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Modeling and Identification of a Class of Servomechanism Systems With Stick-Slip Friction

This paper describes a technique for modeling and identifying a class of nonlinear servomechanism systems with stick-slip friction. The physics of the stick-slip friction is considered in modeling the process. Identification of the system parameters is formulated as a nonlinear optimization problem. A modified simplex algorithm is proposed as the optimization procedure. The difficulties encountered in choosing identification algorithm and input signals for the problem are discussed. A simulation example of a servomotor system is provided.

I Introduction

When operating with small amplitude and low frequency signal input, the presence of stick-slip friction in a servomechanism cannot be ignored. To accurately predict or simulate the performance of the system, the effects of friction on the motor behavior should be taken into consideration by appropriate modeling of the friction. This paper describes a modeling and identification technique for a class of servomechanism systems with stick-slip friction.

In the paper [1], Karnopp presented a novel model for describing the effect of stick-slip friction in mechanical dynamic systems. The model proposed by Karnopp takes the physics of the stick-slip friction process into account by considering the rate of change of momentum in the mechanical system. The main parameters in the model include the inertia of the system, the slip coefficient, the threshold for the momentum rate and the saturation stick force. The stick-slip friction model of Karnopp is adapted to a class of servomechanism in this paper. For the present discussion, the dc servomotor is considered. The proposed adaptation can be extended to other classes of servomechanisms.

This paper next addresses an identification technique and the choice of input signals for estimating the parameters associated with the nonlinear stick-slip model. The identification problem is posed as an optimization problem to be solved using nonlinear programming methods [2]. A modified simplex algorithm is suggested for speeding up the optimization procedure. Simulation results show that the algorithm is effective and suitable for the problem considered. A summary of the simplex method and its modifications are given in the Appendix for completeness. The paper also briefly discusses the difficulties encountered in applying other nonlinear programming methods to the problem under investigation.

II Modeling

A typical armature-controlled dc motor is shown in Fig. 1. The schematic block diagram for the system is given in Fig. 2, where the stick-slip friction model for the system is shown as NL1 and NL2 [1]. In the system, u is the control voltage, $\dot{\theta}_m$ is the motor angular velocity, T is the motor torque, K_a is the amplifier gain, and L_m and R_m are the armature inductance and resistance, respectively; K_t and K_b are the torque and back EMF constants, respectively; J is the inertia; B is the viscous damping coefficient; P is the momentum; T_f, T_{slip}, and T_{stick} denote the total, slip and stick friction torques, respectively; T_s is the saturation torque of stick friction; D_p is the limiting momentum in the stick region, and $D_{\dot{\theta}}$ is the limiting angular velocity in the stick and slip regions; and S_c is the switching variable. A simplified block diagram of the model is shown in Fig. 3.

The friction torque T_f consists of two components, T_{slip} and T_{stick}. In the slip region where $|\dot{\theta}_m| > D_{\dot{\theta}}$, the switching

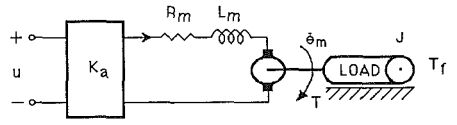

Fig. 1 Servomotor system with stick-slip friction

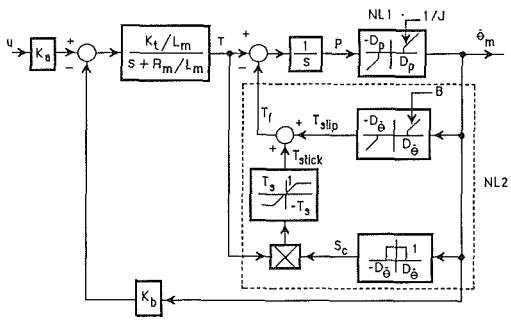

Fig. 2 System schematic black diagram

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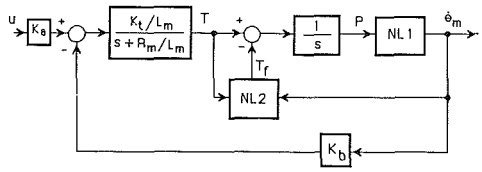

Fig. 3 Simplified system block diagram

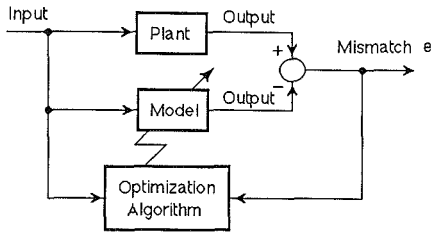

Fig. 4 Identification model

variable S_c vanishes, so that $T_{\text{stick}}=0$. Therefore, the torque $T_f=T_{\text{slip}}$. In the stick region where $|\dot{\theta}_m| \leqq D_{\dot{\theta}}$, $T_{\text{slip}}=0$ and $S_c=1$ so that $T_f=T_{\text{stick}}$. The stick friction T_{stick} is limited to the saturation level T_s.

The nonlinearities can be expressed mathematically as follows:

NL1:

$$\dot{\theta}_m = \begin{cases} 0, & -D_p < P < D_p; \\ P/J, & \text{Otherwise.} \end{cases} \tag{1}$$

NL2:

$$T_f = T_{\text{slip}} + T_{\text{stick}}, \tag{2}$$

where

$$T_{\text{slip}} = \begin{cases} 0, & -D_{\dot{\theta}} < \dot{\theta}_m < D_{\dot{\theta}}; \\ B\dot{\theta}_m, & \text{Otherwise,} \end{cases} \tag{3}$$

and

$$T_{\text{stick}} = \begin{cases} -T_s, & T'_{\text{stick}} < -T_s; \\ T'_{\text{stick}}, & -T_s < T'_{\text{stick}} < T_s; \\ T_s, & T_s < T'_{\text{stick}}, \end{cases} \tag{4a}$$

with

$$T'_{\text{stick}} = \begin{cases} T, & -D_{\dot{\theta}} < \dot{\theta}_m < D_{\dot{\theta}}; \\ 0, & \text{Otherwise.} \end{cases} \tag{4b}$$

The relationship between D_p and $D_{\dot{\theta}}$ is given by [1]

$$D_p = JD_{\dot{\theta}}. \tag{5}$$

It is noted that the limiting augular velocity $D_{\dot{\theta}}$ should be sufficiently wide so as to allow T_{stick} to reach saturation torque T_s.

The nominal values of parameters K_a, K_t, L_m, R_m, and K_b are usually specified by the manufacturer, or can often be determined a priori. However, the parameters J, B, D_p, $D_{\dot{\theta}}$, and T_s are not generally known since they vary considerably with operating conditions and environment.

III Identification

This section presents an identification scheme for estimating the unknown parameters. Figure 4 shows the identification configuration in which the system parameters are identified by minimizing the mismatched output error between the plant and the model. General description and development on this method can be found, for example, in Richalet [3], Brogan [4] and Christopher et al. [2].

The idea is to compare the behaviors between the physical plant and the mathematical model. If the behaviors of both plant and model, under the conditions of experiment, are almost identical by some measure [3], then we may claim that we have identified a model which mathematically describes the plant. If not, the parameters or even the structure of the model may be modified in such a way that a greater conformity of the model behavior to that of the plant is achieved. The identification of the model can be formulated in terms of minimizing the mismatched output errors as follows.

Let z denote the set of unknown plant parameters to be identified or estimated, and $\hat{z}$ denote an estimate of z. Define the mismatched error between the outputs of the plant and model as

$$e(\hat{z},t_i) = y_p(t_i) - y_m(\hat{z},t_i), \tag{6}$$

where $y_p(t_i)$ is the plant output at sample instants t_i, $y_m(\hat{z},t_i)$ is the sampled model output which depends on the estimated parameters $\hat{z}$, and $i=1,2,\ldots,N$ with N being the total number of samples. Consider the objective function of the weighted squared errors given by

$$J_e(\hat{z}) = \sum_{i=1}^{N} q_i e^2(\hat{z},t_i), \tag{7}$$

where q_i is a non-negative weighting factor which may vary with t_i. The parameter identification problem may be posed as an optimization problem as follows:

$$\text{Minimize } J_e \tag{8}$$

subject to the dynamic model constraint

$$y_m = G(\hat{z},t), \tag{9}$$

where G is a nonlinear mathematical description of the plant shown in Fig. 2. When the objective function J_e is less than a predetermined small value, one may say both plant and model are identical in the sense of the weighted least squared error, and that parameters $\hat{z}$ identifies with z.

In the case where a number of experimental responses are to be fitted simultaneously, a multiresponse objective function of the form (10) may be used:

$$J_e = \sum_{j=1}^{L} w_j J_{ej}(\hat{z}_j), \tag{10}$$

where

$$J_{ej}(\hat{z}_j) = \sum_{i=1}^{N_j} q_{ji}[y_{pj}(t_i) - y_{mj}(\hat{z}_j,t_i)]^2,$$

L is the number of experimental responses, w_j is the weighting factor for each of the responses, $j=1,2,\ldots,L$, and N_j is the number of data in each experiment. y_{pj} and y_{mj} are the plant and model outputs, respectively, for the jth experiment.

IV Optimization Method

Nonlinear programming techniques [2, 5] may be applied to the optimization process of the identification scheme. We may classify nonlinear programming methods as two types: the direct search method such as simplex and Powell's methods, and the gradient method such as Davidon-Fletcher-Powell (D-F-P) method [5]. Since the latter uses information from the derivative or gradient of the objective function in determining the search direction, one generally expects a more stable and faster convergence in the optimization process. However, our experience has shown that the gradient method like D-F-P method is not suitable for the identification problem under investigation. This may be due to a number of factors: 1) an explicit mathematical gradient relationship for the nonlinear

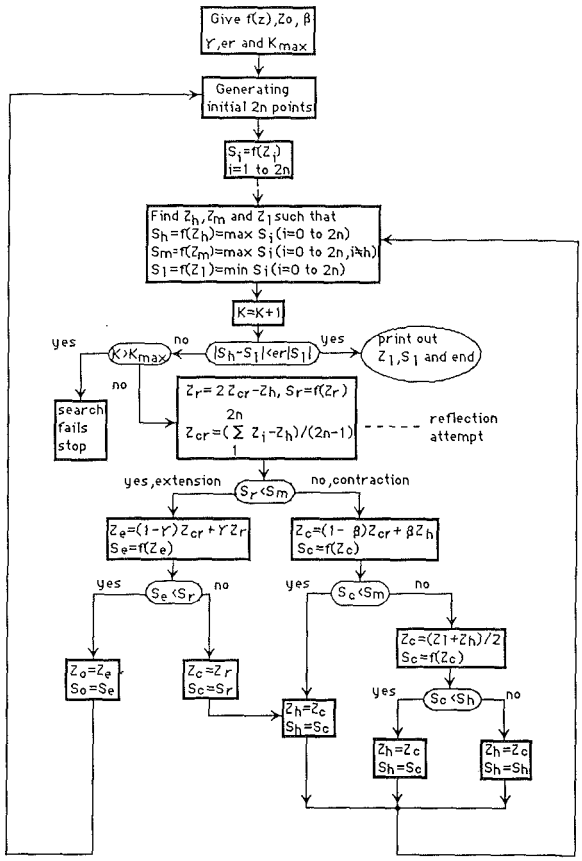

Fig. 5 Flowchart of the modified simplex algorithm

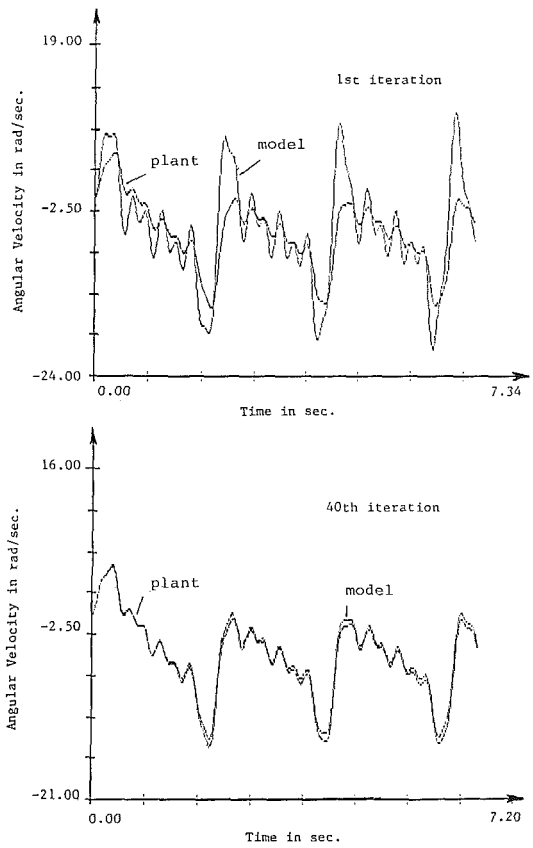

Fig. 6 Comparison of responses of plant and model

system model response could not be found; 2) the objective function (7) or (10) is generally not convex over the parameter space; and 3) the objective function varies very slowly with the variations of some of the parameters under certain system excitation conditions.

The difficulties in computing the gradients of the objective function with sufficient accuracy turn our attention to the direct search method. From extensive programming experiments, it is found that the simplex method [5–8] with some modifications is effective in overcoming the above difficulties.

The work of Hanus et al. [6] presents a description of the classical simplex method introduced by Splendley et al. [7], and its improvements. The classical simplex procedure consists of choosing $(n+1)$ points in a space of n independent variables. Based on the $(n+1)$ points, it sets out to find a new point in the n-dimensional space which produces a better value for the objective function. The optimum point is found by a sequence of elementary geometric transformations: *reflection*, *contraction*, and *extension*. A comparison of the simplex method with other nonlinear programming methods is found in [8]. The classical simplex algorithm is relatively slow in convergence and the final point is not necessary the global optimum. Modifications that help to speed up the rate of convergence and to increase the chance of finding a global optimum have been proposed [5, 6].

In this paper, we introduce a new modification in the extension procedure:

If the new extension point is better than the reflection point, then take this new point as the initial point, and begin a new Simplex search iteration. If not, continue in the usual way.

This modification helps to increase the convergence rate of the search algorithm. Since the modified procedure explores new directions, the chances of finding the global optimum are also increased. Simulation results show that these improvements have been very effective. The modified simplex algorithm is summarized in the Appendix for completeness, and its flowchart is shown in Fig. 5. The reflection coefficient α is chosen as 1, the contraction and extension coefficients β and γ may be chosen as some values around 0.5 and 2.0, respectively [6]; *er* is the scale of convergence criterion which is a small number whose value depends on the accuracy desired; $K_{\max}$ is the maximum iteration time; and $f(z)$ is the objective function to be minimized, such as J_e given by (10).

V Identifiability of Parameters and Choice of Excitation Input Signals

Successful identification of system characteristics requires the understanding of the identifiability of the parameters and the right choice of excitation input signals. It is necessary to excite the transient behavior associated with the parameters to be identified in order to study their characteristics. The information which one can draw from any experiment thus depends on the excited characteristics and the nature of the excitation inputs. There are two different kinds of parameters in the model being investigated: one associated with linear characteristics and other with nonlinear characteristics. In this paper, we will concentrate on the problem of finding a proper input which simultaneously excites both the linear and nonlinear characteristics of the system.

A solution to the suitable choice of excitation inputs would be a set of input signals which have high and low frequency components as well as small and large magnitude. The small magnitude low frequency signal excites the nonlinear stick slip friction characteristics while moderately large magnitude high frequency signal incites mostly the linear behavior. The multiresponse objective functions (8)–(10) may be used to

estimate the optimum parameters that identify the linear and nonlinear characteristics. The effectiveness of the proposed approach is verified by simulation studies shown in the next section.

The choice of input signal is an interesting aspect of both linear and nonlinear system identification problems. A comprehensive discussion on this subject is given in [9].

VI Analyses of Results

In this section, we apply the method suggested in the preceding sections to identify the parameters of a simulated servomotor system in which the stick-slip friction is considered. The aim is to verify that the proposed parameter identification technique works on a simulation basis. A future goal is to test the scheme on an experimental basis.

Satisfactory simulation results were obtained and are shown in Fig. 6. It is found that the response of the model agrees closely with that of the simulated plant at the 40th iteration of the modified Simplex algorithm. The objective function reduces from 4.36×10^{3} at the 1st iteration to approximately 10^{-10} at the final iteration when the algorithm converges.

In the simulation, it is assumed that K_a, K_b, K_t, L_m, and R_m are known, and J, B, D_p, D_ϕ, and T_s are to be identified. A summary of the results is as follows: the actual values of the parameters used for simulating the plant are: $J=3.0\times10^{-3}$, $B=1.2\times10^{-4}$, $D_p=8.5\times10^{-2}$, and $T_s=1.0\times10^{-3}$; the estimated values recovered from the simulated plant output data are: $J=3.000\times10^{-3}$, $B=1.199\times10^{-4}$, $D_p=8.519\times10^{-2}$, and $T_s=9.995\times10^{-4}$. Note that D_ϕ is determined by using the relationship between D_p and D_ϕ given in (5). Hence, the parameters in the proposed model of a servomotor system with stick-slip friction can be obtained. The results could be considered to be quite satisfactory.

VII Conclusions

A nonlinear model for a class of servomechanism system with stick-slip friction is presented. An effective technique has been developed to identify the parameters of this model. The main difficulties in the identification problem are: the formulation of the identification model, the identifiability of the system parameters, the choice of excitation inputs and the choice of the nonlinear programming method. The modified simplex method has proved to be effective as an optimization procedure in the identification problem investigated. It is expected that the technique proposed here is applicable to system parameter identification problems associated with a wide class of nonlinear systems. Experimental aspects and verification of the method is currently underway.

Acknowledgment

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APPENDIX

Modified Simplex Algorithm

Let Z denote the set of n independent variables; $f(z)$, $z\epsilon Z$, be the objective function to be minimized. Then, after choosing a first set of $(n+1)$ points $z_i\epsilon Z$, $i=1, 2, \ldots, n+1$, the classical simplex procedure [7] basically includes a sequence of elementary geometric transformations: reflection, contraction and extension, to coverge to an optimum. Define the following:

A *reflection* point z_r is given by

$$z_r=(1+\alpha)z_{cr}-\alpha z_h, \quad \alpha>0, \tag{11}$$

where z_h is the worst point which yields the highest value of $f(z_i)$, and z_{cr} is the centroid:

$$z_{cr}=\left[\sum_{\substack{i=1\\ i\neq h}}^{n+1} z_i\right]\Big/n. \tag{12}$$

The value of α is usually 1. It is a reflection of the worst point z_h through the centroid z_{cr}. A *contraction* point z_c is given by

$$z_c=(1-\beta)z_{cr}+\beta z_h, \quad 0.0<\beta<1.0. \tag{13}$$

The usual value of β is 0.5. It is a contraction of the worst point z_h towards the centroid z_{cr}. A *general contraction* is a procedure given by

$$z_i=(1-\beta)z_l+\beta z_i, \quad i=1,2, \ldots, n+1, \tag{14}$$

with z_l the best point which yields the lowest value of $f(z_i)$. All the points are contracted towards the best point. An *extension* point z_e is given by

$$z_e=(1-\gamma)z_{cr}+\gamma z_r, \quad \gamma>1. \tag{15}$$

It is an extension of the reflection beyond the reflection point. The usual value of the parameter γ is 2.

The simplex iteration begins with reflection. The *reflection* procedure consists of trying to replace the worst point by its reflection through the centroid of all other points. If the reflection point is better than the current best point, the *extension* procedure is tried; else the reflection point is compared to all the other points except the worst one. If the reflection point is better than one of other points, then it takes the place of the worst one and a new iteration can be repeated; else the *contraction* procedure is tried.

The *contraction* procedure consists of trying to replace the worst point z_h by a contraction point z_c. If the contraction point is better than the worst point, then it takes the place of that point and a new iteration starts; else a *general contraction* of all the points is executed towards the best one, and a new simplex iteration may be started.

When the reflection point is better than the current best one, the *extension* procedure is applied. If this new extension point is better than the reflection point, it replaces that point in the simplex, and a new simplex iteration may be started again.

Since each transformation replaces the current worst point with a better one, an optimum would finally be reached.

In order to overcome the disadvantages of the classical simplex algorithm [8], different improvements have been proposed [6, 10]. Some of the main improvements include:

(1) Regarding the simplex size, Lesuisse [10] has shown that the optimal number of points is $2n$ and not the minimum number $(n+1)$. The choice of $2n$ points assures a maximum speed of convergence and a maximum chance to find a global optimum.

(2) Regarding the contraction procedure, Hanus et al. [6] modified the simplex method as follows. For the contraction procedure, if the contraction point is not better than the current worst one, do the following operation: a new point z_n is chosen halfway between the worst point and the best one:

$$z_n = (z_l + z_h)/2. \tag{16}$$

If the new point z_n is better than the current worst point z_h, it takes its place in the simplex with the associated cost given by $f(z_n)$. If not, it also takes the place of the worst point but with the associated cost given by $f(z_h)$, and a new iteration is started.

In our approach, we introduce a new modification in the extension procedure: If the new point z_e after extension is better than the reflection point z_r, then take this point as initial point, and a new simplex algorithm is started with new set of initial points around z_e. If not, continue in usual way.

The modifications in both contraction and extension procedures, and the use of $2n$ points assure the convergence of the algorithm. Since they explore new directions, the chance of finding the global optimum increases. Moreover, when the simplex has reached an optimum, the entire procedure can be restarted with a new simplex for exploring the vicinity around this newly found optimum.