R. A. Hargreaves
Sunderland Polytechnic, England

G. R. Higginson
University of Durham, England

Some Effects of Lubricant Starvation in Cylindrical Roller Bearings

Experiments on cylindrical roller bearings of 50-mm bore lubricated by an air/oil mist show that friction torque and operating temperatures are much reduced by running at low lubricant supply rates.

Introduction

The last few years have seen much experimental and theoretical work on several aspects of the performance of roller bearings [1–8].[1] Hydrodynamic and elastohydrodynamic theories have been used to interpret, with varying success, the influence of such important variables as load, speed and lubricant properties upon performance, particularly friction torque and life of the bearing; slip, i.e., failure to maintain near-epicyclic motion, has also attracted a good deal of attention.

The effect of lubricant supply rate has been studied in some of this earlier work, but nearly always at supply rates which could be described as copious (of the order of litres per min). The very low end of the scale of supply rate, nowadays described as "starvation" conditions, which is the subject of this paper, has not figured in the literature on roller bearings, but a good starting point has been given by the experimental work of Wedeven, Evans and Cameron [9] and the theoretical papers by Wolveridge, Baglin and Archard [10] and Castle and Dowson [11]. The work of Wolveridge, et al. (described henceforth as WBA) is particularly appropriate as a starting point to the work described in this paper, which is entirely devoted to starvation conditions. They adapt the Grubin model of ehl contact to the starvation condition.

In the experiments described later in the paper, the customary lubricating and cooling roles of the lubricant are separated by employing an air/oil mist system of lubrication. With this arrangement the rates of supply of air and lubricant can be varied independently.

Theoretical Considerations

A Single Rolling Contact. The Grubin model of an ehl line contact relies on two simplifying assumptions: (1) that the deformed shape of the cylinders is the same as that in a dry contact, and (2) that a very high pressure is developed in the entry region to the Hertzian zone. This simple model was conspicuously successful in predicting the film thickness in flooded conditions. WBA modify the model in two ways: first they use a simple approximate expression to describe the Hertzian deformation; and second they take, as the starting point to the pressure curve, that point on the common tangent to the two surfaces at which a full film is formed across the gap, as shown in Fig. 1. In the following paragraphs, the WBA evaluation of film thickness is very briefly summarized, in the notation of the present paper, and then the analysis is extended to predict rolling viscous traction.

If it is assumed that the viscosity varies exponentially with pressure only,

$$\eta = \eta_0 \exp(\alpha p)$$

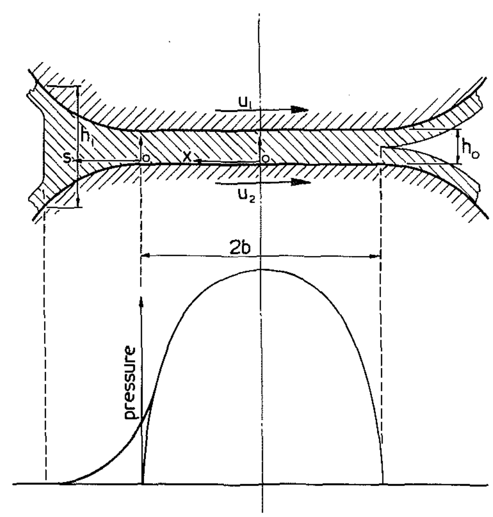

Fig. 1 Geometry and coordinates

[1] Numbers in brackets designate References at end of paper.

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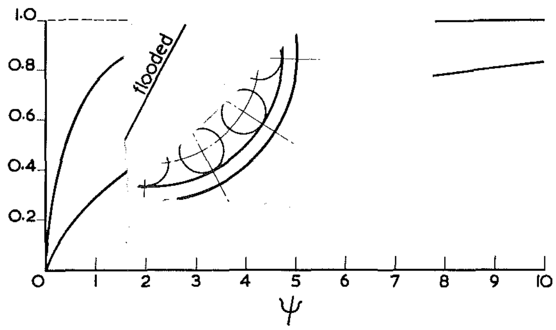

Fig. 2 Variation of film thickness and rolling traction with position of inlet boundary

the Reynolds equation for plane flow can be written in terms of "reduced pressure" q

$$\frac{dq}{dx} = -12\eta_0 u\left(\frac{h - h_0}{h^3}\right) \tag{1}$$

which become in dimensionless terms

$$\frac{dQ}{dx} = -48\left(\frac{W}{2\pi}\right)^{1/2} U\left(\frac{H - H_0}{H^3}\right) \tag{2}$$

Crook [12] has shown that a fairly good approximation to the dry Hertzian shape outside the contact zone is

$$\begin{aligned} H - H_0 &= \frac{4W}{\pi}\left(\frac{4\sqrt{2}}{3}\right)\{|X| - 1\}^{3/2} \\ &= 2.4\ WS^{3/2} \end{aligned} \tag{3}$$

if we put

$$\begin{aligned} \tau &= \left(\frac{2.4W}{H_0}\right)^{2/3} S, \\ \frac{dQ}{d\tau} &= -10.68\frac{U}{W^{1/6}H_0^{4/3}}\ \frac{\tau^{3/2}}{(1 + \tau^{3/2})^3} \end{aligned} \tag{4}$$

The integration of the last term is given by WBA

$$\begin{aligned} \mathrm{I} = \int\frac{\tau^{3/2}}{(1 + \tau^{3/2})^3}\,d\tau &= \frac{(2\tau^{3/2} - 1)\tau}{9(1 + \tau^{3/2})^2} \\ &- \frac{2}{27}\left[\frac{1}{2}\ \ln\left\{\frac{(1 + \tau^{1/2})^2}{\tau - \tau^{1/2} + 1}\right\} + \sqrt{3}\ \tan^{-1}\left\{\frac{(2 - \tau^{1/2})}{\sqrt{3}\tau^{1/2}}\right\}\right] \end{aligned} \tag{5}$$

Hence

$$Q_s = 10.68\frac{U}{W^{1/6}H_0^{4/3}}\,[I]_s^{\,i} \tag{6}$$

where the suffix i refers to the value of s at which pressure generation starts.

The second Grubin assumption is that at the inlet edge of the Hertzian zone, $Q \rightarrow 1/G$.

so

$$\frac{1}{G} = \frac{10.68U}{W^{1/6}H_0^{4/3}}\,[I_i - I_0] \tag{7}$$

Remembering that for the case of copious lubricant supply, $i \rightarrow \infty$, and noting that numerically

$$I_\infty - I_0 = 0.2867$$

we see that

$$[H_0]_\infty = 2.205(GU)^{3/4}/W^{1/8} \tag{8}$$

the central film thickness for the flooded condition.

For a given set of G, U and W, the central film thickness for a starved condition is given by

$$\left(\frac{H_0}{H_{0\infty}}\right)^{4/3} = \frac{I_i - I_0}{I_\infty - I_0} \tag{9}$$

This must be evaluated in terms of τ, but it can most usefully be plotted against the variable Ψ used by WBA. In the present notation,

$$\Psi = 4S_i\left(\frac{W}{2\pi H_{0\infty}}\right)^{2/3} = 0.693\frac{S_i W^{3/4}}{(GU)^{1/2}}$$

The plot is shown in Fig. 2, and is a universal curve within the limitations of the assumptions made, for all values of the external variables.

Rolling Friction. Crook [13] has shown that the tractions per unit face width on the surfaces in an elastohydrodynamic contact are given by

$$t_{1,2} = t_R \mp t_s \tag{10}$$

where

$$t_R = -\frac{1}{2}\int_i^0 h\frac{dp}{dx}dx$$

Nomenclature

- b = half Hertzian zone width = $4R(W/2\pi)^{1/2}$
- E_1, E_2 = Young's modulus for solids 1 and 2
- E^1 = effective modulus, $\frac{1}{E^1} = \frac{1}{2}\left[\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}\right]$
- G = αE^1
- h = film thickness
- h_1 = inlet film thickness, see Fig. 1
- h_0 = centerline film thickness
- H = h/R
- H_1 = h_1/R
- H_0 = h_0/R
- $H_{0\infty}$ = value of H_0 with inlet boundary at infinity
- I = integral defined by equation (5)
- N = shaft speed, rev/min
- p = pressure
- P = p/E^1
- q = "reduced pressure" = $\frac{1}{\alpha}\,[1 - \exp\text{-}\,(-\alpha p)]$
- Q = q/E^1
- r = roller radius
- R = "equivalent radius," defined by $1/R = 1/R_1 + 1/R_2$
- R_1, R_2 = radii of solids in contact
- S = $X - 1$
- S_i = value of S at inlet point
- t = viscous traction per unit width
- t_R = the rolling component of t
- t_s = the sliding component of t
- T_R = t_R/E^1R
- $T_{R\infty}$ = value of T_R with inlet boundary at infinity
- u = $\frac{1}{2}(u_1 + u_2)$
- u_1, u_2 = surface velocities of solids
- U = $\eta_0 u/E^1R$
- w = load per unit length of roller
- W = w/E^1R
- W_1 = value of W for most heavily loaded roller (at $\theta = 0$)
- W_θ = value of W for roller at angle θ, see Fig. 3
- x = coordinate
- X = x/b
- α = pressure coefficient of viscosity defined by $\eta = \eta_0\exp(\alpha p)$
- η = viscosity
- η_0 = viscosity at ambient pressure
- θ = angular coordinate
- ν_1, ν_2 = Poisson's ratio for solids 1 and 2
- τ = $(2.4W/H_0)^{2/3}S$
- Ψ = $4\,S_i(W/2\pi H_{0\infty})^{2/3}$
- Ψ_1 = value of Ψ for most heavily loaded roller
- Ψ_θ = value of Ψ for roller at angle θ

$$t_s = (u_1 - u_2) \int_i^0 \frac{\eta}{h} dx$$

If, in a rolling bearing, the other torques on a loaded rolling element are very small compared with this viscous traction at the contacts, then the rolling element will run with just sufficient slip to maintain the condition $t_s = t_R$; or more precisely the sum for the two contacts will be equal. For rollers loaded into the "ehl range," this slip is usually extremely small, and the rollers run in a condition which is practically pure rolling, giving epicyclic motion to the whole bearing assembly.

To evaluate t_R, we first write it in terms of our dimensionless parameters.

$$T_R = \frac{t_R}{E^1 R} = -\frac{1}{2} \int_i^0 H \frac{dP}{dx} dX$$

This can be recast

$$\int_i^0 H \frac{dP}{dX} \; dX = [HP]_i{}^0 - \int_i^0 P \frac{dH}{dX} dX$$

P = 0 at the points i and o, so $[HP]_i{}^0 = 0$

Hence
$$T_R = \frac{1}{2} \int_i^0 P \frac{dH}{dX} \; dX \tag{11}$$

Clearly a region of constant H, or one which is symmetrical in P and H, makes no net contribution to the rolling friction; so we would expect the Hertzian zone to make no substantial contribution to T_R. There will be a small contribution in the region of the outlet constriction, but that will be neglected in this simple treatment. So we can write

$$T_R \approx \frac{1}{2} \int_i^{X=1} P \frac{dH}{dX} dX$$

or in terms of S

$$T_R = \frac{1}{2} \int_{S=S_i}^{S=0} P \frac{dH}{dS} dS \tag{12}$$

From (3) $\quad \dfrac{dH}{dS} = 3.6 \; WS^{1/2}$

From (6) and (7) $\quad GQ_s = \dfrac{I_i - I_s}{I_i - I_0}$

and by definition $\quad GP_s = -\ln(1 - GQ_s)$

The integration of (12) has been performed numerically, and the result is shown in Fig. 2. It is easily shown that $T_R/T_{R\infty}$ is, like $H_0/H_{0\infty}$, a function of Ψ only. Clearly the derivation of T_R is very speculative, but it agrees closely with the one accurately computed case which has been published [11].

Fig. 2 shows that, whereas the film thickness retains very nearly its flooded value until the inlet boundary is extremely close to the Hertzian zone, the rolling traction, as would be expected, falls steadily as the inlet moves in towards the Hertzian zone. When the film thickness has 80 percent of its flooded value, the traction has fallen to below 30 percent of its flooded value. Although the variable Ψ contains load speed, etc., the results can be described in a simple general way. If one thinks in terms of the thickness of lubricant at the point in the inlet film where the pressure buildup starts (h_1 in Fig. 1), and expresses it in terms of the calculated ehl film thickness for the prevailing load, speed, viscosity, etc. (assuming flooded lubrication), then the results can be described thus: the rolling traction starts to fall significantly when the supply film falls below about 30 times the calculated film thickness, but the film thickness still has 80 percent of its flooded value when the supply film is as little as 3 times the full ehl film thickness. The film thickness has 95 percent of its flooded value when the supply film is 11 times the full ehl film thickness; Wedeven, Evans and Cameron [9] give a corresponding figure of 9 for a starved point contact.

The Bearing Assembly. The analysis so far has been wholly concerned with a single rolling contact, but it can be extended to cover all the outer track/roller contacts in a bearing, provided the value of the film inlet parameter can be estimated for each roller. It is therefore possible to estimate the total traction due to the rolling contacts within a bearing assembly operating under minimal lubricant conditions.

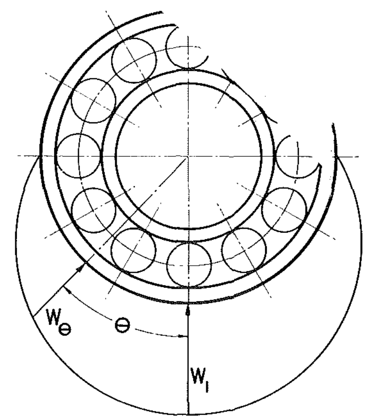

Fig. 3*(a)* The bearing assembly

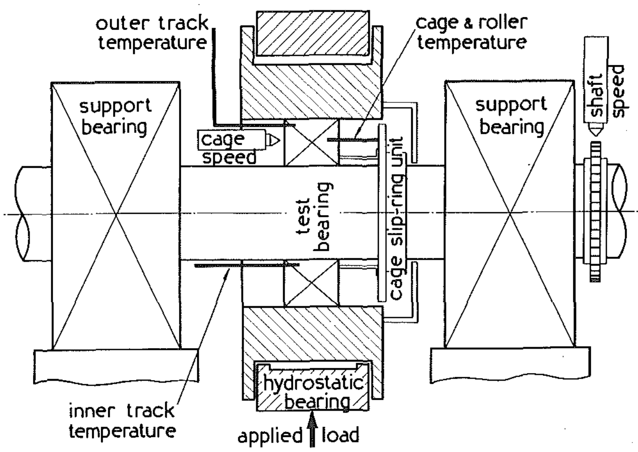

Fig. 3*(b)* The test rig

The value of the film inlet parameter Ψ for each roller depends on load and speed at the contact and also on the point at which the pressure build-up starts, described earlier as the point $S = S_1$. In order to relate the value of Ψ for any roller in the assembly to that for the most heavily loaded roller, perfect bearing geometry is assumed, together with a simple model of lubricant distribution.

The deflection against load of a cylindrical roller and its tracks is nearly linear [2], so in the assembly shown in Fig. 3*(a)* the load on a roller at point θ is approximately $W_\theta = W_1 \cos\theta$. The point within the inlet film at which the pressure buildup starts is assumed to be the point where $h = h_1$ (arbitrarily defined) and this value h_1—illustrated in Fig. 1—is taken to have the same value for all the loaded rollers; the assumption is described by Allen in a contribution to the discussion of [10].

In nondimensional terms we have, from (3),

$$H_1 - H_0 = 2.4 \; WS_i^{3/2} \tag{13}$$

H_0 is not much dependent on W, so the variation of S_i among the rollers will be, to a good approximation

$$S_i \alpha (1/W)^{2/3}$$

Substitution into the expression for Ψ gives

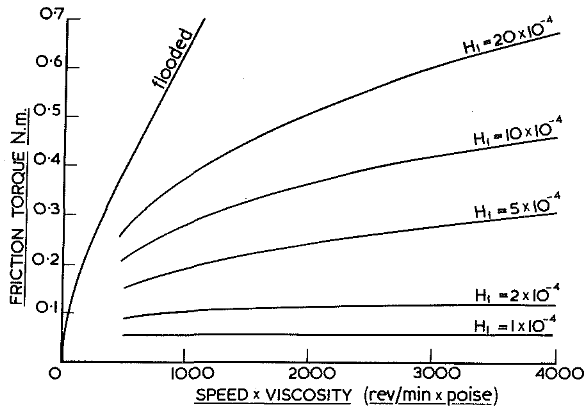

Fig. 4 Theoretical friction torque curves for test bearing due to rolling and sliding at roller/track contacts, 1kN load; based on assumption H_1 = constant

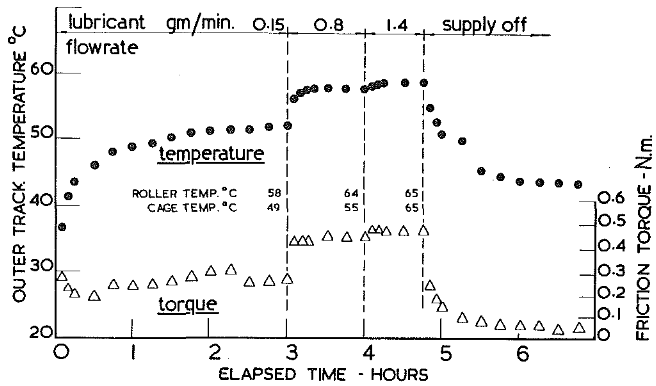

Fig. 5 Preliminary test results 10kN load, 3750 rev/min

$$\Psi_\theta/\Psi_1 = (\cos\,\theta)^{1/12} \tag{14}$$

This ratio can be taken as unity for the values of θ for which ehl theory can be considered to apply, so Ψ can be regarded as constant for all the loaded rollers in the assembly. This suggests that the total rolling traction in the bearing can be estimated simply by multiplying the value for the most heavily loaded roller by the number of loaded rollers (as was done in [3] for a flooded bearing). The friction torque calculated on this basis is shown in Fig. 4 for a load of 1kN.

Experiments

The aims of the experiments were twofold. Firstly, to provide data concerned with the operation of a roller bearing under minimal lubricant conditions to which the analysis could be related. Secondly, to investigate the effects of minimal lubricant conditions on the operating temperatures of the bearing components (particularly those of the cage and rollers) thus allowing a better estimate of lubricant viscosity at each of the contacts in the assembly.

The results which follow were obtained using 50 mm bore roller bearings (SKF NU310 and SKF N310), fitted with machined brass cages. Lubrication was effected by an air/oil mist: the lubrication was a base mineral oil of approximately 1 poise (0.1 Ns/m^2) viscosity at normal temperature, and a wide range of oil supply rates was covered. The air supply was from a main at 4 bars and 20–22 deg C; the supply was held constant at 0.15 kg/min throughout the experiments. The test loads were 1kN and 10kN (0.1 and 1.0 ton).

Measurements included load, shaft speed, cage and roller speeds, friction torque; air and lubricant supply rates; various temperatures and the voltage drop between the inner and outer tracks. A sketch of the test rig is shown in Fig. 3*(b)*.

A series of preliminary tests were carried out before the main experimental program, and the results of these tests are of interest since they indicate in a general way the effects of low lubricant flowrate on friction torque and component temperatures in a roller bearing. A typical curve is shown in Fig. 5 and it will be seen that both friction torque and outer track temperature show noticeable variation with lubricant flowrate at the low rates investigated and that a reduction in flowrate provides a reduction in measured friction torque. This reduction is certainly not due to a reduction in lubricant viscosity. On the contrary, the reduction in torque is accompanied by a reduction in temperature, suggesting that the reduction in friction torque due to a reduced quantity of lubricant is more significant than would at first appear.

Friction Torque. The experimental program yielded a range of results summarized in Fig. 6 and typified by the bold lines in Fig. 7. All the results, when plotted to a base of (1n. lubricant flowrate) show a linear variation with this parameter, an increase in speed effecting only a modest translation of the experimental curve.

The experimental results can be compared with theoretical curves of which Fig. 4 is an example. Although the theoretical values include only the viscous tractions at the track/roller contacts it has been shown [8] that these contacts provide the major contribution to the total friction torque, estimated to be in excess of 70 percent in these experiments; a direct comparison is therefore of some value. Fig. 7*(a)* shows the experimental curves for a 1kN load and a number of flowrates, superimposed by a reproduction of Fig. 4, and Fig. 7*(b)* shows the corresponding curves for a 10kN load.

These experimental results offer some support for the inlet boundary assumption. The values of H_1 in Fig. 7 suggest that in the experiments the inlet boundary point ranged from less than one Hertzian zone width to about 5 Hertzian zone widths from the centre line over the whole range of flowrates.

Temperature. The temperature measurements employed 12 thermocouples: nine under the surface of the outer (stationary) track, one under the surface of the inner track, one at a point in the cage midway between two roller centers, and one in the center of a roller. The last two necessitated a slip-ring unit which prevented simultaneous measurement of friction torque, so two complete sets of tests with the same bearing were conducted.

Figs. 8 and 9 indicate the main trends, but a few features should be noted, keeping in mind that in other situations these trends may be modified by the nature of the bearing installation.

(i) Inner track temperature remains below that of the outer track at low flow rates over a wide speed range, but at high flowrate and high speed the inner track gets hotter than the outer.

(ii) The cage which is outer-ring guided (SKF NU 310) runs always at a significantly lower temperature than the outer track, typically 10–15 deg C. In contrast, the cage which is inner-ring guided (SKF N 310) runs at a much higher temperature than that

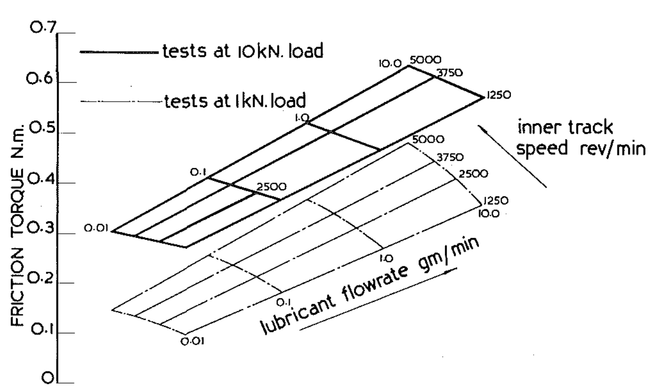

Fig. 6 Variation of friction torque with shaft speed and flowrate; bearing NU 310

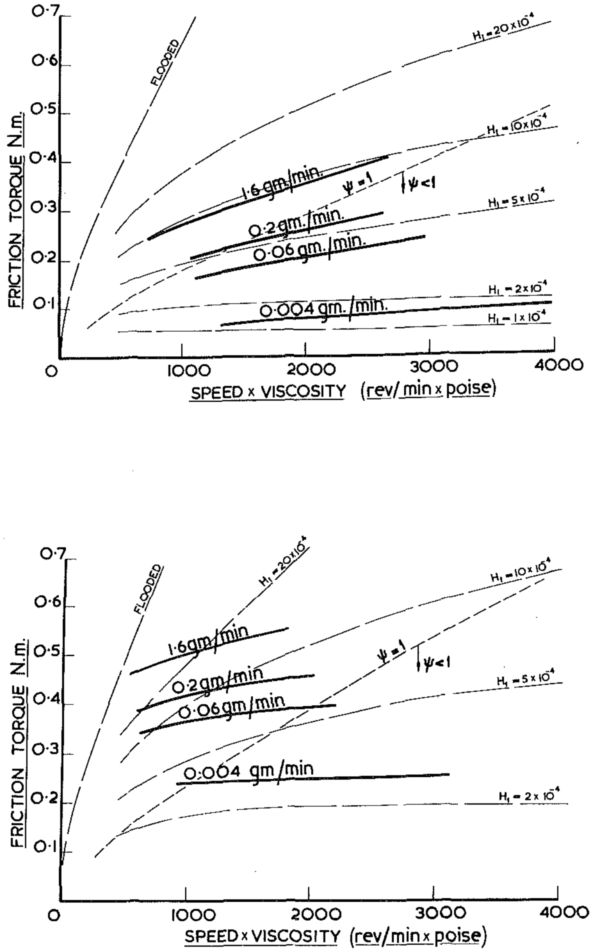

Fig. 7 Comparison of experimental and theoretical torque; *(a)* 1kN load, *(b)* 10kN load

which is outer-ring guided, and in some conditions gets hotter than the outer track. This is very probably due to the lower cage/ring clearance in the N310 (about half that of the NU310) used in these experiments; in addition the sliding speed at the interface is about 25% higher in the N310 than in the NU310.

(iii) The roller is always the hottest component in the assembly, but at low flowrates its temperature is much the same as that of the outer track, irrespective of speed. At the higher flowrates and speeds, the rollers run about 10–15 deg C higher than the outer track.

The friction and temperature results have been compared with those of Boness [6] on a similar bearing but operating at much higher lubricant flowrates. The higher flowrate results approach those of Boness.

Finally, it should be noted that in none of these tests was there any indication of breakdown of hydrodynamic lubrication. The only conditions in which partial failure appeared (in the form of rising friction torque with falling speed) were at low speeds with a very low viscosity lubricant-paraffin; it seemed likely that this partial failure was in the cage pockets or at the cage/flange contact, because the oil film voltage-drop measurements indicated that the rolling contact films were intact.

Conclusions

General conclusions cannot be drawn from measurements taken on one test rig; plainly much further work must be done on bearings of different sizes working under a range of operating conditions before firm statements can be made. But from these limited tests a number of noteworthy tentative conclusions emerge.

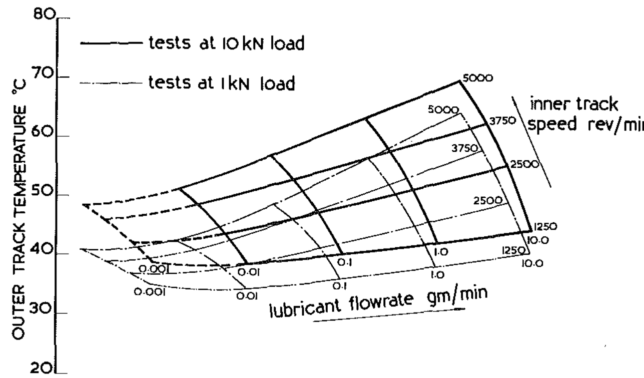

Fig. 8 Variation of outer track temperature with shaft speed and flowrate; bearing Nu 310

There appears to be a wide range of operating conditions where the benefits of very low lubricant supply rate (with air cooling) can be enjoyed, without the danger of breakdown of the hydrodynamic film through paucity of lubricant. The reduction of torque and operating temperature with lubricant supply rate is plain to see. At higher flowrates and speeds, the temperatures of the components differ, the rollers being substantially hotter than the tracks; but at low flowrates the components run uniformly cool over a wide range of speed.

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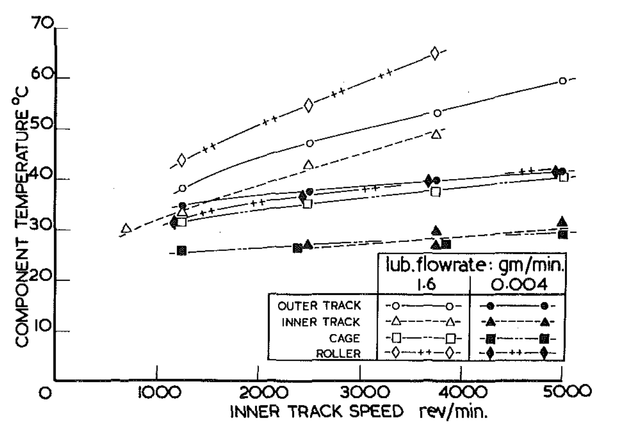

Fig. 9 Variation of component temperatures with speed; bearing NU 310

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