Time-domain detection of superluminal group velocity for single microwave pulses

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(Received 17 May 2000)

Single microwave pulses centered at 9.68 GHz with 100-MHz (full width at half maximum) bandwidth are used to evanescently tunnel through a one-dimensional photonic crystal. In a direct time-domain measurement, it is observed that the peak of the tunneling wave packets arrives (440 ± 20) ps earlier than the companion free space (air) wave packets. Despite this superluminal behavior, Einstein causality is not violated since the earliest parts of the signal, also known as the Sommerfeld forerunner, remain exactly luminal. The frequency of oscillations and the functional form of the Sommerfeld forerunner for any causal medium are derived.

PACS number(s): 42.25.Bs, 03.65.Bz, 73.40.Gk, 42.25.−p

I. INTRODUCTION

In their authoritative work, Sommerfeld and Brillouin [1] considered the problem of electromagnetic wave propagation in a dispersive medium. In part, this study was intended to explain the abnormal behavior of the group velocity in the regions of anomalous dispersion since at the time it was known that for these frequency ranges the group velocity exceeds the speed of light in vacuum (or is ''superluminal''). Considering the propagation of a sinusoidally modulated step function through a Lorentzian medium, their delineation of the concept of wave velocity into such terms as phase, group, energy, and forerunner (both Sommerfeld and Brillouin forerunners) continues to be the standard description today. (To be complete one has to add the term ''signal velocity'' defined as the velocity of the half maximum point to the list. However, by their own admission such a definition is arbitrary ([1], p. 79), and as we will see this velocity also may be superluminal.) While phase, group, and even energy velocities are discussed in many undergraduate and graduate electromagnetic books, the velocity of the forerunners has not received much attention. (While Jackson was one of the few authors to treat this subject in the earlier additions of his well-respected book, ''*Classical Electrodynamics*,'' to our dismay we noticed that in the latest edition [2] this subject has been omitted.) This is of particular importance since, as will be shown below, the Sommerfeld forerunner velocity (also referred to as the front velocity) is the only physical velocity which must satisfy the requirements of special relativity.

The recent interest in the subject of superluminal group velocities was rekindled from consideration of the electron tunneling time. Since ''analogies'' between photon and electron tunneling in particular [3,4], and between Maxwell-Helmholtz and the Schrödinger wave equations in general [5], are well established, one may hope that experimental results from the more manageable photon tunneling experiment can be used to gain some insight into the more difficult problem of electron tunneling time.

Working in the *optical* regime, Chiao and co-workers [6] used conjugate pairs of photons emitted simultaneously in the process of spontaneous parametric downconversion, and found the tunneling velocity for a single photon through a one-dimensional photonic crystal (1DPC) to be superluminal. In this quantum-domain measurement, they observed superluminal velocities 1.7 times greater than c. Similarly, Spielman *et al.* used a Ti:sapphire laser capable of generating 10–15-fs optical pulses at $\lambda = 0.8\ \mu$m to study tunneling through a 1DPC [7]. Using mirror-dispersion control and a nonlinear background free-correlation technique, they were able to measure advances up to 6 fs in the autocorrelated signal.

To obtain larger advances in time, tunneling experiments can be performed in the microwave regime. In a series of experiments with different optical barriers such as undersized waveguide, misaligned horn antennas, and two side-by-side prisms, Ranfagni *et al.* investigated the superluminal tunneling for microwave frequencies [8–10]. Another series of microwave experiments were performed by Nimtz and his co-workers [11–16]. While they were able to improve on Ranfagni's original work with an undersized waveguide, their frequency domain experiments in general and their brief description of the 1DPC inserted inside an undersized waveguide in particular suffer from interpretation and measurement errors. The correct frequency domain measurement procedures are described in Ref. [17], and an attempt to correctly interpret the results, particularly in light of comments in Refs. [18,19], is undertaken here.

This paper is organized as follows. In Sec. II, the experiment with single microwave pulses evanescently propagating through a 1DPC is described. It is seen that a pulse tunneling through a 1DPC arrives (440 ± 20) ps earlier than a pulse traveling an equivalent physical distance in free space. In Sec. III, a proof is given that for a signal propagating a distance x, no detection is possible for times less than $t_0 = x/c$. Additionally, we have attempted to address the most common misunderstandings and misinterpretations associated with the subject of superluminal group velocities. In light of the importance of the Sommerfeld forerunner, particularly in relation to the requirements of special relativity, the frequency of oscillation and the functional form of these early fields for any causal medium are discussed in Sec. IV. Section V contains the conclusions and our summary.

II. EXPERIMENTAL RESULTS

The considerable difficulties associated with defining a unique tunneling time have been well documented [20,21].

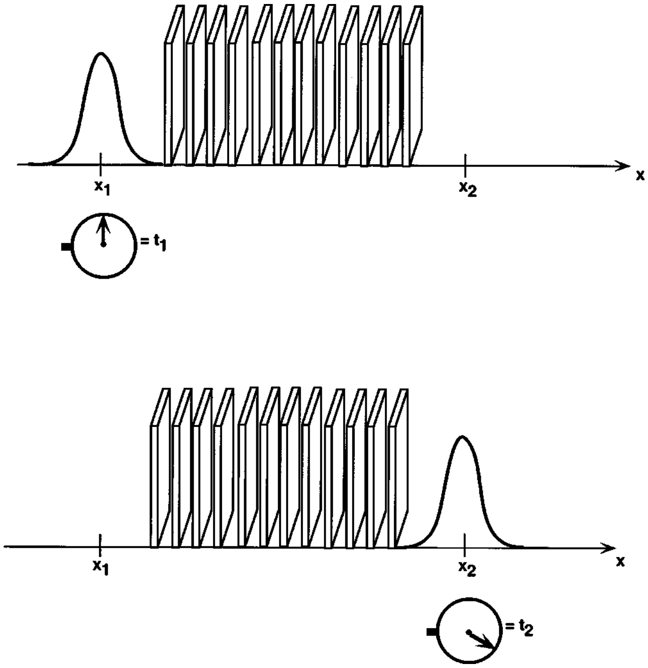

FIG. 1. Scheme used to define the ''operational'' time of flight.

However, regardless of the various definitions of tunneling time, one can, to use a phrase from Chiao [21], propose an ''operational'' definition of the time of flight. This idea is depicted in Fig. 1.

When the wave peak reaches the point x_1, we start our stop watch (t_1). Some time later (t_2), the wave maximum reaches the point x_2. The ''operational'' time of flight is then given by $t_2 - t_1$. In a more elegant version of the same idea, the stop watch is replaced with a companion pulse that travels the same distance ($x_2 - x_1$) in vacuum. In this manner, the time of flight for a pulse traversing a medium (here a 1DPC) can directly be compared to the time required to cover the same distance in free space. With the above prescription, one should be able to measure the time of flight for either electronic or photonic waves. In this paper we have concentrated on the electromagnetic wave packet tunneling and its time of flight.

Figure 2 shows the experimental setup used in the time-domain measurements. A BWO is used to generate the microwave pulse and a mode converter (MC) changes the TM_{01} mode (an annular pattern) to a TE_{11} mode (a central-lobe pattern), which is then radiated via a conical horn antenna (CHA).

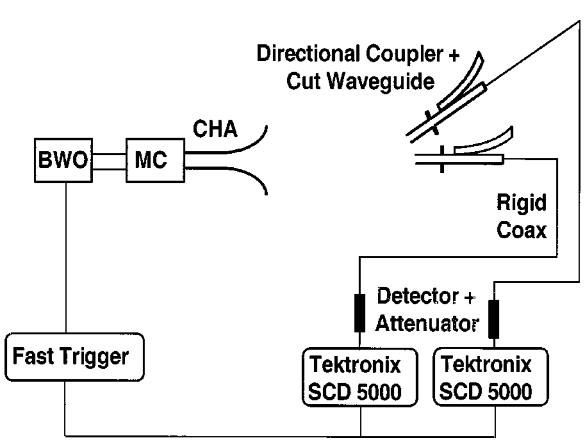

FIG. 2. Time-domain experimental setup.

The diameter of the CHA is 15 cm, which by conservative estimates places the antenna's far field at approximately 114 cm for 9.68 GHz. Two directional couplers attached to a series of attenuators and a HP 8470-B, low-barrier Schottky diode detector (provided in pairs) were used to detect the microwave pulse at two distinct points in the antenna's radiation intensity pattern. These two points will be referred to as the ''center'' (at the center of the antenna's pattern) and the ''side'' (at the side of the antenna's pattern). The signals from the HP detectors are then routed to two fast Tektronix SCD-5000 single channel oscilloscopes. Each Tektronix scope has a 4.5-GHz bandwidth and was set to record 1024 points over a 50-ns window. This means that the time interval between two adjacent points on the pulse trace was approximately 48.9 ps.

In order to reduce the scope trigger jitter as much as possible, a line from the Sinus-6 accelerator section of the BWO was routed to a PSPL fast picosecond pulse generator, model 4500E. This pulse generator is capable of producing triggering pulses with very sharp raise times (a 10–90 % rise time of roughly 100 ps), which in turn is used to trigger both Tektronix scopes. Such an effort reduces the uncertainty associated with the triggering jitter to approximately 20 ps. The frequency of the microwave pulse is measured by heterodyning the signal against a known oscillator (not shown in Fig. 2) [22]. Repeated measurements on the microwave signal indicated that the pulse frequency content is centered at 9.68 GHz with a 100-MHz bandwidth [full width at half maximum (FWHM)].

For our setup described above, a series of single shots were fired in order to measure the delay between the ''center'' and ''side'' paths. This delay is due to the fact that the cable length, the attenuators and detectors, and the internal response of the two scopes are not exactly identical. However, such a systematic and repeatable delay is readily measurable and its effect is easily removed by electronically introducing a delay or advancement for one of the two paths. [For example, the trigger delay option of the SCD-5000 can be used to introduce the appropriate delay such that the peaks of the two traces (''center'' and ''side'') arrive at the same time. Equally well, a data acquisition software such as LABVIEW or a plotting package can be used to shift one of the two traces by the measured delay such that their peaks arrive at the same time.] After synchronizing the two paths such that the peaks of the ''center'' and the ''side'' pulses arrive at the same time, a 1DPC with its band gap tuned to the main frequency component of the incident pulse (9.68 GHz) is inserted along the ''center'' path. The 1DPC used consisted of five polycarbonate sheets of thickness 1.27 cm and an index of 1.66 separated by regions of air of thickness 4.1 cm and index of unity. The details describing the design of this 1DPC will be discussed elsewhere. The insertion of the 1DPC along the ''center'' path allows us to measure the advancement or the delay of the tunneling pulse as compared to the companion free-space pulse (''side'').

Figure 3(a) shows the synchronized ''center'' and ''side'' pulses, without the 1DPC present. In order not to crowd the

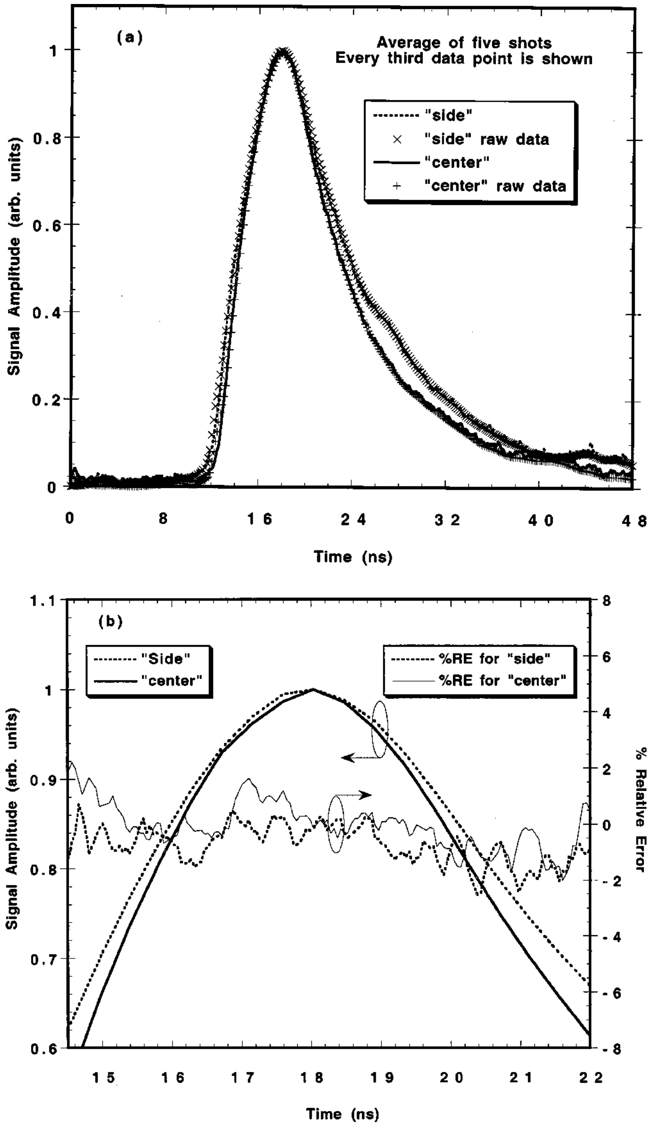

FIG. 3. Synchronized pulses propagating along ''center'' and ''side'' paths: (a) every third experimental raw data along with the locally weighted least-squares fit is shown, (b) an expanded view of (a) where the least-squares fit and the percent relative difference between the fit and the raw data is shown.

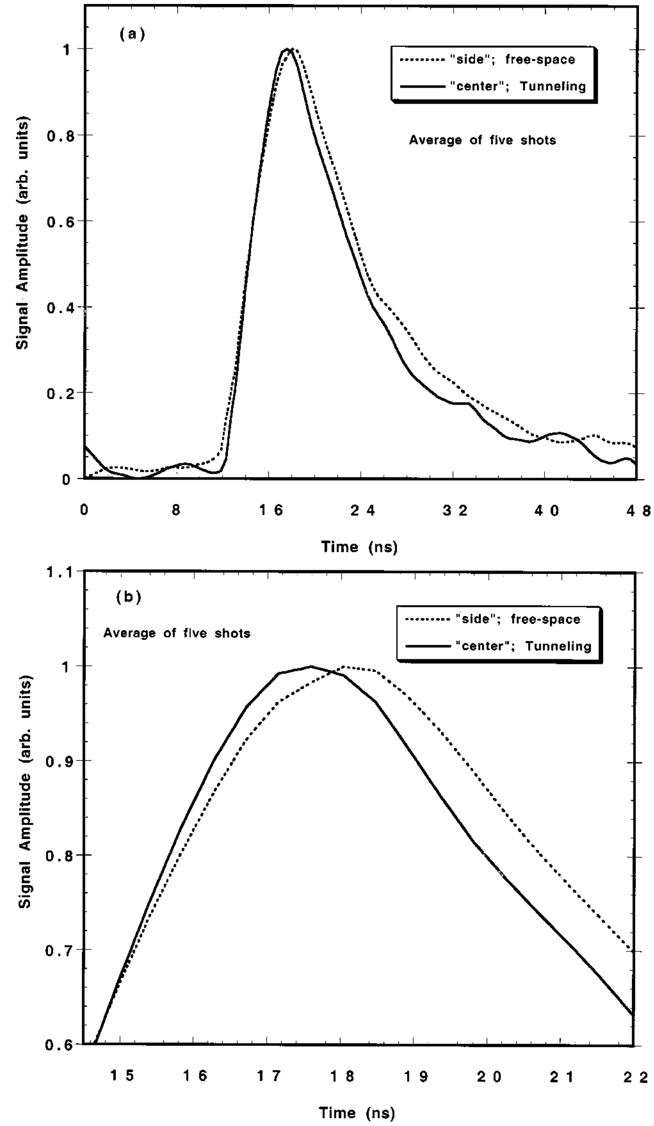

FIG. 4. The pulse propagating along the ''center'' path and tunneling through the 1DPC, and the pulse propagating along the ''side'' path in free space: (a) the normalized wave packets, (b) the expanded view of (a) in the vicinity of the pulses maxima.

figure, only every third experimental data point (the + and × signs) are shown. The solid curves are the locally weighted least-square fit, used to obtain the best smooth curves through the experimental data ([23], pp. 246 and 247), and they match the raw data (including those not shown here) well.

An expanded view of Fig. 3(a) in the vicinity of the pulse maxima is shown in Fig. 3(b). Also, on the right axes we have plotted the percent relative difference between the actual raw experimental data and the least-square fit. From this figure it is clear that the match between the raw data and the fit is good to less than 2.5%. Since the fit is of similar quality for the remaining figures presented in this paper, we display only the fitted curves for clarity of presentation. As is evident from this figure, the peaks of the ''center'' and the ''side'' pulses arrive at the same time. We note that the main reason for the difference between the two pulses' shapes is the fact that they were sampled at two different points of the radiation intensity pattern. Slight frequency response mismatches among the components used along the two paths are also contributing factors.

At this point, the 1DPC is inserted in the ''center'' path while leaving the ''side'' path unchanged. Figure 4(a) shows the result. This figure and its expanded view in the vicinity of the pulse maxima [Fig. 4(b)] indicate the pulse propagating along the ''center'' path, and tunneling through the 1DPC arrives sooner than the companion free-space ''side'' pulse. For the peak of the pulse, this shift to earlier time is measured to be 440 ± 20 ps. Although Figs. 3 and 4 display the normalized (with respect to the maximum) wave packets, it is important to note that due to the evanescent tunneling, the ''center'' wave packet has been attenuated by a factor of 2.8.

The traditional view of pulse propagation through a region with high attenuation (regions of anomalous dispersion) held that the extreme attenuation (coupled with the dispersion) would distort the signal to such an extent that the origi-

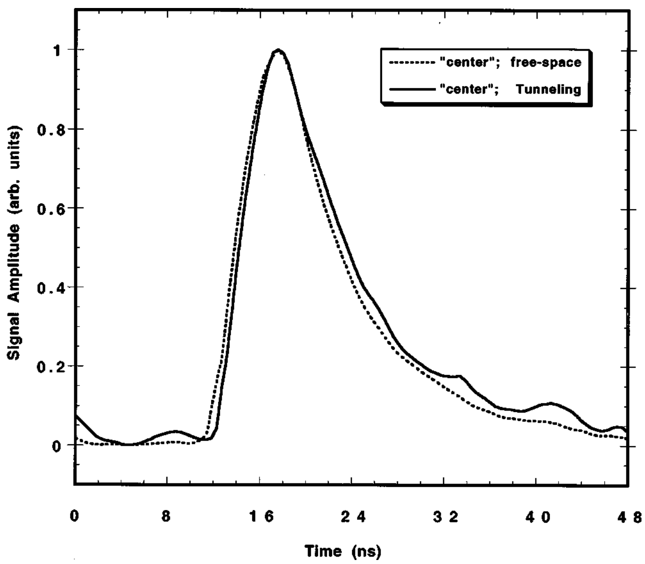

FIG. 5. A measure of the pulse broadening due to tunneling through 1DPC. The two pulses have propagated along the same path (''center'') in free space and through the 1DPC. The free-space pulse is manually shifted to an earlier time to make the comparison clearer.

nally well defined wave packet and its peak would not be recognizable upon emergence. For example, Landau and Lifshitz write, ''When considerable absorption occurs, the group velocity cannot be used, since in absorbing medium wave packets are not propagated but rapidly ironed out'' ([24], p. 285). In a similar manner, Sommerfeld, citing Laue, states, ''. . . with anomalous dispersion, due to the strong absorption which destroys the significance of a characteristic wavelength after a short path length, one can no longer sharply define the velocity of propagation of the energy'' ([1], p. 22). It is the same understanding which compelled Brillouin to write, ''. . . but if absorption also occurs, *a* [the wave vector] becomes complex or imaginary and the group velocity ceases to have a clear physical meaning'' ([25], p. 75). (The expression inside of the brackets is ours.) In light of the above, it is important to emphasize the following two points. First, for a sufficiently narrowband pulse centered in a region of minimal frequency dispersion, it is possible to propagate an evanescent mode through an optical barrier such that, while the transmitted wave packet is reduced in magnitude, it suffers negligible dispersion or distortion. Second, if group velocity is a useful physical parameter in describing the wave packet propagation through the ''center'' path without the 1DPC present (i.e., pulse marked ''center'' in Fig. 3, which propagates through free space), and if upon insertion of the 1DPC in the ''center'' path the emerging tunneling wave packet envelope (pulse marked ''Tunneling'' in Fig. 4), though reduced in amplitude, closely resembles the nontunneling wave packet, then the concept of group velocity is still valid for the latter case.

Figure 5 demonstrates these two points. It shows the tunneling (the solid curve) and the free-space (the dashed curve) pulses along the same ''center'' path. The tunneling pulse was obtained with the 1DPC inserted in the ''center'' path, whereas the free-space pulse was acquired without the presence of the PC for the same path. In order to make the comparison easier, the free-space pulse was manually shifted to earlier times to coincide with the tunneling pulse.

Measurements show that the FWHM of the free-space wave packet is approximately 9.1 ns, while the FWHM of the tunneling wave packet is 9.3 ns, indicating a 2.2% increase. Considering such a small broadening, we have to accept that if group velocity is a good parameter for the free-space pulse, it also must be a good parameter for the tunneling pulse.

The shift to earlier time displayed in Fig. 4 after synchronizing the two paths can be calculated according to

$$\Delta t = \frac{L_{\mathrm{PC}}}{c} - \tau_g , \tag{1}$$

where L_{PC} is the physical length of the PC and τ_g is the time associated with traversing the 1DPC, also known as group delay. (Group delay is the angular frequency derivative of the 1DPC transmission phase.) Since the structural parameters for the 1DPC are known, the PC length and the group delay (at 9.68 GHz) can be evaluated to be 22.75 cm and 320 ps, respectively [17,26]. Substituting these values in Eq. (1) results in the calculated time shift (Δt) of 438 ps, which is in good agreement with the measured value of 440 ± 20 ps. The group velocity of the wave packet propagating through a 1DPC of length L_{PC} is given by $\nu_g = L_{\mathrm{PC}}/\tau_g$ and is related to the time shift (Δt)

$$\nu_g = \frac{L_{\mathrm{PC}}}{(L_{\mathrm{PC}}/c) - \Delta t}. \tag{2}$$

Equation (2) implies that for the measured $\Delta t = 440 \pm 20$ ps, the microwave pulse group velocity traveling through the 1DPC is approximately $(2.38 \pm 0.15)c$. This agrees well with the calculated group velocity of $2.37c$.

Finally, let us consider the velocity by which the half maximum of the signal propagates. This velocity is of some historical importance since it was used by Sommerfeld and Brillouin to define the ''signal velocity'' ([1], p. 74). These authors used this velocity, hereafter referred to as the Sommerfeld signal velocity, as a velocity equal to the group velocity away from the regions of anomalous dispersion, which also remained subluminal within the region of anomalous dispersion ([1], p. 76). However, by their own admission such a definition is rather arbitrary ([1], p. 79). To directly cite them, Brillouin writes, ''In general the signal velocity measured depends on the sensitivity of the detecting apparatus used. With a very sensitive detector, even the forerunners, or certain parts of them, might be detected . . . But if the sensitivity of the detector is restricted to a quarter or half the final signal intensity, then an unambiguous definition of the signal velocity can, in general be given'' ([1], p. 100). A comparison of Figs. 3 and 4 shows that the half maximum point of the pulse propagating along the ''center'' path and tunneling through the 1DPC has shifted to earlier time by 303 ps, indicating that the Sommerfeld signal velocity is also superluminal. In light of this fact, and other problems associated with the definition of energy velocity in the case of an inverted medium as discussed in Refs. [21,27–32], the true ''signal velocity'' as it is to be used in connection with the theory of special relativity must refer to the velocity by which the front or Sommerfeld forerunner propagates. However, it must be pointed out that due to high frequency and small amplitude of the forerunner, this new definition of

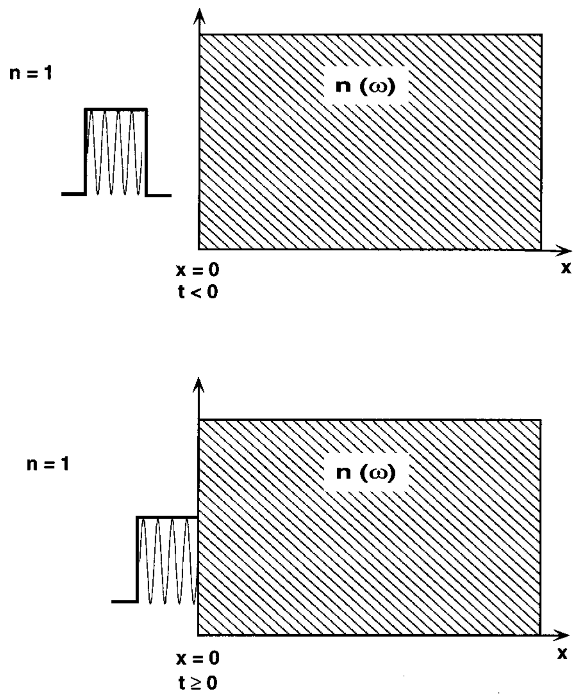

FIG. 6. A pulse impinging upon a causal medium characterized by index (or effective index) $n(\omega)$.

''signal velocity,'' although necessitated by Einstein causality, is probably not a practical definition under all circumstances. The subject of the front or Sommerfeld forerunner is discussed in the next section.

III. WHY EINSTEIN CAUSALITY IS NOT VIOLATED

A. Causal signals or signals with front

In this section a simple proof that no signal can travel faster than c is given. Although parts of this proof can be found elsewhere [21] ([33], pp. 315 and 316), in light of recent objections to Einstein causality for evanescent modes [19] and previous concerns regarding the feasibility of generating a front and its relevance to ''signal velocity'' [18], we intend to provide a more complete and coherent description of the underlying physics and the mathematical formalisms.

Figure 6 shows an incident electromagnetic pulse traveling in vacuum from left to right. At the time $t=0$, the pulse reaches the boundary of a medium characterized by the index of refraction $n(\omega)$, given by

$$n(\omega)=c\,\frac{k(\omega)}{\omega}. \tag{3}$$

This medium can be a dielectric slab, an undersized waveguide, a 1DPC, or any material or structure for which the dispersion is described by $n(\omega)$. For the sake of simplicity, here we only consider the case of one-dimensional propagation. However, the results presented here can easily be extended to higher-dimensional situations, although in some cases (for example, polarization effects or finite transverse size limitations) this extension may require more rigorous reasoning. For the pulse impinging on the boundary at normal incidence, the electric field at position x and time t is given by ([2], p. 336)

$$u(x,t)=\int_{-\infty}^{+\infty}\frac{2}{1+n(\omega)}A(\omega)e^{ik(\omega)x-i\omega t}\,d\omega$$
$$=\int_{-\infty}^{+\infty}g(\omega)e^{i\phi(\omega)}\,d\omega, \tag{4}$$

where $A(\omega)$ is the signal spectrum, given by

$$A(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}u(x=0,t)e^{i\omega t}\,dt. \tag{5}$$

We require that the signal have a well-defined front,

$$\begin{aligned} u(0,t)&=0 \quad \text{for } t<0,\\ u(0,t)&\neq 0 \quad \text{for } t\geqslant 0. \end{aligned} \tag{6}$$

The above condition is a requirement for any ''true signal.'' In other words, for any physically realizable electromagnetic pulse, there must be a point in time prior to which the amplitude of the field is identically zero. (Throughout this work, the effect of noise is neglected. Clearly in the presence of the noise, statistical considerations regarding noise and its effect on the front must be included.) For example, in our experiment the times prior to the discharge of the capacitor in the BWO result in zero amplitude for the microwave pulse. In a similar manner, for Chiao's single-photon experiment [6], one can always point to the times prior to the process of photon downconversion as times for which no tunneling photon exists. Stated differently, in contrast to the view set forth by the authors in Refs. [18,19], *strictly time limited* signals and not *strictly frequency band limited* signals are the norm of the physical universe. (Even the microwave background radiation, which has presumably started with the Big Bang, is by definition a strictly time-limited signal. Needless to say, this radiation was never utilized in any of the experiments concerned with superluminal group velocities.) Perhaps the arbitrary convention of defining the frequency content of a given electromagnetic pulse in terms of a pair of numbers (be it FWHM or any other) has caused confusion in thinking that a ''true signal'' generated at a given point in space and time cannot or should not have frequency components outside the interval defined by the aforementioned pair of numbers.

The fact that the value of the field is zero up to a given time [Eq. (6)], along with a few other reasonable assumptions (discussed in the following sections), is sufficient to show that the value of the integral in Eq. (4) is identically zero for $t<t_0$. In contrast to the opinion expressed by Nimtz *et al.* that ''there is no experimental condition known by which such a well defined front could be generated,'' Eq. (6) is justified for any ''true'' as opposed to a mathematically constructed signal. More importantly, the manner in which the field does turn on has no effect on the result expressed above. A signal can be turned on as slowly (a linear function of time) or as quickly (exponentially with time) as possible, and the value of the integral in Eq. (4) remains zero for all times less than t_0.

Before proving the above statement, in light of the controversies surrounding superluminal velocities it is important to discuss objections raised in Refs. [18,19]. In describing their frequency domain measurements, Nimtz *et al.* write, ‘‘The Fourier transform $F(t)=\int_{\nu_1}^{\nu_2}d\nu\, A(\nu)T(\nu)e^{2\pi i\nu t}$ yields the time response of the measured regions. As this signal is frequency band limited, it extends from $-\infty$ to $+\infty$ in the time domain. Since there is no defined front, such a signal cannot be used to check Einstein causality.’’ Aside from the authors’ unphysical assumption that such a ‘‘signal’’ has existed for all time in the past and will continue to exist for all time to come, their frequency domain measurements suffer from both experimental and interpretation errors. The correct frequency domain approach, particularly for the case of a 1DPC as the optical barrier, is discussed in Ref. [17]. In the Nimtz *et al.* experiments the fact that one is able to set a frequency sweep range (strictly frequency band limited to $\nu_1-\nu_2$) with a network analyzer does not mean that an actual signal extending in time from $-\infty$ to $+\infty$ has been generated. The NA and its synthesized source measure *a portion* of the frequency domain transmission coefficient, which are then used in a Fourier transform by Nimtz *et al.* and assumed to describe the *complete* time-domain results. Moreover, this approach requires assuming an incident pulse [$A(\nu)$, a Gaussian or Kaiser-Bessel function] which in reality has not been generated. As was discussed above, man-made signals must begin at a point in space and time and hence by definition are strictly time-limited.

The author in Ref. [19] also claims, ‘‘In this letter I shall show that frequency band limitation is a fundamental property of signals and that such signals containing only evanescent modes can violate Einstein causality.’’ He then argues that, ‘‘In theory switching on a signal generates infinitely high frequencies . . . However, signals with an infinite spectrum are impossible, since Planck has shown in 1900 that the minimum energy of a frequency component is $\hbar\omega$ . . . Since a signal has a finite energy (may be as small as of the order of 100 photons only), it follows that its spectrum has also to be finite.’’

In view of the above, it is important to note that the concept of infinity is a mathematical construct that physical reality can only approximate. For example, in all of the superluminal experiments in the microwave regime, one can safely say that frequencies in the range of tens or hundreds of GHz are a good approximation to the idea of infinitely high frequencies. [For the wave packet used in our experiment (centered at 9.68 GHz with a FWHM of 100 MHz), one can easily say that frequency components of ten or tens of GHz are indeed high frequencies which can be employed in the formation of the signal’s front.] More precisely, neglecting frequency components higher than these results in quantifiable errors as small as desired. More importantly, in the theory of signal processing and communication it is rather a well known fact that the energy of a signal [$u(t)$] is given in terms of its spectral density [$U(\nu)$] according to ([34], p. 38)

$$E=\int_{-\infty}^{+\infty}U(\nu)U^{*}(\nu)d\nu=\int_{-\infty}^{+\infty}|U(\nu)|^{2}d\nu. \tag{7}$$

Clearly, the function $U(\nu)$ does not have to be strictly frequency bandwidth limited for the above integral to remain finite.

Furthermore, in regard to the ‘‘minimum energy of a frequency component’’ alluded to by the author of Ref. [19], the following two points need consideration. First, it is universally accepted that a genuinely monochromatic plane wave (a frequency domain δ function) with energy $\hbar\omega$ is never physically realizable, and even the most narrow wave packet must contain some frequency spread. Second, with respect to the citation of the original work by Planck, it should suffice to say that the spectral energy density (energy per unit bandwidth per unit volume) is the product of the average energy per mode and the modal density. For example, in the case of blackbody radiation, this is given by ([35], p. 452)

$$\rho(\nu)=\frac{8\pi h\nu^{3}}{c^{3}}\frac{1}{\exp(h\nu/k_{B}T)-1}, \tag{8}$$

where T, h, and k_B are the temperature, Planck constant, and Boltzmann’s constant, respectively. It is a matter of simple exercise to show that the integration of Eq. (8) for all frequencies and over a physical volume yields a finite energy.

Upon returning to Eq. (4), in order to show that the value of the integral is identically zero for all times less than $t_0=x/c$, we need one more requirement. Stated simply, this requirement reads as follows: we shall not expect to measure a response from the medium characterized by $n(\omega)$, in the absence of a stimulus. [The reader may note that at this point nothing has been said with regard to the maximum speed by which a stimulus can propagate. The limitation on the stimuli propagation speed is set by special relativity and will be confirmed as the result of the proof that the value of the integral in Eq. (4) is identically zero for times less than x/c.] This is merely the description of a causal medium for which the effect cannot proceed the cause. This condition is mathematically expressed as ([2], p. 330)

$$\frac{\varepsilon(\omega)}{\varepsilon_0}-1=\int_{0}^{+\infty}G(\tau)e^{i\omega\tau}d\tau, \tag{9}$$

with $G(\tau)=0$ for $\tau<0$. $G(\tau)$ is what is commonly referred to as the susceptibility kernel and is given by

$$G(\tau)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}[\varepsilon(\omega)/\varepsilon_0-1]e^{-i\omega\tau}d\omega. \tag{10}$$

B. Titchmarsh theorem

Using the Titchmarsh theorem ([36], p. 426), it is straightforward to show that $u(x,t)$ is identically zero for $t<t_0=x/c$. This theorem states that any one of the concepts of causality, analyticity, or the Hilbert transform implies the other two. In our particular example, the requirement of a signal with a front [also known as a causal signal, Eq. (6)] implies that $A(\omega)$ from Eq. (5) must be analytical in the upper half plane (UHP) of the complex ω plane. Similarly, the requirement of the causal medium [$G(\tau)=0$ for $\tau<0$] implies that $n(\omega)$ and consequently $2A(\omega)/[1+n(\omega)]$ must also be analytical on the UHP. Now, consider the phase term in Eq. (4) away from the real ω axis ($\omega \to z = \eta + i\xi = |\omega| e^{i\vartheta}$) and in the limit of $|\omega| \to \infty$. This is given by

$$\exp(i\phi) = \exp\left[i\frac{\eta}{c}(x-ct)\right]\exp\left[-\frac{\xi}{c}(x-ct)\right] \quad \text{as } |\omega| \to \infty, \tag{11}$$

where $n(\omega) \to 1$ as $|\omega| \to \infty$ ([2], p. 333).

If $x - ct > 0$, then, in the UHP (i.e., $\xi > 0$) we have

$$\exp(i\phi) \to 0 \quad \text{as } |\omega| \to \infty. \tag{12}$$

At this point we use contour integration to evaluate the integral in Eq. (4). We can then write

$$\begin{aligned} u(x,t) &= \int_{-\infty}^{+\infty} g(\omega) e^{i\phi(\omega)} d\omega \\ &= \oint g(z) e^{i\phi(z)} dz - \lim \int_0^{\pi} g(|\omega| e^{i\vartheta}) \\ &\quad \times e^{i\phi(|\omega|, \exp(\vartheta))} i |\omega| e^{i\vartheta} d\vartheta, \\ &\quad |\omega| \to \infty, \end{aligned} \tag{13}$$

where the counterclockwise semicircle contour is closed in the UHP. From the results in Eq. (12), the last integral in Eq. (13) vanishes as $|\omega| \to \infty$ (this is Jordan's lemma). Furthermore, since we have seen that $g(\omega) e^{i\phi(\omega)}$ is analytical in the UHP, from the Cauchy-Goursat theorem the closed contour integral is also zero, therefore $u(x,t)$ is zero for $x - ct > 0$. This completes the proof that no signal can travel faster than c; the result is summarized as follows:

$$\begin{aligned} u(x,t) &= \int_{-\infty}^{+\infty} \frac{2}{1+n(\omega)} A(\omega) e^{ik(\omega)x - i\omega t} d\omega \\ &= \int_{-\infty}^{+\infty} g(\omega) e^{i\phi(\omega)} d\omega = 0 \\ &\quad \text{for } x - ct > 0 \equiv t_0 > t \equiv V > c \end{aligned} \tag{14}$$

with $t_0 = x/c$ and $V = x/t$.

IV. SOMMERFELD FORERUNNER

A. Stationary phase approximation

In the preceding section we showed that at the position x, for times less than the time for light in vacuum to travel the distance x (i.e., t_0), there will be no field. For times larger than t_0, the contour must be closed in the lower-half-plane LHP in order for the contribution from the infinite semicircle to be vanishingly small. Now a question can be asked: Is there some general behavior of the earliest parts of the pulse (the Sommerfeld forerunner) which can be ascertained without the need for a specific model of the index? The answer to this question is yes. In other words, a qualitative description of the Sommerfeld precursor's field can be obtain which would be applicable to any dispersive system such as a Lorentzian medium, a 1DPC, or an undersized waveguide, and so on. Thus far in the literature, whenever such a problem has been discussed, a Lorentzian model of the index has been chosen [1] ([33], pp. 313–326). We show here that the results derived for a Lorentzian model can be generally applied to any spatially or temporally dispersive system with only a change in the definition of a constant.

Sommerfeld and his student Brillouin used the steepest-descent method (SDM) for a Lorentzian dispersion with a sinusoidally modulated step-function input, to calculate the first (Sommerfeld) and second (Brillouin) precursors ([1], pp. 23–83). Here we apply the less rigorous yet simpler stationary phase method (SPM). A comparison between the two methods is provided by Brillouin ([1], pp. 81–83). However, in the case of a purely real index, the SPM and SDM are equivalent, and fortunately in the limit of $\omega \to \infty$ (the situation relevant to the Sommerfeld precursor), any index behaves as purely real to within $1/\omega^2$.

The stationary phase condition (SPC) $\partial\phi/\partial\omega = 0$ can be used to write

$$n + \omega \frac{dn(\omega)}{d\omega} = \frac{ct}{x} = c\frac{dk}{d\omega} = \frac{c}{\nu_g} = \frac{t}{t_0} \quad \text{for } t \geqslant t_0, \tag{15}$$

where Eq. (3) was used, and ν_g is the group velocity ($\nu_g = d\omega/dk$). Equation (15) provides us with the locations of the stationary points at different times. Using Eq. (15), it is easy to show that the earliest contributions to the integral in Eq. (4) come from the values of the index at large frequencies ($\omega \to \infty$). To see this, let us evaluate the left-hand side of Eq. (15) for the value of the index at $\omega \to \infty$. We have

$$n(\omega \to \infty) + \omega \frac{dn(\omega \to \infty)}{d\omega} = 1 + 0 = 1. \tag{16}$$

However, Eq. (16) is also equal to t/t_0, which implies that the first intersection of the horizontal line t/t_0 with $n + \omega dn/d\omega$ occurs for $t = t_0$. But, this is merely the onset of pulse propagation.

B. Forerunner frequency of oscillations

In the preceding section we showed that for the time equal to t_0 the stationary point is at $\omega = \infty$. Now, let us evaluate the stationary phase points for times *immediately* after t_0. Using integration by parts and neglecting terms of order $1/\omega^3$ and higher, it can be seen that the index of refraction in the limit of large frequencies is purely real and given by [26, ([2], p. 333)

$$n(\omega) \approx 1 - \frac{G'(0)}{2\omega^2}, \tag{17}$$

where the prime denotes the derivative of the susceptibility kernel with respect to time. Substituting Eq. (17) in Eq. (15) and solving for $\omega = \omega_s$ results in

$$\omega_s = \sqrt{G'(0)} \Bigg/ \left[2\left(\frac{t}{t_0} - 1\right)\right]^{1/2}. \tag{18}$$

For Lorentzian dispersion, $G'(0)$ is equal to the square of the plasma frequency ([2], p. 331), so that Eq. (18) can be rewritten as

$$\omega_s = \omega_p \Bigg/ \left[2\left(\frac{t}{t_0} - 1\right) \right]^{1/2}. \tag{19}$$

Our Eq. (19) is identical to the expression obtained by Sommerfeld ([1], p. 54) for the Lorentzian medium. Equation (18) or equivalently Eq. (19) imply that at a given observation point, for times immediately after t_0, the points of the stationary phase (Sommerfeld forerunners) only depend on the gross properties of the medium [e.g., ω_p or $\sqrt{G'(0)}$].

C. Forerunner functional form

In order to calculate the functional form of the Sommerfeld forerunner, the input signal must be known. Following Jackson's ([33], p. 314) consideration of a Lorentzian medium, we choose to model the earliest parts of the input signal by a polynomial of order *m*; hence

$$u(0,t) = \frac{a t^m}{m!} \leftrightarrow A(\omega) = \frac{a}{2\pi}\left(\frac{i}{\omega}\right)^{m+1}, \tag{20}$$

where *a* is a constant and *m* is an integer. Equation (20) can be used to emulate the earliest parts of a variety of input functions. By increasing the order of the polynomial, input signals with increasingly sharper rise times can be modeled.

Once again, let us use contour integration in order to evaluate the integral in Eq. (4). For times greater than t_0, the contour must be closed in the LHP ($\xi < 0$), and in a manner similar to the previous discussion [Eq. (11)] the contributions from the infinite semicircle tend to zero for $x - ct < 0$. Therefore, the value of the field at position *x* and time *t* is given by

$$u(x,t) = \int_{-\infty}^{+\infty} g(\omega) e^{i\phi(\omega)}\, d\omega$$

$$= -\oint g(z) e^{i\phi(z)}\, dz$$

$$\text{for } x - ct < 0 \equiv t_0 < t \equiv \nu < c. \tag{21}$$

Now, let us substitute Eq. (20) in Eq. (21) and replace the index in $2/[1 + n(\omega)]$ with unity and the index in $\phi(\omega)$ with $n(\omega) \approx 1 - G'(0)/(2\omega^2)$, where $\omega \to z = \eta + i\xi = |\omega| e^{i\vartheta}$. These two separate approximations, one for the amplitude and the other for the phase, are similar to the Fresnel approximation in diffraction theory where higher-order terms in the expansion of the phase are retained in order not to generate errors much greater than 2π radians ([37], pp. 58 and 59). After some mathematical manipulation, we have

$$u(x,t) \approx a\left(\frac{t - t_0}{\gamma}\right)^{m/2} J_m[2\sqrt{\gamma(t - t_0)}] \quad \text{for } t > t_0. \tag{22}$$

where

$$\gamma = \frac{G'(0)}{2c} x = \frac{G'(0) t_0}{2}, \tag{23}$$

and J_m is the Bessel function of the first kind of order *m*. As before, in the case of Lorentzian dispersion, $G'(0)$ is replaced with the square of the plasma frequency (ω_p^2). Equation (22) for $m = 1$ is identical to the expression obtained by Sommerfeld ([1], p. 41). Equation (22) implies that for input signals with sharper rise times (larger *m*), the order of the Bessel function will increase and the forerunner amplitude will decrease.

V. CONCLUSIONS

In this paper we have described an experiment with single microwave pulses tuned to the band gap of a 1DPC. It is observed that the peak of these tunneling wave packets arrives 440 ± 20 ps sooner than the accompanying wave packet traversing the same distance in free space. This implies that the wave packet has propagated through the 1DPC 2.38 ± 0.15 times faster than the speed of light in a vacuum. Despite this abnormal behavior, there is no violation of Einstein causality since the Sommerfeld forerunner (also referred to as the front) remains exactly luminal. In response to objections raised by some authors, a proof that no detection of a signal at the point *x* is possible for tunes less than x/c is provided, and the universality of the strictly time limited signal (signals with fronts) is discussed. Since propagation of the Sommerfeld forerunner is ultimately associated with the propagation of information, the frequency of oscillation and the functional form of these fields for any causal medium are presented.

ACKNOWLEDGMENTS

The authors would like to thank Dr. Raymond Chiao for his helpful discussion and Gregory P. Park for his help with the experiment. This material is based on work supported by the National Aeronautics and Space Administration under Grant No. NRA-99-LeRC-1, and in part by an AFOSR/DoD MURI Grant on compact sources of high energy microwaves.

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