed by using the least squares method.

To conclude this section, the complete procedure for the proposed model updating technique is stated as follows. The process begins with a proper selection of the parameters to be updated. The updating parameters should be chosen such that the computed eigenvalues and eigenvectors are sensitive to them. Moreover, the selection should be based on good engineering judgment and reflect the uncertainties in the FE model [1,2]. The next step is to perform repeated FE analyses to obtain $2^n+2\boldsymbol{n}+1$ sets of frequencies and mode shapes according to FCCD on the normalized updating parameters. To ensure a better curve fitting result for RSAs, move limits are also imposed on the parameters in this step. Then, the test data are introduced and the experimental and analysis modes are paired using MAC, after which the relative differences in Eq. (4) and the length of the error vector defined in Eq. (3) are calculated for all $2^n+2\boldsymbol{n}+1$ sets of configurations. The coefficients of the RSA for $\|e\|$ are obtained by curve fitting the data sets to a second order polynomial. Lastly, Eq. (3) is solved using a standard constrained optimization routine to yield a set of updated parameters, and the results are checked for convergence. If converged, the process is stopped; otherwise, a new set of design points centered at the updated parameters and bounded by shrunken move limits are created based on FCCD, and the process is continued in an iterative way.

APPLICATION

The model updating procedure mentioned above was applied to an H-shaped frame structure in this paper. The H-frame [13] consists of three steel rectangular tubes, four aluminum end plates of 25.4 mm in thickness, and four steel plates of 12.7 mm in thickness that join the long tubes and the aluminum plates. Each steel rectangular tube has a nominal shell thickness of 6.35 mm. All structural members are connected either by welding or by bolting. The geometry of the H-frame is shown in Fig. 2 and the test model, which consists of 79 measurement points, can be seen in Fig 3. The experimental data containing eight modes of natural frequencies, damping ratios, and mode shapes of the H-frame tested under a free-free condition with shaker excitations are adopted from Chen [13]. The test frequencies and damping ratios of the H-frame are given in Table 1.

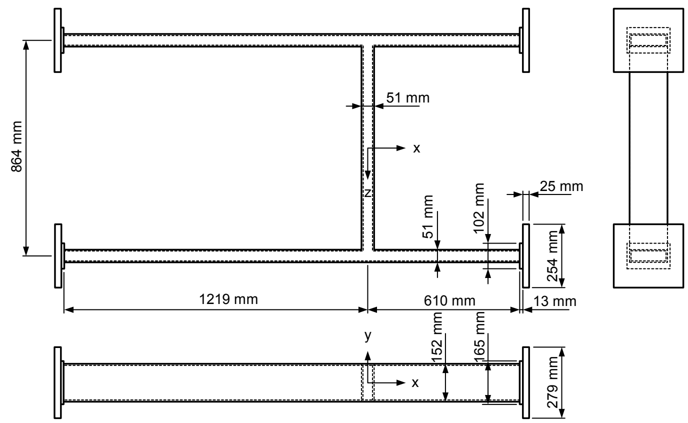

Figure 2: Geometry of the H-frame

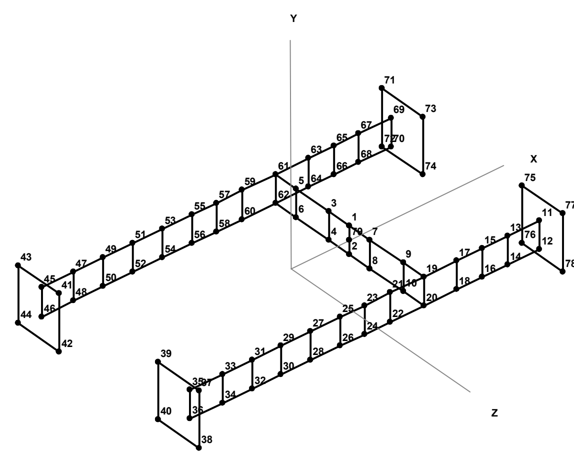

Figure 3: The H-frame test model

Table 1: Test frequencies and dampings of the H-frame

Mode no.	1	2	3	4
Frequency (Hz)	14.952	23.904	36.447	55.048
Damping (%)	0.738	0.141	0.104	0.112
Mode no.	5	6	7	8
Frequency (Hz)	77.201	157.905	162.070	168.669
Damping (%)	0.103	0.050	0.119	0.125

The FE model of the H-frame structure shown in Fig. 4 consists of quadrilateral and triangular shell elements and four rigid elements connecting the aluminum and steel plates to the four free ends of the tubes. The computed results of this initial FE model using MSC.Nastran [14] are compared with those from the test in Table 2. It is clearly shown in Table 2 that the frequencies and MAC values are rather good even for the initial, before updating FE model. In fact, not only the diagonal terms of the MAC matrix have high values, but also all the offdiagonal terms are very small. This means that the test and the FE analysis modes are well matched and that the H-frame's mode shapes are cleanly shaped, which is typical for a relatively simple and lightly damped structure.

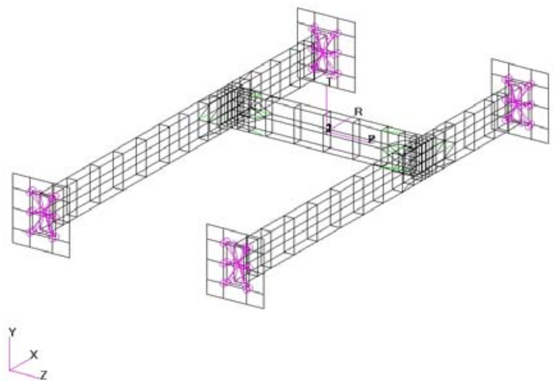

Figure 4: the H-frame FE model

Table 2: Comparison of the results from the test and initial FE model

Mode no.	Test freq. (Hz)	FEA freq. (Hz)	Relative difference (%)	MAC
1	14.952	15.035	0.554	0.991
2	23.904	23.881	-0.098	0.983
3	36.447	36.947	1.287	0.995
4	55.048	56.835	3.246	0.944
5	77.201	78.917	2.223	0.927
6	157.905	161.938	2.554	0.994
7	162.070	167.860	3.573	0.930
8	168.669	175.034	3.774	0.956

To begin the updating procedure, five groups of elements were first defined and they are depicted in Fig. 5. The Young's moduli of the first four groups, denoted by DV1 through DV4, and the mass density of the fifth group, denoted by DV5, were selected as the updating parameters (design variables) and their initial values are given in Table 3. The grouping of the elements can ensure a symmetrical FE model and reduce the number of updating parameters, whereas the selection of the parameters is to reflect the fact that welding, drilling, and bolting may alter the structure's local stiffness and mass properties. Since the aluminum plates were drilled and bolted together by four steel bolts and nuts to the steel plates, which were also welded to the tubes, updating the mass density of the aluminum plates (DV5) is equivalent to finding the effective mass of the plates. The upper and lower bounds for the updating parameters DV1~DV4 were set as to allow a maximum variation of 60% above and below their initial values, and 30% for DV5.

Move limits were also applied to the updating parameters and they were set in the following way:

$$x_i^u = x_i^c + 0.5\left(\gamma^{1/n} d_i^0\right) \qquad i = 1,\ldots,n \tag{8}$$

$$x_i^l = x_i^c - 0.5\left(\gamma^{1/n} d_i^0\right) \qquad i = 1,\ldots,n \tag{9}$$

and

$$d_i^0 = x_i^{u0} - x_i^{l0} \qquad i = 1,\ldots,n \tag{10}$$

where x_i^u and x_i^l represent the upper and lower limits for the ***i***th normalized design parameter at current iteration; x_i^c is the normalized design parameter obtained from previous iteration and is always equal to 1; ***n*** denotes, again, the number of design parameters; and γ is a coefficient for reducing the design space as the iteration proceeds and is defined as

$$\gamma = \begin{cases} 0.5 & \text{for } k \le 2 \\ (0.5)^{k-1} & \text{for } k \ge 3 \end{cases} \tag{11}$$

with ***k*** being the number of iterations. If x_i^u gets higher than x_i^{u0} or x_i^l moves lower than x_i^{l0}, the move limit in question will be replaced by the absolute upper or lower bound, i.e., x_i^{u0} or x_i^{l0}. For the first two iterations, $\boldsymbol{k} \le 2$, the reduction coefficient γ, set to be 0.5, defines a design space that is one half in size of the original space and that will be searched for an optimal set of updating parameters. For $\boldsymbol{k} \ge 3$, γ further reduces the design space by 50% each time as the updating process moves to the next iteration.

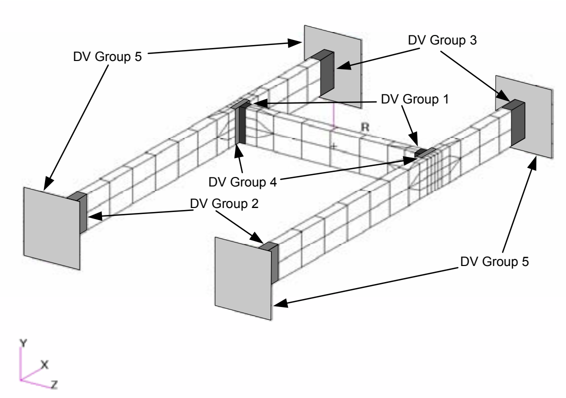

Figure 5: Grouping of the Design Parameters

Table 3: Initial values for the updating parameters

Updating para.	DV1 (GPa)	DV2 (GPa)	DV3 (GPa)	DV4 (GPa)	DV5 (kg/m^3)
Initial value	206.8	206.8	206.8	206.8	2768

Beside MSC.Nastran for FE analyses, the RSA phase, including the move limits, and the optimization phase of the proposed updating procedure were implemented in a MATLAB [15] code. After five iterations, the updating process appeared to have converged. Table 4 shows a comparison of the final updated natural frequencies with the test results and the MAC values for the matched modes, and Table 5 presents the corresponding updated parameters. The iteration history for the length (2-norm) of the error vector and the frequency differences can be seen in Fig. 6. The updated FE mode shapes for all eight modes, compared with their experimental counterparts, are also shown in Fig. 7.

It can be observed from Table 4 that most modes have been improved to be within 1% of frequency difference relative to the test modes and all eight pairs of matched modes attain high MAC values. Comparing Table 5 with Table 3 shows that the change of the effective mass density of the aluminum plates (DV5), increased from 2768 kg/m^3 to 2975 kg/m^3, indicates the higher density effect from the steel bolts and nuts fastening the plates to the tubes. Additionally, the updated Young's moduli

(DV1~DV4) in Table 5 suggest that welding changes the H-frame's local stiffness and that the masses of the aluminum plates tend to "weaken" the joints modeled as rigid connections connecting the aluminum plates to the steel tubes (see DV2 and DV3).

While a logical explanation can be provided to the decreases in DV2, DV3, and DV4, the updated value of DV1 (301.8 GPa) shows unrealistically high stiffness around the welded joint area at both ends of the center tube in the updated model. This reflects that the selection of DV1 and its grouping (see Figure 5) may require modifications. There are many ways that may improve the physical meaningfulness of the updated model. One of them is to combine both groupings of DV1 and DV2 into one single grouping with Young's modulus as the updating parameter. This group of elements includes the entire, welded joint area connecting the leg tubes and the center tube. Additionally, the shell thickness of the three tubes can be selected as a global updating parameter to further improve the result. These two alterations and other possibilities are currently under investigation, and their results will be reported in the future.

Table 4: Comparison of the results from the test and final updated FE model

Mode no.	Test freq. (Hz)	Updated FEA freq. (Hz)	Relative difference (%)	MAC
1	14.952	14.831	-0.812	0.994
2	23.904	23.697	-0.867	0.976
3	36.447	36.509	0.088	0.991
4	55.048	55.427	0.689	0.957
5	77.201	77.481	0.362	0.948
6	157.905	159.335	0.906	0.959
7	162.070	164.442	1.464	0.942
8	168.669	171.207	1.500	0.989

Table 5: The final updated parameters

Updating para.	DV1 (GPa)	DV2 (GPa)	DV3 (GPa)	DV4 (GPa)	DV5 (kg/m^3)
Updated value	301.8	82.7	107.5	158.9	2975

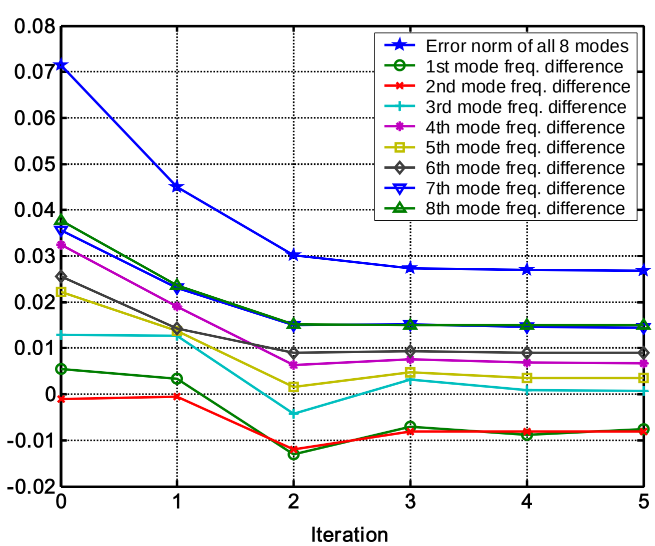

Figure 6: Iteration history for the error norm and the frequency differences

CONCLUSIONS

A model updating problem can be defined as to minimize the length of an error vector containing the relative differences between the experimental and FE natural frequencies, while during the updating process the FE and experimental modes are to be checked using MAC to ensure that they are correctly matched. The length of the error vector can be approximated, based on the design of experiments of FCCD, by a second order polynomial, which is a common form of RSA. With RSA, the proposed model updating procedure requires no sensitivity calculations and is much easier to implement with a general-purpose FE code. However, to ensure a better curve fitting result for RSA, move limits are imposed on the updating parameters. Then, the updating problem is solved in an iterative way.

Beside MSC.Nastran for FE analyses, the proposed updating procedure was implemented in a MATLAB code for performing RSA and optimization. The H-frame example has demonstrated the effectiveness of the proposed method. The updating parameters were selected to reflect the uncertainties in the joints of the FE model. Although one updated parameter gave an unexpected high value, the updated results clearly showed the higher density effect of the steel bolts and nuts on the aluminum plates, local stiffness altering on the welded joints, and weakening on the joints due to mass loading from the aluminum plates.

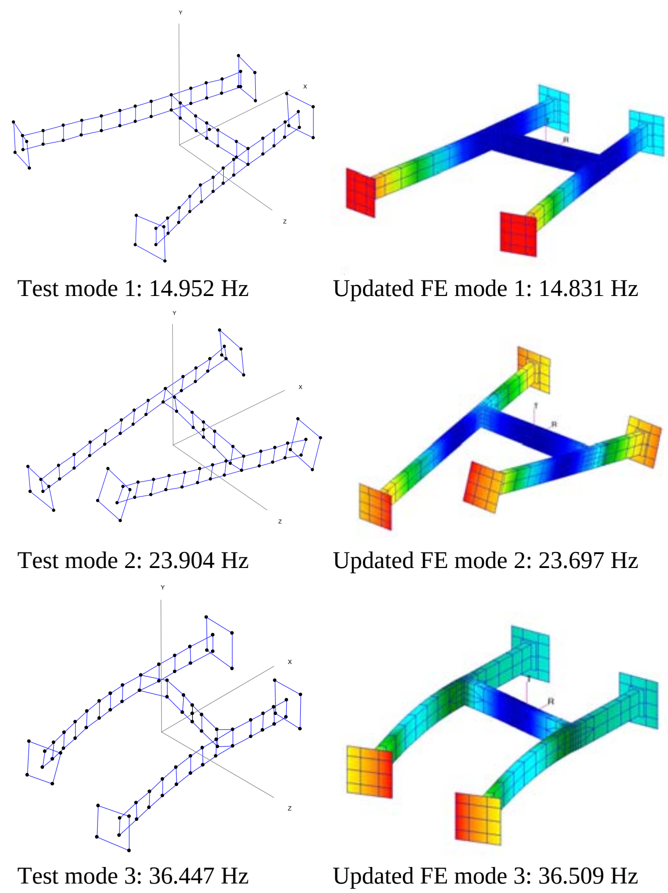

Figure 7: Comparison of the updated FE and the experimental mode shapes

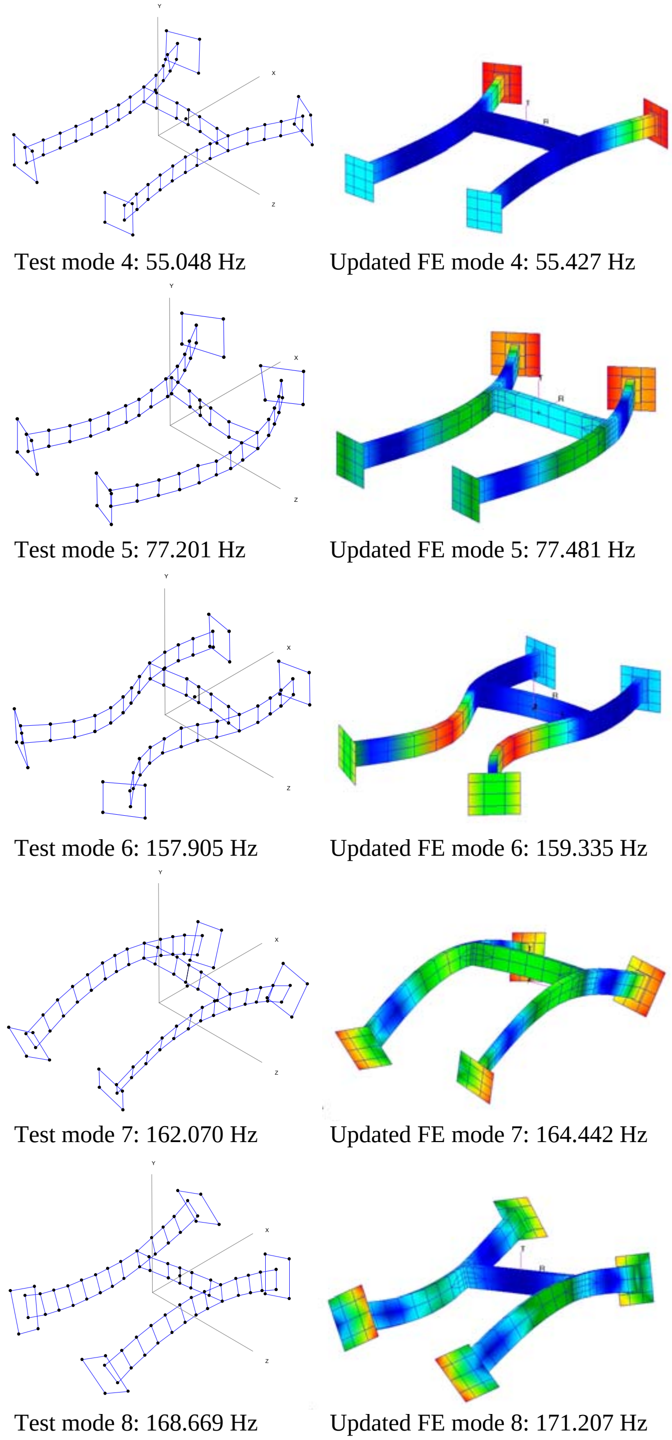

Figure 7: Comparison of the updated FE and the experimental mode shapes (continued)

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