Double Weighted Fourier Transform for the Wave Field in Inhomogeneous Medium: Some Results and Prospects

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Abstract

This paper discusses results of the application of double weighted Fourier transform (DWFT) to problems of communication and diagnostics of inhomogeneous media. An agreement is noted between DWFT results and results of geometrical optics, Rytov, and phase screen approximations. A relation has been found between DWFT and path integrals. Numerical simulation has been used to demonstrate the possibility of spatial signal processing based on inverse DWFT with resolution exceeding the Fresnel one during strong field fluctuations. The numerical simulation has also shown the potential applicability of this processing for reducing wave intensity fluctuations in inhomogeneous random media.

1. Introduction

The efficiency of signal processing in diagnostics and communication systems depends largely on the signal model employed which describes the relation between signal and medium parameters. The widely used geometrical optics (GO) model can not account correctly for inhomogeneities of sizes less than the Fresnel radius. Born and Rytov models can not be applied during strong field fluctuations. The phase screen method requires some prior information about the location of the plane in the vicinity of which there are inhomogeneities under study. Thus, the models listed above give no way of describing the field structure of the wave propagating in a multiscale inhomogeneous medium. Double (in coordinates of sources and receivers) weighted Fourier transform (DWFT) has been suggested previously [1] to describe the wave field in multiscale inhomogeneous media. This paper discusses results of investigations [1-5] into different problems of communication and diagnostics of inhomogeneous media by the DWFT method.

2. DWFT and its Relation to Other Methods

Let a point source emitting the harmonic oscillation with a frequency of ω be located at a point $\mathbf{r}_0 = \{z_0, x_0, y_0\} = \{z_0, \boldsymbol{\rho}_0\}$ on the plane $z = z_0$, and the field $U(z_t, \boldsymbol{\rho}; z_0, \boldsymbol{\rho}_0)\exp\{ik(z_t - z_0)\}$ be determined on the plane $z = z_t$ at a point $\mathbf{r} = \{z_t, x, y\} = \{z_t, \boldsymbol{\rho}\}$ where $\boldsymbol{\rho}_0 = \{x_0, y_0\}$ and $\boldsymbol{\rho} = \{x, y\}$ are two-dimensional vectors indicating the source and receiver location on the corresponding planes. Between emission and observation planes is inhomogeneous plasma with a refractive index $n^2(\mathbf{r}) = 1 + \tilde{\varepsilon}(\mathbf{r})$, $|\tilde{\varepsilon}(\mathbf{r})| << 1$. Next we will examine wave propagation along the z axis and apply a small-angle approximation. In [1, 3], DWFT was exploited to derive a solution to this problem which may be written as follows

$$U(z_t, \boldsymbol{\rho}; z_0, \boldsymbol{\rho}_0) = \frac{-Ak^2}{4(\pi Z)^3}\exp\{ik(\boldsymbol{\rho}-\boldsymbol{\rho}_0)^2/(2Z)\}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\xi d^2\xi_0 \exp\{2ik(\boldsymbol{\rho}-\boldsymbol{\xi})(\boldsymbol{\rho}_0-\boldsymbol{\xi}_0)/Z + ik\tilde{\Phi}(\boldsymbol{\xi}, \boldsymbol{\xi}_0, z_t, z_0)\}, \quad (1)$$

where $k = \omega / c = 2\pi / \lambda$; c and λ are the velocity of light and the wavelength in a free space;; $Z = z_t - z_0$; A is the wave amplitude determined from the transmitter power and the transmitting antenna diagram;

$$\tilde{\Phi}(\boldsymbol{\xi}, \boldsymbol{\xi}_0, z_t, z_0) = 0.5\int_{z_0}^{z_t} \tilde{\varepsilon}[\mathbf{s}(\boldsymbol{\xi}, \boldsymbol{\xi}_0, z'), z'] dz', \quad (2)$$

$$\mathbf{s}(\boldsymbol{\xi}, \boldsymbol{\xi}_0, z') = [\boldsymbol{\xi}(z' - z_0) + \boldsymbol{\xi}_0(z_t - z')]/Z. \quad (3)$$

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Expression (2) is the increment of phase path along the line (3) joining points $\boldsymbol{\xi}$ and $\boldsymbol{\xi}_0$ in the receiving ($z'=z_t$) and transmitting ($z'=z_0$) planes respectively. By calculating integrals in (1), when the minimum size l of inhomogeneity exceeds the Fresnel radius l_f

$$l > l_f = l_f(z') = \sqrt{\lambda(z_t - z')(z' - z_0)/Z} \tag{4}$$

we can derive the GO approximation using the stationary phase method. During weak phase disturbances when $\left|k\tilde{\Phi}(\boldsymbol{\xi},\boldsymbol{\xi}_0,z_t,z_0)\right| << 1$, setting in integrand (1) $\exp\left\{k\tilde{\Phi}(\boldsymbol{\xi},\boldsymbol{\xi}_0,z_t,z_0)\right\} \simeq 1 + k\tilde{\Phi}(\boldsymbol{\xi},\boldsymbol{\xi}_0,z_t,z_0)$ yields (see [1]) the Rytov approximation accounting for diffraction effects. Thus, Expression (1) simultaneously takes into account diffraction effects of small-scale inhomogeneities and considerable phase variations described by the GO approximation. Besides, as was shown in [1], integral expression (1), when the inhomogeneity is localized in the vicinity of the plane $z'=z_b$, conforms to results of the phase screen method and hence accounts for multipath propagation and caustics during wave propagation in an inhomogeneous medium.

In [6], the path integral method was employed to obtain a strict representation for the Green function for the parabolic equation which we write as follows:

$$U(z_t,\rho;z_0,\rho_0) = \frac{-ikA}{2(\pi Z)^2}\exp\left\{ik(\rho-\rho_0)^2/(2Z)\right\}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\xi d^2\xi_0 \exp\left\{2ik(\boldsymbol{\rho}-\boldsymbol{\xi})(\boldsymbol{\rho_0}-\boldsymbol{\xi_0})/Z - ik(\boldsymbol{\rho}-\boldsymbol{\xi})^2/(2Z)\right\}$$
$$\times\int D^2v(\tau)\,\delta\left(\int_{z_0}^{z_t} v(\tau)d\tau + \boldsymbol{\rho} - \boldsymbol{\xi}\right)\exp\left\{ik\left[\int_{z_0}^{z_t} v^2(\tau)d\tau + \int_{z_0}^{z_t}\tilde{\varepsilon}\left(\tau,\mathbf{s}(\boldsymbol{\xi},\boldsymbol{\xi}_0,\tau) - \int_{z_0}^{\tau} v(\alpha)d\alpha\right)d\tau\right]/2\right\}. \tag{5}$$

Here the symbol $\int D^2v(\tau) = \lim_{N\to\infty}(2\pi i)^{-N}\int_{R^2} k\Delta z dv_{N-1}\ldots\int_{R^2} k\Delta z dv_0$ corresponds to the continual integration over angular variables defined by the relationship $\rho(\tau) = \rho + \int_{\tau}^{z_t} v(\alpha)d\alpha$. By taking into account the equality

$$\int D^2v(\tau)\,\delta\left(\int_{z_0}^{z_t} v(\tau)d\tau + \boldsymbol{\rho} - \boldsymbol{\xi}\right)\exp\left\{ik\int_{z_0}^{z_t} v^2(\tau)d\tau/2\right\} = \frac{k}{2\pi iZ}\exp\left\{ik(\boldsymbol{\rho}-\boldsymbol{\xi})^2/(2Z)\right\}$$

we can see that the solution (5) transforms to (1), when $\int_{z_0}^{\tau} v(\alpha)d\alpha$ is neglected in the last exponent of (5). Hence the approximate DWFT representation for the Green function for a parabolic equation can be obtained from the strict continual integral representation, if we ignore the propagation path deviation as compared to a straight path. This gives an insight into the said agreement between DWFT results and known methods.

If the solution of (1) is acted on by the operator L

$$L[U(\boldsymbol{\rho},\boldsymbol{\rho}_0)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\rho d^2\rho_0 U(\boldsymbol{\rho},\boldsymbol{\rho}_0)\exp\left\{-ik\left[(\boldsymbol{\rho}-\boldsymbol{\rho}_0)^2/(2Z) + 2(\boldsymbol{\rho}^*-\boldsymbol{\rho})(\boldsymbol{\rho}_0^*-\boldsymbol{\rho}_0)/Z\right]\right\}, \tag{6}$$

then we obtain

$$\hat{U}(\boldsymbol{\rho}^*,\boldsymbol{\rho}^*_0) = L[U(\boldsymbol{\rho},\boldsymbol{\rho}_0)] = -A\pi Z(4k^2)^{-1}\exp\left\{ik\tilde{\Phi}(\boldsymbol{\rho}^*,\boldsymbol{\rho}^*_0,z_t,z_0)\right\}. \tag{7}$$

Thus, the spatial processing of the field $U(\boldsymbol{\rho},\boldsymbol{\rho}_0)$ in accordance with (6) yields a wave field $\hat{U}(\boldsymbol{\rho}^*,\boldsymbol{\rho}^*_0)$ with a phase increment $\tilde{\Phi}(\boldsymbol{\rho}^*,\boldsymbol{\rho}^*_0,z_t,z_0)$ equal to linear integral (2) employed in many inhomogeneous medium diagnostics methods. It has the form of a phase increment in the GO approximation, but it was derived from model (1), which, as we already mentioned, accounts for diffraction effects, and therefore, after spatial processing of (6), we can obtain linear integral (2) for inhomogeneities with transversal scales less than the Fresnel radius. Let us draw attention to the fact that, in contrast to the GO approximation, the received signal has no first-order amplitude fluctuations after processing with algorithm (6).

3. Simulation of spatial processing based on inverse DWFT

Here we shall address some possibilities of inverse DWFT (6). Note that inverse DWFT (6) performs integration in receiving and transmitting planes to infinite limits. In real conditions, both receiving and transmitting apertures have finite sizes and (6) can be written as

$$\hat{U}(\boldsymbol{\rho}^*,\boldsymbol{\rho}_0^*)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U(\boldsymbol{\rho},\boldsymbol{\rho}_0)\exp\left\{-(\boldsymbol{\rho}-\boldsymbol{\rho}_m)^2/\left(2D^2\right)-(\boldsymbol{\rho}_0-\boldsymbol{\rho}_{m0})^2/\left(2D_0^2\right)\right\}$$
$$\times\exp\left\{-2ik\left(\boldsymbol{\rho}^*-\boldsymbol{\rho}\right)\left(\boldsymbol{\rho}_0^*-\boldsymbol{\rho}_0\right)/Z/Z-ik(\boldsymbol{\rho}-\boldsymbol{\rho}_0)^2/(2Z)\right\}d^2\rho d^2\rho_0. \qquad (8)$$

Here we take the finiteness of the apertures into account by putting weighted functions $\exp[-(\boldsymbol{\rho}_0-\boldsymbol{\rho}_{m0})^2/(2D_0^2)]$ and $\exp[-(\boldsymbol{\rho}-\boldsymbol{\rho}_m)^2/(2D^2)]$ into the kernel of integral operator L, where $\boldsymbol{\rho}_m$ and $\boldsymbol{\rho}_{m0}$ are centers of weighted functions on the receiving and transmitting planes respectively; further they are taken to be $\boldsymbol{\rho}_m=\boldsymbol{\rho}^*$ and $\boldsymbol{\rho}_{m0}=\boldsymbol{\rho}_0^*$ to simplify the calculations.

Under weak fluctuations, as $U(\boldsymbol{\rho},\boldsymbol{\rho}_0)$ at $|k\Phi|<2\pi$ we used the Rytov's approximation for a spherical wave. Fig. 1a presents the simulation results for the phase $\Phi(x^*)=\arg\hat{U}(x^*=x_0^*,y^*=y_0^*=0)$. The inhomogeneity model is assumed to be Gaussian $\tilde{\varepsilon}(\boldsymbol{\rho},z)=\varepsilon_m\exp\left\{-\left[(\boldsymbol{\rho}-\boldsymbol{\rho}_c)^2+(z-z_c)^2\right]/\left(2l^2\right)\right\}$. In the simulation, we set the following parameter values: $\boldsymbol{\rho}_c=z_c=0$, $l=1\text{cm}$, $\varepsilon_m=-0.1$, $z=3\text{m}$, $z_0=-3\text{m}$; the wavelength $\lambda=2\text{mm}$. In this case, the Fresnel radius l_f at the location of inhomogeneity is 5.4 cm. Widths of the weighted functions $D=D_0$ on the receiving and transmitting planes are taken to be equal to 2 cm (dotted line), 3 cm (dashed), 10 cm (dash-and-dot line). The solid line depicts the ideal projection. The simulation results show that as the aperture increases the resolution exceeds the Fresnel resolution.

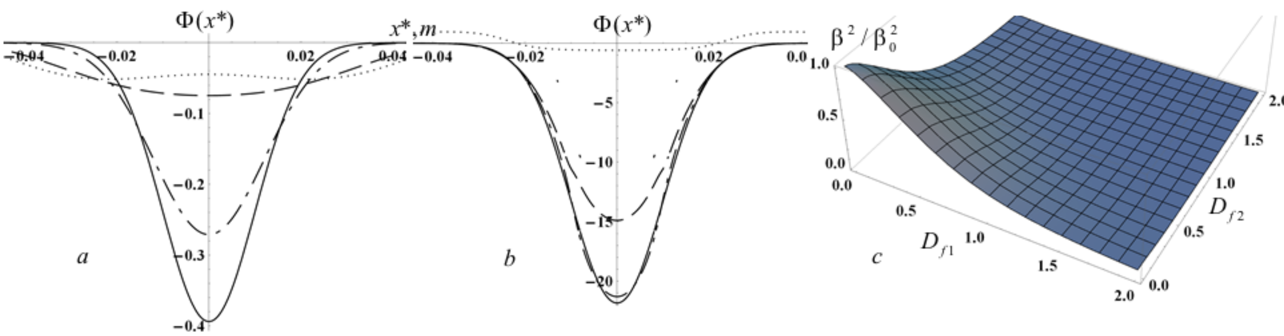

Fig. 1. The influence of antenna sizes on resolution during weak (a) and strong (b) phase disturbances and on the intensity fluctuation decrease (c).

Now let us consider the case of strong fluctuations, when $|k\Phi|\geq 2\pi$. Here the wave field behind the phase screen is taken to be the field under processing. Fig. 1b presents the results of calculations of the phase $k\Phi(x^*)$ as a function of $x^*=x_0^*$ in the cross-section $y^*=y_0^*=0$ at the same parameters as in Fig. 1a, except for antenna sizes: $D=D_0=1\text{cm}$ (dotted line), 28 cm (dashed line), 35 cm (dash-and-dot line) and the disturbance amplitude $\varepsilon_m=-0.55$ that caused a stronger phase disturbance. As is evident from the simulation results, sufficiently large antennas provide super Fresnel resolution both at small and considerable phase variations.

Since the inverse DWFT (8) reduces amplitude fluctuations, the spatial processing algorithm discussed above has been put forward as a method for reducing amplitude fluctuations [5]. To examine the efficiency of this method, we can simulate the ratio of scintillation indices β^2, β_0^2 after and before processing (8). Fig. 1c presents the dependence of ratio β^2 / β_0^2 on D_{f1}, D_{f2} for weak fluctuations, where $D_{f1} = D_0\left(z_t - z_b\right) / \left(Zl_f(z_b)\right)$, $D_2 = D\left(z_b - z_0\right) / \left(Zl_f(z_b)\right)$, z_b is the z-th coordinate of the inhomogeneity. Here the power spectrum $\Phi_\varepsilon(\kappa, 0, z') = 0.033 C_\varepsilon^2 \kappa^{-p}, p = 11/3$ is taken as the turbulent spectrum. For strong fluctuations, the phase screen approximation was selected as the field under processing in [5]. It has been found that for sufficiently large apertures the scintillation index goes into the expression derived for weak fluctuations. This implies that the processing of (8) reduces both weak and strong intensity fluctuations.

4. Conclusion

Here we shall formulate main lines of further investigations.

1. We must determine boundaries of applicability of DWFT. This requires as accurate field model as possible. In this case one can use numerical simulation or the above mentioned relation between DWFT and path integrals. Moreover, this relation may define further development of DWFT. Thus taking trajectory fluctuations in the first approximation in (2) into account, it is possible to derive (see [2]) the asymptotic expression for the scintillation index which agrees with the results obtained with the aid of the path integral method.

2. The DWFT variant discussed in this paper can be applied only in the approximation of small-angle scattering. The results should be generalised in case of the wrong condition of this approximation, for example, in back scattering and for reflection from inhomogeneous layer. One of the variants of such generalisation is the combination of DWFT and the Fock's method of the fifth parameter [2].

3. Since the Fourier transform is an ill-conditioned (unstable) problem, it is very important to evaluate noise effects on results of processing, using the algorithm discussed in this paper.

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