Proceedings of the ASME 2011 International Mechanical Engineering Congress & Exposition
IMECE2011
November 11-17, 2011, Denver, Colorado, USA

IMECE2011-63108

ANALYTICAL SOLUTIONS FOR THE STATIC INSTABILITY OF MICRO/NANO MIRRORS UNDER THE COMBINED EFFECT OF CAPILLARY FORCE AND CASIMIR FORCE

Hamid Moeenfard
PhD Candidate, Department of mechanical engineering, Sharif University of Technology, Tehran, Iran.
hamid_moeenfard@mech.sharif.edu

Ali Darvishian
M.Sc. Student, School of Mechanical Engineering, Sharif University of Technology, Tehran, Iran.
alidarvishian@gmail.com

Mohammad Taghi Ahmadian
Center of Excellence in Design, Robotics and Automation, Department of Mechanical Engineering, Sharif University of Technology, Tehran, Iran.
ahmadian@mech.sharif.edu

ABSTRACT
This paper deals with the problem of static instability of Micro/Nano mirrors under the combined effect of capillary force and Casimir force. At the First the governing equations of the statical behavior of Micro/Nano mirrors under the combined effect of capillary force and casimir force is obtained. The dependency of the critical tilting angle on the physical and geometrical parameters of the nano/micromirror and its supporting torsional beams is investigated. It is found that existence of casimir force can considerably reduce the stability limits of nano/micromirror. It is also found that rotation angle of the mirror due to capillary force highly depends on the casimir force applied to the mirror. Finally analytical tool Homotopy Perturbation Method (HPM) is utilized for prediction of the mirror's behaviour under combined capillary and casimir forces. It is observed that a sixth order perturbation approximation accurately predicts the rotation angle and stability limits of the mirror. Results of this paper can be used for successful fabrication of nano/micromirrors using wet etching process where capillary force plays a major role in the system.
.

Keywords: Nano/micromirror, capillary force, casimir force, HPM.

1) Introduction
The technology of MEMS devices has experienced a lot of progress recently. Their low manufacturing cost, batch production, light weight, small size, durability, low energy consumption and compatibility with integrated circuits, makes them extremely attractive [1, 2]. Successful MEMS devices rely not only on well developed fabrication technologies, but also on the knowledge of device behavior, based on which a favorable structure of the device can be forged [3]. The important role of MEMS devices in optical systems has initiated the development of a new class of MEMS called MicroOptoElectroMechanical Systems (MOEMS). MOEMS mainly includes micromirrors and torsional micro-actuators. These devices has found variety of applications such as optical cross connects [4, 5], optical switches [6], digital micromirror devices (DMD) [7], micro scanning mirrors [8], and etc.
Existence of a liquid bridge between two objects results in forming capillary force [9]. The existence of capillary force even in low relative humidity is observed experimentally [10]. Parallel plate MEMS actuators are conventionally fabricated by forming a layer of a plate or beam material on the top of a sacrificial layer of another material and wet etching the sacrificial layer. In this process, capillary force can be easily formed and in

the case of poor design, the structure will collapse and adhere to the substrate. So investigating the effect of capillary force on micromirrors is extremely important in their design and fabrication.

Many researchers investigated the effect of capillary force on MEMS devices. Mastrangelo and Hsu [11, 12] studied the stability and adhesion of thin micromechanical structures under capillary force, theoretically and experimentally. Moeenfard et al [13] studied the effect of capillary force on the static pull-in instability of fully clamped micromirrors. The effects of capillary force on the static and dynamic behaviors of atomic force microscopes (AFM) are widely assessed [14-16]. Recently, the instability of torsional MEMS/NEMS micro-actuators under capillary force was investigated by Guo et al [17].

When the size of a structure is sufficiently small, casimir and van der Waals forces play an important role and in the case of poor design, can lead to the collapse of the structure. VdW force is the interaction force between neutral atoms and it varies from covalent and ionic bondings in that it is caused by correlations in the fluctuating polarizations of nearby particles [18]. Casimir effect is understood as the longer distances range analog of the vdW force, resulting from the propagation of retarded electromagnetic waves, whose distance ranges from a few nanometers up to a few micrometers [19].

Tahami et al [20] discussed Pull-in Phenomena and Dynamic Response of a Capacitive Nano-beam Switch by considering casimir effect**.** Casimir effect on the pull-in parameters of nanometer switches has been studied by Lin and Zhao [21]. They [22] also studied Nonlinear behavior of nano-scale electrostatic actuators with casimir force. Ramezani et al [23, 24] investigated the two point boundary value problem of the deflection of nano-cantilever subjected to casimir and electrostatic forces using analytical and numerical methods to obtain the instability point of the nanobeam. Modelling and simulation of electrostatically actuated nano-switches under the effect of casimir forces have been studied by mojahedi et al [25]**.** Sirvent et al [26] theoretically studied pull-in control in capacitive microswitches actuated by Casimir forces using external magnetic fields. Effect of the casimir force on the static deflection and stiction of membrane strips in MEMS have been studied by Serry et al [27]. Guo and Zhao [28] discussed the effect of casimir force on the pull-in of electrostatic torsional actuators. But statical behavior and pull-in of single sided nano/micromirrors under effect of capillary and casimir forces has not been investigated. So in this paper, the combined effect of casimir and capillary forces on the tilting angle and stability of torsional nano/micromirror is studied. In this study, HPM is used as a perturbational based analytical tool.

Perturbation methods have been used to analytically solve the nonlinear problems in MEMS. Younis and Nayfeh [29] investigate the response of a resonant microbeam to an electric actuation using the multiple-scale perturbation method. Abdel-Rahman and Nayfeh [30] used the same method to model secondary resonances in electrically actuated microbeams. Since perturbation methods are based upon the assumption that there is a small parameter in the equations, they have some limitations in problems without involvement of small parameters. In order to overcome this limitation a new perturbational based method, namely Homotopy Perturbation Method (HPM) was developed by He et al [31]. His new method takes full advantages of the traditional perturbation methods and homotopy techniques. Homotopy perturbation method has also been used for solving the nonlinear problems encountered in N/MEMS. For example, Moeenfard et al [32] used HPM to model the nonlinear vibrational behavior of Timoshenko micobeams. Mojahedi et al [33] applied the HPM method to simulate the static response of nano-switches to electrostatic actuation and intermolecular surface forces. But so far no analytic solution has been presented to model the behavior of nano/micromirrors.

In the current paper, the equations governing the statical behavior of rectangular nano/micromirrors are obtained using the minimum potential energy principle. Then pull-in parameters of nano/micromirrors under effect of casimir and capillary forces are investigated. At the end, tilting angle of a nano/micromirror under casimir and capillary forces is calculated both numerically and analytically using HPM.

2) Theoretical model

The micromirror shown in figure (1) is considered.

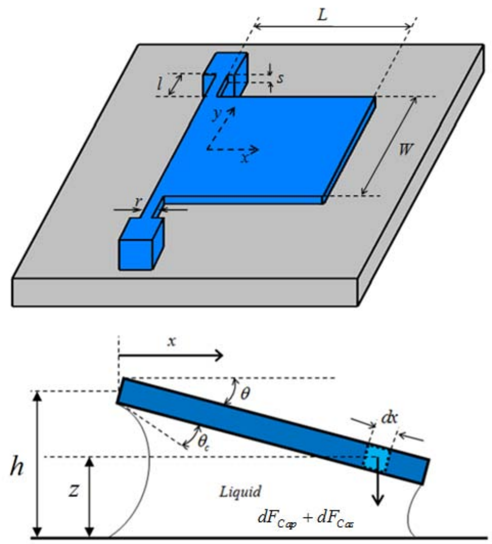

Fig. 1: Schematic view of a nano/micromirror.

The capillary pressure, P_{cap} underneath the mirror is [13]:

$$P_{cap} = \frac{2\gamma \cos\theta_c}{h - x \sin\theta} \tag{1}$$

where h is the initial distance between the mirror and the substrate, θ is the tilting angle of the mirror, γ is

surface energy of liquid and θ_c is contact angle between liquid and solid surface.
Furthermore the differential casimir force applied to a differential surface element of the mirror shown in figure (1) is [34]:

$$dF_{Cas} = \frac{\pi^2 \hbar c}{240(h - x\sin\theta)^4} W dx \tag{2}$$

Where c is speed of light, $\hbar$ is Plank's constant divided by 2π and W is width of mirror as illustrated in figure (1).
The minimum total potential energy principle [35] is utilized here to obtain equilibrium equation and to investigate the stability of the equilibrium points. The total potential energy of the system can be divided into two parts: the potential strain energy of the torsion beams and the potential energy of applied loads which is equal to the minus of work done by external forces.

$$\Pi = U + V = U - W_e \tag{3}$$

Where Π is the total potential energy of the system, U is the potential strain energy of the torsion beams, V is the potential energy of applied loads and W_e is the work done by external forces. In equilibrium points, variation of the total potential energy of the system is zero.

$$\delta\Pi = \delta U - \delta W_e = 0 \tag{4}$$

The potential strain energy of system can be calculated as:

$$U = \frac{1}{2} K \theta^2 \tag{5}$$

Where

$$K = \frac{2GI_p}{l} \tag{6}$$

In this equation, G is the shear modulus of elasticity of the beam's material, l is length of each torsion beam and I_p is the polar momentum of inertia of the beams cross section which can be calculated using equation (7) [36].

$$I_p = \frac{1}{3} r s^3 - \frac{64}{\pi^5} s^4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \tanh\frac{(2n-1)\pi r}{2s} \tag{7}$$

Where r and s are the width and length of the torsion beams cross section respectively as illustrated in figure (1).
The variation of U would be as

$$\delta U = K\theta\delta\theta \tag{8}$$

The total external work done on nano/micromirror to rotate it from angle θ to angle $\theta + \delta\theta$ can be calculated as follows.

$$\begin{aligned}\delta W_e &= \int_0^L (dF_{Cap} + dF_{Cas})(x\,\delta\theta) \\ &= \int_0^L \left(\frac{2\gamma\cos\theta_c}{h - x\sin\theta} W dx + \frac{\pi^2 \hbar c}{240(h - x\sin\theta)^4} W dx\right)(x\,\delta\theta)\end{aligned} \tag{9}$$

Where L is length of mirror as illustrated in figure (1). Since $\frac{h}{L} << 1$, the tilting angle is small, and $\sin\theta$ can be closely approximated by θ. For simplification purpose, the following dimensionless variable is introduced.

$$\Theta = \frac{\theta}{\theta_0} \tag{10}$$

Where $\theta_0 \approx \sin\theta_0 = \frac{h}{L}$ is the maximum physically possible rotation angle of the mirror.
At equilibrium points equation (4) must be satisfied. So by performing the integration the equilibrium equation is obtained as follows.

$$\frac{\eta}{\Theta}\left(1 + \frac{1}{\Theta}\ln(1-\Theta)\right) - \frac{\lambda}{\Theta^2}\left(\frac{1}{6} - \frac{3\Theta - 1}{6(\Theta - 1)^3}\right) + \Theta = 0 \tag{11}$$

where η and λ are called instability numbers and are defined as equations (12) and (13) respectively.

$$\eta = \frac{2\gamma\cos\theta_c W L^3}{K h^2} \tag{12}$$

$$\lambda = \frac{\pi^2 \hbar c W L^3}{240 h^5 K} \tag{13}$$

Performing the second variation operator on equation (3) and using equilibrium equation yields:

$$\begin{aligned}\delta^2\Pi = \frac{(\delta\Theta)^2 h^2 K}{L^2}\Bigg[1 - \frac{\eta}{\Theta^2}\left(1 + \frac{2\ln(1-\Theta)}{\Theta} + \frac{1}{1-\Theta}\right) \\ + \frac{\lambda}{\Theta^3}\left(\frac{8}{3}\frac{1}{(1-\Theta)^3} - \frac{2}{(1-\Theta)^2} - \frac{1}{(1-\Theta)^4} + \frac{1}{3}\right)\Bigg]\end{aligned} \tag{14}$$

According to minimum total potential energy principle an equilibrium point is stable when $\delta^2\Pi > 0$ and is unstable when $\delta^2\Pi < 0$. So the stability condition is reduced to:

$$\begin{aligned}\mathrm{I}(\eta,\lambda,\Theta) = 1 - \frac{\eta}{\Theta^2}\left(1 + \frac{2\ln(1-\Theta)}{\Theta} + \frac{1}{1-\Theta}\right) \\ + \frac{\lambda}{\Theta^3}\left(\frac{8}{3}\frac{1}{(1-\Theta)^3} - \frac{2}{(1-\Theta)^2} - \frac{1}{(1-\Theta)^4} + \frac{1}{3}\right) > 0\end{aligned} \tag{15}$$

Finding η from equation (11) and substituting it in equation (15) leads to:

$$I(\lambda,\Theta)=1-\frac{\frac{\lambda}{\Theta^{3}}\left(\frac{1}{6}-\frac{3\Theta-1}{6(\Theta-1)^{3}}\right)-1}{1+\frac{1}{\Theta}\ln(1-\Theta)}\left(1+\frac{2\ln(1-\Theta)}{\Theta}+\frac{1}{1-\Theta}\right) \\ +\frac{\lambda}{\Theta^{3}}\left(\frac{8}{3}\frac{1}{(1-\Theta)^{3}}-\frac{2}{(1-\Theta)^{2}}-\frac{1}{(1-\Theta)^{4}}+\frac{1}{3}\right) \quad (16)$$

Figure (2) shows the function $I(\lambda,\Theta)$ versus Θ at some values of λ.

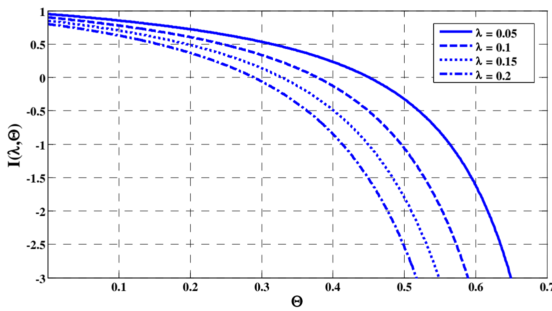

Fig. 2: Function $I(\lambda,\Theta)$ versus Θ.

An equilibrium point is stable if $I(\lambda,\Theta)>0$ and unstable if $I(\lambda,\Theta)<0$. It is observed that in certain value of Θ called Θ_P, which relates to the pull-in state, $I(\lambda,\Theta)$ becomes zero. When $\Theta<\Theta_P$, $I(\lambda,\Theta)$ would be positive and the resulting equilibrium point is stable and when $\Theta>\Theta_P$, $I(\lambda,\Theta)$ would be negative and the resulting equilibrium point is unstable.

At the pull-in state the following equation is satisfied.

$$I(\lambda,\Theta_p)=0 \quad (17)$$

Figure (3) shows the values of pull-in angle versus λ_p where λ_p is the value of λ at pull-in. It is observed that with increasing the value of λ_p the pull-in angle of the mirror is reduced.

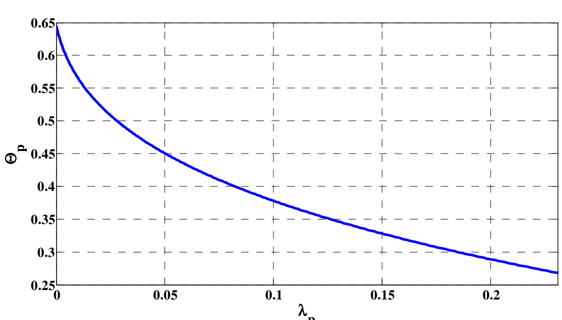

Fig. 3: pull-in angle of mirror versus λ_p.

Using equations (11) and (17), pull-in angle can be plotted versus η_P as illustrated in figure (4) where η_P is the value of η at pull-in state.

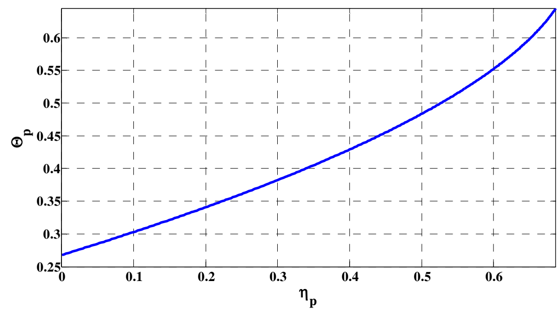

Fig. 4: pull-in angle of mirror versus η_P.

This figure shows that by increasing η_P, pull-in angle of the mirror is increased. By eliminating Θ_P between equations (11) and (17), η_P can be obtained versus λ_P as plotted in figure (5).

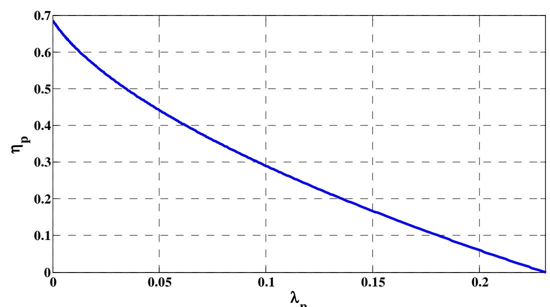

Fig. 5: η_P versus λ_P.

It is observed that with increasing λ pull-in occurs at lower values of η. In fact this figure shows that casimir force can significantly reduce the maximum allowable value for η and as a result, the stability limits of the nano/micromirror are reduced. In addition it can be concluded that even in the absence of capillary force, casimir force can lead to the occurrence of pull-in. So, in order to have a successful and stable design for nano/micromirrors fabricated using wet etching process where capillary force plays a major role, the inequalities given in equation (18) has to be satisfied.

$$\eta=\frac{2\gamma\cos\theta_c WL^3}{Kh^2}<\eta_p \\ \lambda=\frac{\pi^2\hbar cWL^3}{240h^5K}<\lambda_p \quad (18)$$

In order to investigate the mirror's behavior under combined capillary and casimir loading, the dimensionless rotation angle has been plotted versus η in figure (6).

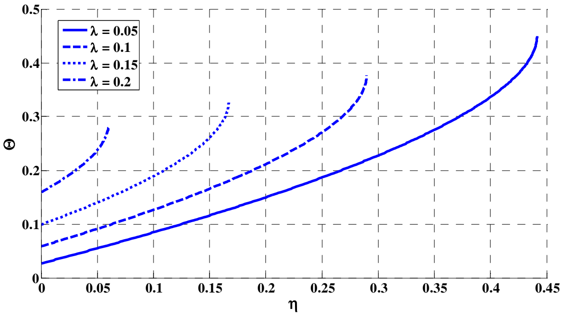

Fig. 6: Stable equilibrium angle versus η.

It is observed that by increasing the value of η the rotation angle of the nano/micromirror is increased, but the maximum value of η at pull-in, highly depends on the value of λ and it is verified that by increasing λ, the maximum allowable value for λ is reduced. Furthermore it is concluded that at a constant η, larger values of λ would lead to larger values for stable equilibrium angle.

3) Analytical solution of equilibrium equations

In this section, it is tried to find the value of the rotation angle of the nano/micromirror analytically in terms of η and λ. For this purpose, the analytical tool, HPM is utilized.

The linear part of equation (11) can be found by using Taylor series expansion of the equilibrium equation (11) as follows.

$$L(\Theta,\eta,\lambda) = -\frac{\eta+\lambda}{2} + \left(\frac{3-4\lambda-\eta}{3}\right)\Theta \tag{19}$$

Where $L(\Theta,\eta,\lambda)$ is the linear part of equation (11). Obviously the nonlinear part of equilibrium equation is obtained by subtracting $L(\Theta,\eta,\lambda)$ from equation (11).

$$N(\Theta,\eta,\lambda) = \frac{\eta}{\Theta}\left(1+\frac{1}{\Theta}\ln(1-\Theta)\right) - \frac{\lambda}{\Theta^2}\left(\frac{1}{6}-\frac{3\Theta-1}{6(\Theta-1)^3}\right) + \left(\frac{4\lambda+\eta}{3}\right)\Theta + \frac{\eta+\lambda}{2} \tag{20}$$

Now, the homotopy form is constructed as follows.

$$\Im(\Theta,\eta,\lambda,P) = L(\Theta,\eta,\lambda) + P.N(\Theta,\eta,\lambda) = 0 \tag{21}$$

In equation (21), $\Im(\Theta,\eta,\lambda,P)$ is the homotopy form and P is an embedding parameter which serves as perturbation parameter. When $P=1$, the homotopy form would be the same as the equilibrium equation and when $P=0$, homotopy form would be the linear part of equilibrium equation. The value of the dimensionless rotation angle Θ can also be expanded in terms of the embedded parameter P as follows.

$$\Theta = \Theta_0 + P\Theta_1 + P^2\Theta_2 + P^3\Theta_3 + \ldots \tag{22}$$

Substituting equation (22) into homotopy form yields:

$$\Im(\Theta,\eta,\lambda,P) = L\left(\Theta_0 + P\Theta_1 + P^2\Theta_2 + \ldots,\eta,\lambda\right) + P.N\left(\Theta_0 + P\Theta_1 + P^2\Theta_2 + \ldots,\eta,\lambda\right) = 0 \tag{23}$$

The Taylor series expansion of right hand side of equation (23) in terms of P would be as

$$\begin{aligned}\Im(\Theta,\lambda,P) &= L(\Theta_0,\eta,\lambda) + \left(\Theta_1\frac{\partial L(\Theta_0,\eta,\lambda)}{\partial\Theta_0} + N(\Theta_0,\eta,\lambda)\right)P \\ &+ \left(\Theta_2\frac{\partial L(\Theta_0,\eta,\lambda)}{\partial\Theta_0} + \Theta_1\frac{\partial N(\Theta_0,\eta,\lambda)}{\partial\Theta_0}\right)P^2 \\ &+ \left(\Theta_3\frac{\partial L(\Theta_0,\eta,\lambda)}{\partial\Theta_0} + \Theta_2\frac{\partial N(\Theta_0,\eta,\lambda)}{\partial\Theta_0} + \frac{1}{2}\Theta_1^2\frac{\partial^2 N(\Theta_0,\eta,\lambda)}{\partial\Theta_0^2}\right)P^3 \\ &+ \ldots = 0\end{aligned} \tag{24}$$

Since the homotopy form must be unified with zero, the coefficients of all powers of P must be zero. This, leads to the following equations.

$$L(\Theta_0,\eta,\lambda) = 0 \tag{25}$$

$$\Theta_1\frac{\partial L(\Theta_0,\eta,\lambda)}{\partial\Theta_0} + N(\Theta_0,\eta,\lambda) = 0 \tag{26}$$

$$\Theta_2\frac{\partial L(\Theta_0,\eta,\lambda)}{\partial\Theta_0} + \Theta_1\frac{\partial N(\Theta_0,\eta,\lambda)}{\partial\Theta_0} = 0 \tag{27}$$

$$\Theta_3\frac{\partial L(\Theta_0,\eta,\lambda)}{\partial\Theta_0} + \Theta_2\frac{\partial N(\Theta_0,\eta,\lambda)}{\partial\Theta_0} + \frac{1}{2}\Theta_1^2\frac{\partial^2 N(\Theta_0,\eta,\lambda)}{\partial\Theta_0^2} = 0 \tag{28}$$

With solving equations (25) to (28) , the parameters Θ_i $0 \le i \le 3$ are found as follows.

$$\Theta_0 = \frac{3(\eta+\lambda)}{2(3-\eta-4\lambda)} \tag{29}$$

$$\Theta_1 = -N(\Theta_0,\eta,\lambda)\Big/\left(\frac{\partial L(\Theta_0,\eta,\lambda)}{\partial\Theta_0}\right) \tag{30}$$

$$\Theta_2 = -\Theta_1\left(\frac{\partial N(\Theta_0,\eta,\lambda)}{\partial\Theta_0}\right)\Big/\left(\frac{\partial L(\Theta_0,\eta,\lambda)}{\partial\Theta_0}\right) \tag{31}$$

$$\Theta_3 = -\left(\Theta_2\frac{\partial N(\Theta_0,\eta,\lambda)}{\partial\Theta_0} + \frac{1}{2}\Theta_1^2\frac{\partial^2 N(\Theta_0,\eta,\lambda)}{\partial\Theta_0^2}\right)\Big/\left(\frac{\partial L(\Theta_0,\eta,\lambda)}{\partial\Theta_0}\right) \tag{32}$$

The value of Θ can be found by substituting Θ_i $0 \le i \le 3$ and $P = 1$ in equation (22). In figure (7) the results of the numerical simulations is compared with those of analytical HPM results for the special case of $\lambda = 0.1$. It is observed that HPM closely approximates the rotation angle of the mirror. Obviously increasing the order of perturbation approximation would lead to more precise results, but increasing the order of the perturbation approximation more than 6 will not improve the accuracy of the obtained response significantly. As a result, a sixth order perturbation approximation used in HPM can precisely predict the nano/micromirror behaviour under the combined effects of capillary and casimir force.

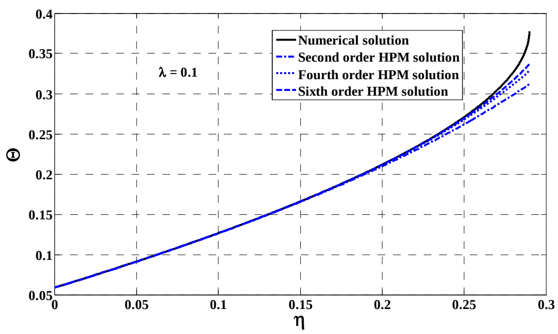

Fig. 7: Estimation of nano/micromirror's rotation angle using HPM.

Conclusion

The dimensionless equilibrium equation of nano/micromirror under capillary force was obtained considering casimir force. The dependency of the critical tilting angle on the instability numbers defined in the paper was investigated. Results show that neglecting casimir effect on the static equilibrium of nano/micromirrors under capillary force may lead to considerable error in predicting stability limits of the mirror and can lead to an unstable design.

It was observed that rotation angle of the mirror due to capillary force highly depends on the casimir effect applied to the mirror. HPM was utilized to analytically predict the rotation angle and stability limits of the nano/micromirrors. It was found that a sixth order perturbation approximation can accurately estimate the rotation angle of the mirror due to capillary and casimir loading. Presented results in this paper can be used for stable design and fabrication of nano/micromirrors using wet etching process where the gap between the mirror and the underneath substrate is less than a few micrometers and as a result, both capillary and casimir forces have significant effects on the system.

References

[1] N. Maluf, K. Williams, 1999, "An Introduction to Microelectromechanical Systems Engineering, Second Edition, MicroelectromechanicalSystems (MEMS) Series, Boston, London, Artech House Inc.

[2] M.I Younis, 2004, "Modeling and Simulation of Microelectromechanical Systems in Multi-Physics Fields", PhD thesis, Virginia Polytechnic Institute and State University.

[3] P.C.P Chao, C.W. Chiu, C.Y. Tsai, 2006, "A novel method to predict the pull-in voltage in a closed form for micro-plates actuated by a distributed electrostatic force", J. Micromech.Microeng. 16, pp.986–998.

[4] L.J.Hornbeck, 1991 "Spatial light modulator and method", US Patent 5,061,049.

[5] J.E. Ford, V.A. Aksyuk, D.J. Bishop, J.A. Walker, 1999, "Wavelenghth add-drop swiching using tilting micromirrors", J. Lightwave technol. 17, 904-911.

[6] D.L. Dickensheets, and G.S. Kino, 1998, "Silicon-micromachined scanning confocal optical microscope", Journal of Microelectromechanical Systems, vol. 7, no. 1 , pp. 38-47.

[7] P.M. Zavracky, S. Majumber, E. McGruer, 1997, "Micromechanical switches fabricated using nickel surface micromachining", J. Microelectromech. Syst. 6, pp. 3–9.

[8] H. Toshiyoshi, H. Fujita, 1996, "Electrostatic micro torsion mirrors for an optical switch matrix", J. Microelectromech. Syst. 5, 231–237.

[9] Z. Wei, and Y.P. Zhao, 2007, "Growth of liquid bridge in AFM", Journal of Physics D: Applied Physics, vol. 40, no. 14, pp. 4368-4375.

[10] P. J. van Zwol, G. Palasantzas, J. Th. M. De Hosson, 2008, "Influence of roughness on capillary forces between hydrophilic surfaces", Physical Review E 78, 03160.

[11] C. H. Mastrangelo, C. H. Hsu, 1993, "Mechanical Stability and Adhesion of Microstructures Under Capillary Forces-Part I: Basic Theory", Journal of Microelectromechanical Systems 2 (1), pp. 33-43.

[12] C. H. Mastrangelo, C. H. Hsu, 1993, "Mechanical Stability and Adhesion of Microstructures Under Capillary Forces-Part 2: Experiments", Journal of Microelectromechanical Systems 2 (1), pp. 44-55.

[13] H. Moeenfard, M.H.Kahrobaiyan, M.T. Ahmadian, 2010, "Aplication of the extended Kantorovich method to the static deflection of microplates under capillary force", ASME International Mechanical Engineering Congress and Exposition, IMECE2009-12186.

[14] L. Zitzler, S. Herminghaus, F. Mugele, 2002, "Capillary forces in tapping mode atomic force microscopy", PHYSICAL REVIEW B 66, 155436.

[15] Xide Li, Yun Peng, 2006, "Investigation of capillary adhesion between the microcantilever and the substrate with electronic speckle pattern interferometry", APPLIED PHYSICS LETTERS 89, 234104.

[16] J. Jang, G.C. Schatz, M.A Ratner, 2004, "Capillary force in atomic force microscopy",Journal of Chemical Physics 120 (3), pp. 1157-1160.

[17] J.G. Guo, L.J. Zhou, and Y.P. Zhao, 2009, "Instability analysis of torsional MEMS/NEMS actuators under capillary force", Journal of Colloid and Interface Science, vol. 331, no. 2, pp. 458-462.

[18] G. Xie, J. Ding, S. Liu, W. Xued, J. Luob, 2009, "Interfacial properties for real rough MEMS/NEMS surfaces by incorporating the electrostatic and Casimir effects–a theoretical study", Surf. Interface Anal., 41, pp. 338–346.
[19] A. Gusso, G.J. Delben, 2007, "Influence of the Casimir force on the pull-in parameters of silicon based electrostatic torsional actuators", Sensors and Actuators A 135, pp. 792–800.
[20] F.V. Tahami, H. Mobki, A.A.K.Janbahan, G. Rezazadeh, 2009, "Pull-in Phenomena and Dynamic Response of a Capacitive Nano-beam Switch", Sensors & Transducers Journal, Vol. 110, Issue 11, pp. 26-37.
[21] W.H. Lin, Y.P. Zhao, 2005, "Casimir effect on the pull-in parameters of nanometer switches", Microsystem Technologies 11, pp. 80–85.
[22] W.H. Lin, Y.P. Zhao, 2005, "Nonlinear behavior for nanoscale electrostatic actuators with Casimir force", Chaos, Solitons and Fractals 23, pp. 1777–1785.
[23] A. Ramezani, A. Alasty, J. Akbari, 2007, "Pull-in parameters of cantilever type nanomechanical switches in presence of Casimir force", Nonlinear Analysis: Hybrid Systems 1, pp. 364–382.
[24] A. Ramezani, A. Alasty, J. Akbari, 2008, "Analytical investigation and numerical verification of Casimir effect on electrostatic nano-cantilevers", Microsyst Technol. 14, pp. 145–157.
[25] M. Mojahedi, H. Moeenfard, M. T. Ahmadian, 2009, "A new efficient approach for modeling and simulation of nano- switches under combined effects of intermolecular surface forces and electrostatic actuation", International Journal of Applied Mechanics Vol. 1, No. 2, pp. 349–365.
[26] R.E. Sirvent, M.A. Palomino-Ovando,G. H. Cocoletzi, 2009, "Pull-in control due to casimir forces using external magnetic fields", APPLIED PHYSICS LETTERS 95, 051909.
[27] F. M. Serry, D. Walliser,G. J. Maclay, 1998, "The role of the casimir effect in the static deflection and stiction of membrane strips in microelectromechanical systems „(MEMS) ", J. Appl. Phys., Vol. 84, No. 5, pp. 2501-2506.
[28] J.G. Guo, Y.P. Zhao, 2004, "Influence of van der Waals and Casimir Forces on Electrostatic Torsional Actuators", JOURNAL OF MICROELECTROMECHANICAL SYSTEMS, VOL. 13, NO. 6, pp. 1027-1035.
[29] M.I. Younis, A.H. Nayfeh, 2003, "A study of the nonlinear response of a resonant microbeam to an electric actuation", Nonlinear Dynamics, vol. 31, no. 1, pp. 91-117.
[30] E.M. Abdel-Rahman, A.H. Nayfeh, 2003, "Secondary resonances of electrically actuated resonant microsensors", Vol. 13, pp. 491–501.
[31] [31] J. H. He, 2000, "A coupling method of a homotopy technique and a perturbation technique for non-linear problems", Int. J. Non-Linear Mechanics, 35, pp. 37–43.
[32] H.Moeenfard, M.Mojahedi, M.T.Ahmadian, "A homotopy perturbation analysis of nonlinear free vibration of timoshenko microbeams", Journal of mechanical science and technology 25 (3), 279-285, DOI 10.1007/s12206-010-0130-8, article in press.
[33] M. Mojahedi, H. Moeenfard, and M.T. Ahmadian, 2010, "Analytical solutions for the static instability of nano-switches under the effect of casimir force and electrostatic actuation", ASME International Mechanical Engineering Congress and Exposition, Proceedings, pp. 63-69.
[34] H. Liu, S. Gao, S. Niu, L. Jin, 2010, "Analysis on the adhesion of microcomb structure in MEMS", International Journal of Applied Electromagnetics and Mechanics 33, pp. 979–984.
[35] S.S. Rao, 2007, "Vibration of Continuous Systems", John Wiley & Sons, New Jersey.
[36] J.M. Huang, A.Q. Liu, Z.L. Deng, Q.X. Zhang, J. Ahn, and A. Asundi, 2004, "An approach to the coupling effect between torsion and bending for electrostatic torsional micromirrors", Sensors and Actuators, A: Physical, vol. 115, no. 1, pp. 159-167.