ON THICK DOMAIN WALLS WITH VISCOUS FIELD COUPLED WITH ELECTROMAGNETIC FIELD IN GENERAL RELATIVITY

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Abstract. In this paper, we have obtained plane symmetric cosmological models of domain wall with viscous field coupled with electromagnetic field in general relativity by using equation of state $p = (\gamma - 1)\rho$. And their physical and geometrical properties have been discussed.

Key words: thick domain walls, plane symmetry, viscous field, electromagnetic field.

1. INTRODUCTION

The exact solutions to the Einstein Maxwell equation is important for many physical applications in particular in astrophysics. The formation of galaxies is due to domain wall and cosmic string Vilenkin [1], Hill et al. [10]. The space-time of the cosmological domain walls in general relativity has been studied by Sikivie and Ispser [15], Bonjour [8], Mukherji [14]. The topological object such as domain walls, monopoles and string have an important role in the formation of universe. The domain walls are formed when the universe undergoes a series of phase transition with discrete symmetry being spontaneously broken Vilenkin [1, 2]. After symmetry breaking different regions of the universe can be settling into different parts of the vacuum with domain walls forming boundaries between their regions. In the recent years many of the researchers interested in the study of large scale structure of the universe because of the fact that the origin of the structure in the universe is one of the greatest cosmological mysteries even today. The light domain walls of large thickness may have produced during the late time phase transitions such as those occurring after the decoupling of matter and radiations

Hill and Schramm [12], D R K Reddy [6]. Zel'dovich [5] pointed out the stress energy of the domain walls composed of the surface energy density and string tension into spatial directions with magnitude of tension equal to the surface energy density, this is interesting because there are several indication that tension acts repulsive source of gravity in general relativity. On other hand, bulk viscosity coefficient ξ plays an important role for understanding the field nature of the universe around us. Krori [13] and Mukherji [14] have discussed cosmological model with early universe. A lot of work has been done on thick domain walls in Lyra geometry by Rahamann [9,10,11]. Recently, Pradhan [3, 4] obtained general solution for bulk viscous domain walls in Lyra geometry. Reddy and Rao [7] studied axially symmetric domain walls in Lyra geometry. Pawar et al. [16] has been obtained bulk viscous fluid plane symmetric dust magnetized string cosmological model in general relativity. In present paper, we obtained cosmological models of thick domain walls with viscous field coupled with electromagnetic field in general relativity and also discussed the nature of the existing model.

2. FIELD EQUATIONS

Consider the plane symmetric space-time,

$$ds^2 = dt^2 - A^2\left(dx^2 + dy^2\right) - B^2 dz^2,\tag{1}$$

where the metric potential A and B are function of cosmic time t only.

Thick domain walls are characterized by the energy momentum tensor of a viscous field with electromagnetic field which has the form

$$T_{ik} = \rho\left(g_{ik} + w_i w_k\right) + \tilde{p}\, w_i w_k + E_{ik}\,, \quad \text{with} \quad u_i u^j = -w_i w^i = 1\,,\tag{2}$$

where $\tilde{p} = p - \xi\theta$.

Here $\rho, p, \tilde{p}$ and ξ are the energy density, the pressure in the direction normal to the plane of the walls, the effective pressure and bulk viscous coefficient and w_i is unit space like vector in the same direction. The energy momentum tensor for electromagnetic field E_{ik} be given by,

$$E_{ik} = \frac{1}{4\pi}\left[F_{i\alpha}F_{k\beta}g^{\alpha\beta} - \frac{1}{4}g_{ik}F^{\alpha\beta}F_{\alpha\beta}\right],\tag{3}$$

where F_{ik} is electromagnetic field tensor which satisfy Maxwell equations,

$$F_{[ij;k]} = 0 \quad \text{i.e} \quad \left(F^{ij}\sqrt{-g}\right)_{;j} = 0\,.\tag{4}$$

In commoving co-ordinate system the incident magnetic field is taken along z-axis with the help of Maxwell equations (4), the components of F_{ik} except F_{12} are vanish i.e. F_{12} is only non-vanishing component along z-axis,

$$F_{12} = \text{constant} = H \text{ (say)}.$$

From equation (3) we have,

$$E_1^1 = E_2^2 = \frac{H^2}{8\pi A^4} \quad \text{and} \quad E_3^3 = E_4^4 = -\frac{H^2}{8\pi A^4}. \tag{5}$$

Equation (2) yields,

$$T_1^1 = T_2^2 = \rho + \frac{H^2}{8\pi A^4} \quad , \quad T_3^3 = -\tilde{p} - \frac{H^2}{8\pi A^4} \quad \text{and} \quad T_4^4 = \rho - \frac{H^2}{8\pi A^4}. \tag{6}$$

The Einstein field equations for the space-time (1), choosing gravitational units such that $c = 1$, $8\pi G = 1$ as

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = 8\pi\rho + \frac{H^2}{A^4}, \tag{7}$$

$$-\left[\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2}\right] = 8\pi\tilde{p} + \frac{H^2}{A^4}, \tag{8}$$

$$\frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB} = 8\pi\rho - \frac{H^2}{A^4}. \tag{9}$$

The dot ($\cdot$) over letters indicate differentiation with respect to time t.

3. SOLUTIONS OF THE FIELD EQUATIONS

The set of field equations (7–9) are non-linear differential equations in five unknowns (A, B, ξ, p, ρ), so for solving the field equations completely, we assume following conditions:

(i) The equation of state $p = (\gamma - 1)\rho$;

(ii) Metric potential relation $B = RA^n$, where R and n are constant.

From equation (7) and (9), we get

$$\frac{\ddot{A}}{A} - \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} - \frac{\dot{A}^2}{A^2} = \frac{2H^2}{A^4}. \tag{10}$$

Using second condition $B = RA^n$, equation (10) yields,

$$(n+1)\ddot{A} + (n^2 - 2n - 1)\frac{\dot{A}^2}{A} = \frac{2H^2}{A^3}.$$ (11)

Substituting $\dot{A} = \eta$ and $\eta^2 = w$ in equation (11) we get,

$$\frac{dw}{dA} + \frac{2(n^2 - 2n - 1)}{(n+1)} \frac{w}{A} = \frac{4H^2}{(n+1)A^3}.$$ (12)

On solving equation (12) finally, we get

$$\frac{dA}{dt} = \left[\frac{2H^2}{(n+1)(\alpha - 1)} A^{-2} + cA^{-2\alpha}\right]^{\frac{1}{2}},$$ (13)

$$\Rightarrow dA = \left[\frac{2H^2}{(n+1)(\alpha - 1)} A^{-2} + cA^{-2\alpha}\right]^{\frac{1}{2}} dt,$$

where $\alpha = \frac{(n^2 - 2n - 1)}{(n+1)}$ and c is constant of integration.

Again from equation (8) and (9) by using equation of state,

$$\frac{2\ddot{A}}{A} + \left[\gamma(2n+1) - 2n\right]\frac{\dot{A}^2}{A^2} = 8\pi\xi\theta + (-\gamma)\frac{H^2}{A^4}.$$ (14)

In order to obtain the solution of equation (13), we consider two different cases.

Case (I): The constant of integration $c = 0$, then the equation (13) becomes,

$$dA = \left[\frac{2H^2}{(n+1)(\alpha - 1)} A^{-2}\right]^{\frac{1}{2}} dt.$$ (15)

On solving, we have

$$A = \sqrt{2(Lt + \beta)} \quad \text{and} \quad B = R\left[2(Lt + \beta)\right]^{\frac{n}{2}},$$ (16)

where $L = \frac{\sqrt{2} \ H}{\sqrt{n^2 - 3n - 2}}$ and β is constant of integration.

Then the space-time (1) reduces to,

$$ds^2 = dt^2 - 2(Lt + \beta)(dx^2 + dy^2) - 2^n R^2 (Lt + \beta)^n dz^2. \tag{17}$$

4. THE PHYSICAL AND GEOMETRICAL PROPERTIES OF MODEL

Substituting the values of A and B in equation (9), we have

$$8\pi\rho = \frac{L^2}{4}(Lt + \beta)^{-2} \frac{n(n+1)}{2}. \tag{18}$$

Also, the scalar expansion (θ) is given by

$$\theta = u^{\alpha}_{;\alpha} = u^{\alpha}_{,\alpha} + u^{\beta} \Gamma^{\alpha}_{\beta\alpha}, \ \Rightarrow \theta = \frac{(n+2)}{2} L(Lt + \beta)^{-1}. \tag{19}$$

Using equation (19), equation (14) gives,

$$8\pi \ \xi = \frac{L(Lt + \beta)^{-1}}{2(n+2)} \frac{(n+1)}{2} \big[\gamma(n-4)\big]. \tag{20}$$

And equation (8) yields

$$8\pi p = -\frac{2\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} + 8\pi\xi\theta - \frac{H^2}{A^4}. \tag{21}$$

Using equation (16), (19) and (20) in equation (21) gives

$$8\pi p = \frac{L^2}{4}(Lt + \beta)^{-2} \frac{n(n+1)}{2}(\gamma - 1). \tag{22}$$

Again, $\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$,

$$\sigma^2 = \frac{7}{72}(n+2)^2 L^2 (Lt + \beta)^{-2}. \tag{23}$$

From equation (19) and (23) we have,

$$\lim_{t\to\infty}\left(\frac{\sigma}{\theta}\right) = \sqrt{\frac{7}{18}} \neq 0. \tag{24}$$

Spatial volume $(V) = \sqrt{-g} = 2^{\left(\frac{n+2}{2}\right)} R(Lt + \beta)^{\left(\frac{n+2}{2}\right)}, \tag{25}$

In the presence of magnetic field, the reality conditions $p \geq 0$, $\rho \geq 0$ are satisfied for all n and the coefficient of bulk viscosity ξ is positive for $n \geq 0$. The scalar expansion (θ), pressure (p), density (ρ) are becomes zero, when $t \to \infty$ and they are infinite when $t \to 0$. It means that, the model has initial singularity. The spatial volume (V) is infinite when $t \to \infty$ and which is zero for $t \to 0$. The quantities pressure (p), density (ρ), coefficient of bulk viscosity (ξ) is finite provided $n \neq -1$. This shows that the universe is expanding, shearing and start with big-bang.

Since $\lim\limits_{t \to \infty} \left(\dfrac{\sigma}{\theta} \right) \neq 0$, the model does not approach the isotropy for large values of t. Also from equations (18), (20), (22), we have:

 − If $\gamma = 1$, the dust model of the universe.

 − If $\gamma = 2$, the stiff fluid model of the universe. And for $\gamma = \dfrac{4}{3}$, we obtain the radiating model of the universe.

In **absence of magnetic field** ($H = 0$) the space-time is

$$ds^2 = dt^2 - \beta \left(dx^2 + dy^2 \right) - R\beta dz^2 .$$

This is the flat space-time of special relativity.

In the absence of magnetic field, the quantities pressure (p), density (ρ), coefficient of bulk viscosity (ξ) is zero i.e. we get the vacuum model.

It is interesting to note that the role of magnetic field is vital in this model.

Case (II): The constant of integration $c \neq 0$, then the equation (13) becomes,

$$dt = \left[\frac{2H^2}{(n+1)(\alpha-1)} A^{-2} + cA^{-2\alpha} \right]^{-\frac{1}{2}} dA . \tag{26}$$

For this solution, the geometry of the universe is described by the metric,

$$ds^2 = \left\{ \left[\frac{2H^2}{(n+1)(\alpha-1)} A^{-2} + cA^{-2\alpha} \right]^{-\frac{1}{2}} \right\}^2 dA^2 - A^2 \left(dx^2 + dy^2 \right) - RA^n dz^2 . \tag{27}$$

Under suitable transformation of coordinates of above metric reduces to,

$$ds^2 = \left[\frac{2H^2}{(n+1)(\alpha-1)} T^{-2} + cT^{-2\alpha} \right]^{-1} dT^2 - T^2\left(dx^2 + dy^2\right) - RT^n dz^2. \quad (28)$$

5. THE PHYSICAL AND GEOMETRICAL PROPERTIES OF MODEL

The pressure, density, coefficient of bulk viscosity for the cosmological model (28) are given by,

$$8\pi\rho = \left[\frac{2(2n+1)H^2}{(n+1)(\alpha-1)} + H^2 \right] T^{-4} + (2n+1)cT^{-2(\alpha+1)}, \quad (29)$$

$$8\pi p = \left[\frac{2(2n+1)H^2}{(n+1)(\alpha-1)} + H^2 \right](\gamma-1)T^{-4} + (2n+1)(\gamma-1)cT^{-2(\alpha+1)}, \quad (30)$$

$$8\pi\xi = \frac{1}{(n+2)T^2}\left[\frac{2H^2}{(n+1)(\alpha-1)} T^{-2} + cT^{-2\alpha} \right]^{-\frac{1}{2}} \cdot$$

$$\cdot \left[\frac{-4H^2}{(n+1)(\alpha-1)} T^{-2} - 2\alpha cT^{-2\alpha} + \gamma H^2 T^{-2} \right] + \quad (31)$$

$$+ \left[\gamma(2n+1) - 2n \right] \left[\frac{2H^2}{(n+1)(\alpha-1)} T^{-2} + cT^{-2\alpha} \right]^{\frac{1}{2}}.$$

Spatial volume $\qquad (V) = \sqrt{-g} = RT^{n+2}.$ $\qquad (32)$

In the presence of magnetic field, the energy conditions p, $\rho \geq 0.$, are satisfied provided $n \neq -1$ and $\alpha \neq 1$. The scalar expansion (θ), pressure (p), density (ρ) is become zero when $T \to \infty$, $(\alpha+1) > 0$, and $p, \rho, \theta \to \infty$ as $T \to 0$.

The spatial volume tends to infinite as $T \to \infty$ and it tends to zero as $T \to 0$.

In this case also $\lim\limits_{T \to \infty}\left(\dfrac{\sigma}{\theta}\right) \neq 0$, the model does not approach the isotropy for large values of T. Also from expression of p, ρ, ξ we obtain, for $\gamma = 1$, the dust model of the universe, for $\gamma = 2$, the stiff fluid model of the universe and for $\gamma = \dfrac{4}{3}$ the radiating model of the universe.

In **absence of magnetic field** ($H = 0$) the space-time is

$$ds^2 = cT^{2\alpha}dT^2 - T^2\left(dx^2 + dy^2\right) - RT^{n}dz^2 . \tag{32}$$

The pressure, density, coefficient of bulk viscosity for the cosmological model (28) are given by,

$$8\pi\rho = \left(2n+1\right)cT^{-2(\alpha+1)}, \tag{33}$$

$$8\pi p = \left(2n+1\right)\left(\gamma-1\right)cT^{-2(\alpha+1)}, \tag{34}$$

$$8\pi\xi = \frac{-1}{\left(n+2\right)T^2}\left[cT^{-2\alpha}\right]^{-\frac{1}{2}}\left[2\alpha cT^{-2\alpha}\right]+\left[\gamma\left(2n+1\right)-2n\right]\left[cT^{-2\alpha}\right]^{\frac{1}{2}}. \tag{35}$$

In the absence of magnetic field, the pressure $\left(p\right)$, density $\left(\rho\right)$, coefficient of bulk viscosity $\left(\xi\right)$ is positive, then the model reduces the model with thick domain walls and bulk viscous field in general relativity. The energy conditions $p, \rho \geq 0$ are satisfied provided $n \neq -\dfrac{1}{2}$.

The pressure $\left(p\right)$, density $\left(\rho\right)$ both tend to zero as $T \to \infty$, $\left(\alpha+1\right) > 0$ and p, ρ tends to infinite as $T \to 0$, $\left(\alpha+1\right) > 0$. And as the spatial volume V is infinite when $T \to \infty$ and spatial volume V tends to zero as $T \to 0$, the universe is blow up and shrunk with infinite past and future.

A network of domain walls accelerates the expansion of the universe. An interesting result emerged in this work is that pressure perpendicular to the domain walls is non-zero i.e. at any instant the domain wall density ρ, pressure p is perpendicular direction decreases both side of walls away from the symmetry plane and both vanish as $T \to \infty$ in both cases.

6. CONCLUSION

The plane symmetric cosmological models for thick domain walls with viscous fluid coupled with electromagnetic field are obtained. Also we discussed the models in absence of electromagnetic field. Generally the models are expanding, shearing with big bang. The study of the domain walls in this paper successfully describes various features of the universe.

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