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## **RAPID ENVIRONMENTAL ASSESSMENT WITH AMBIENT NOISE**

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*At frequencies below 5 kHz, shallow rays from distant sources create a strong, near-symmetrical horizontal lobe in the vertical directional density function of ambient noise in shallow water. The angular half-width of the lobe is equal to the critical grazing angle,  $\alpha_c$ , of the seabed. From a two-hydrophone measurement of the vertical coherence of the noise, the critical angle may be estimated, from which the sound speed in the sediment may be determined. Once the sediment sound speed is known, the remaining geo-acoustic parameters may be obtained from the dispersion relations predicted by a recently developed theory of wave propagation in saturated granular media. The combination of the noise measurement and the new theory provides a basis for performing rapid, cost-effective geo-acoustic surveys of shallow water sediments using more or less conventional sonobuoy technology.*

### **1. INTRODUCTION**

The spatial properties of ambient noise in shallow water are influenced by the local environment, in particular, by the proximity of the sea bed. Assuming a fast bottom, in which the speed of sound is greater than that in the water column, there exists a critical grazing angle,  $\alpha_c$ , for radiation incident from above. Typically, for a medium sand sediment in which the speed of sound is in the vicinity of 1750 m/s,  $\alpha_c$  is around 30°. A water-borne ray that is incident on the bottom with a grazing angle greater than  $\alpha_c$  penetrates the interface and is effectively removed from the channel after a few bounces. A shallower ray, propagating at less than the critical grazing angle, undergoes total reflection, thus allowing it to survive multiple bounces with little attenuation. Since this argument holds regardless of the nature of the source, it follows that the field from distant noise sources consists predominantly of shallow rays, with grazing angles less than the critical. Thus, the vertical directional density function[1] of the noise in shallow water often exhibits a strong horizontal lobe with angular half-width  $\alpha_c$ .

It follows that a measurement of the angular width of the horizontal noise lobe returns the critical grazing angle,  $\alpha_c$ . Thus, the speed of sound,  $c_p$ , in the sediment may be inferred, since

$$c_p = \frac{c_w}{\cos \alpha_c} \quad , \quad (1)$$

where  $c_w$ , the speed of sound in the water column, is assumed known. The width of the noise lobe could be determined by sweeping the narrow-band beam of a vertical line array of hydrophones from the upward- to downward-looking vertical. Such an array, however, is likely to be costly and not particularly convenient for rapid-deployment operations. An attractive alternative is a pair of vertically aligned sensors, separated by about 1 m, from which the vertical coherence function of the noise is obtained as a function of frequency[2]. The coherence function is the Fourier transform of the vertical directionality of the noise and both therefore contain the same spatial information. In effect, aperture in the narrow-band vertical array is traded for broader bandwidth in the vertical pair. A crude estimate of the critical angle may be obtained from the first zero crossing of the real part of the coherence function. Naturally, a more refined estimate is available from improved signal processing in which the full coherence function is exploited.

Based on these ideas, a pair of elements mounted in a sonobuoy would appear to provide a first step in a cost-effective system for assessing all the geo-acoustic parameters of the sea bed. However, to advance from an estimate of just the sound speed in the sediment to the values of the remaining geo-acoustic parameters, further information is required. From the empirical relationships of Hamilton[3] and Richardson[4], it has been established that the wave properties and the mechanical properties of sediments are correlated. Recently, Buckingham[5] developed a grain-shearing (GS) theory of wave propagation in sediments, which expresses these correlations through dispersion relationships for the compressional wave and the shear wave. As described below, these new dispersion relationships provide the necessary link between the noise measurement of the sound speed in the sea bed and all the remaining geo-acoustic parameters of the sediment.

## 2. DISPERSION PAIRS

Marine sediments support a compressional wave and a shear wave. According to the GS theory of wave propagation in sediments, the properties of these waves are governed by a certain type of inter-granular sliding, which leads to dissipation and dispersion. The dissipation occurs because wave energy is converted into heat, whilst the dispersion arises because the grain-to-grain interactions make the material stiffer. The enhanced stiffness also accounts for the predicted existence of a shear wave.

The wave properties of the compressional wave and the shear wave are expressed through complex wave speeds, which may be split into real expressions for the two phase speeds and the two attenuations. For the compressional wave, the phase speed,  $c_p$ , is

$$c_p = \frac{c_o}{\text{Re} \left[ 1 + \frac{\gamma_p + (4/3)\gamma_s}{\rho_o c_o^2} (j\omega T)^n \right]^{-1/2}} \quad (2a)$$

and the attenuation,  $\alpha_p$ , is

$$\alpha_p = -\frac{\omega}{c_o} \operatorname{Im} \left[ 1 + \frac{\gamma_p + (4/3)\gamma_s}{\rho_o c_o^2} (j\omega T)^n \right]^{-1/2} . \quad (2b)$$

The corresponding dispersion pair,  $c_s$  and  $\alpha_s$ , for the shear wave is

$$c_s = \sqrt{\frac{\gamma_s}{\rho_o} \frac{(\omega T)^{n/2}}{\cos\left(\frac{n\pi}{4}\right)}} \quad (3a)$$

and

$$\alpha_s = \omega \sqrt{\frac{\rho_o}{\gamma_s} (\omega T)^{-n/2} \sin\left(\frac{n\pi}{4}\right)} . \quad (3b)$$

In these expressions,  $\omega$  is angular frequency,  $\rho_o$  is bulk density,  $T = 1$  s is a normalising time, and  $c_o$  is Wood's sound speed[6] for the equivalent suspension, which equals  $(\kappa_o/\rho_o)^{1/2}$ , where  $\kappa_o$  is the bulk modulus of the medium. Both  $\rho_o$  and  $\kappa_o$  are given by familiar weighted means [see Eqs. (6) and (7) below] of the corresponding values for the individual constituents of the medium and, accordingly, both vary linearly with the porosity,  $N$ . The three coefficients ( $\gamma_p$ ,  $\gamma_s$ ,  $n$ ) are peculiar to the GS theory, characterising the intergranular sliding process. The first two,  $\gamma_p$  and  $\gamma_s$ , are analogous, respectively, to the compressional and shear moduli in elasticity theory. They depend on the mean grain diameter,  $u_g$ , the depth in the sediment,  $d$ , and porosity,  $N$  as follows:

$$\gamma_p = \gamma_{p0} \left[ \frac{(1-N)u_g d}{(1-N_o)u_{g0} d_o} \right]^{1/3} \quad (4a)$$

and

$$\gamma_s = \gamma_{s0} \left[ \frac{(1-N)u_g d}{(1-N_o)u_{g0} d_o} \right]^{2/3} , \quad (4b)$$

where  $\gamma_{p0}$  and  $\gamma_{s0}$  are compressional and shear coefficients, which are independent of the macroscopic properties of the sediment. In Eqs. (4), the parameters with a subscript zero are reference values, chosen for convenience ( $N_o = 0.37$ ,  $u_{g0} = 1000 \mu\text{m}$ ,  $d_o = 0.3$  m) and do not represent additional unknowns. The remaining GS parameter,  $n$ , is a small positive constant representing the non-linear mechanism of strain hardening, an effect that is postulated to occur in the molecularly thin layer of pore fluid separating the grains[5].

The three GS parameters may be evaluated by inserting measurements of the compressional wave speed and attenuation, taken at a spot frequency, into Eqs. (2), and the shear wave speed, also at a spot frequency, into Eq. (3a). Using this procedure with Richardson's 38 kHz compressional-wave data from the SAX99 experiment in the northern Gulf of Mexico, and Richardson's 1 kHz shear-wave data from the Mediterranean, yields  $\gamma_{po} = 3.90 \times 10^8$  Pa,  $\gamma_{so} = 4.65 \times 10^7$  Pa and  $n = 0.085$ .

It should be noted that, although strain-hardening is a non-linear mechanism, it occurs in the GS theory in connection with the stress induced by individual, microscopic sliding events. A random, linear superposition of all such events yields the total stress. It follows that the dispersion relations predicted by the GS theory, Eqs. (2) and (3), are strictly linear. These expressions are also causal, satisfying the Kramers-Kronig relationships.

Another point to note is that the expressions in Eqs. (2b) and (3b) represent, respectively, the *intrinsic* attenuation of the compressional and shear wave, that is to say, the loss due to the conversion of wave energy into heat. In contrast, most measurements return the *effective* attenuation, consisting of the intrinsic attenuation plus any additional loss due, for example, to scattering from inhomogeneities such as shell fragments in the medium. It follows that Eqs. (2b) and (3b) predict the lower bounds to the measured values of the two attenuations.

### 3. GRAIN SIZE AND POROSITY

The GS dispersion relations depend explicitly on the porosity,  $N$ , and the grain size,  $u_g$ . If values for both these variables were available from independent measurements on cores, for example, it is recommended that they be used to evaluate the wave speeds and attenuations from the GS dispersion relations. If, on the other hand, the only available variable were the compressional wave speed, obtained from the vertical coherence of the ambient noise in the water column, the remaining geo-acoustic variables may be derived from the GS dispersion relations. To do this, however, it is necessary to identify a relationship between the porosity and the grain size.

Such a relationship has been developed by Buckingham[7] on the basis of an argument involving the random packing of rough spheres. The resultant expression for the porosity in terms of the grain size is

$$N = 1 - P_s \left\{ \frac{u_g + 2\Delta}{u_g + 4\Delta} \right\}^3, \quad (5)$$

where  $P_s = 0.63$  is the packing factor of a random "close" packing of uniform, smooth spheres and  $\Delta = 1 \mu\text{m}$  is a statistical measure of the departure from sphericity of the grains.

The expression in Eq. (5) is a smooth, monotonic decreasing function of the mean grain diameter,  $u_g$ , which follows the trend of the porosity versus grain size data. In the limits of large and small grain size, Eq. (5) asymptotes, respectively, to  $N_{min} = 1 - P_s = 0.37$  and  $N_{max} = 1 - (P_s/8) = 0.92$ , in good agreement with observations. At intermediate grain sizes, especially those representative of silts and clays, the data show some spread, with Eq. (5) representing reasonable average behaviour.

#### 4. GEOACOUSTIC PARAMETERS FROM AMBIENT NOISE

With the aid of Eq. (5), the GS dispersion relations may be expressed as functions of either the porosity or the grain size. This is illustrated in Fig. 1, which shows the compressional and shear wave speeds and attenuations plotted as functions of the porosity. The sound speed

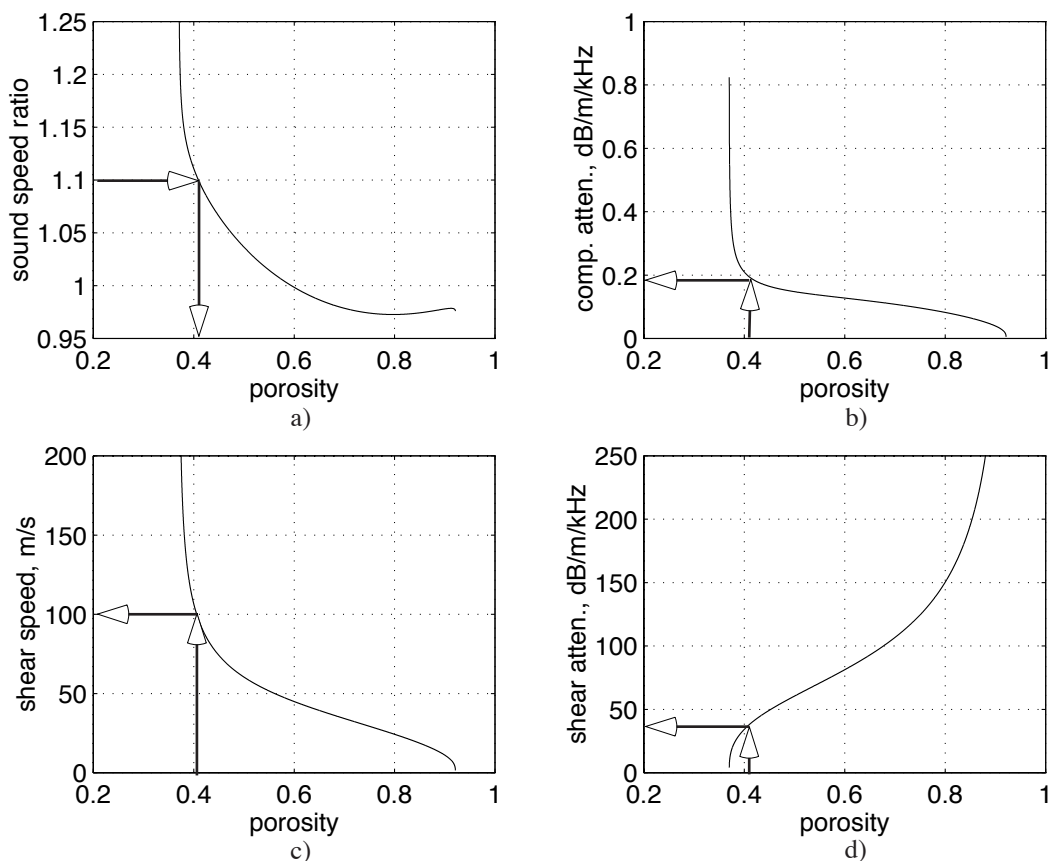


Fig. 1: Wave properties as functions of porosity, from the GS theory. a) Sound speed ratio, b) sound attenuation, c) shear speed and d) shear attenuation.

ratio in Fig. 1a is the wave speed in the sediment normalised to the sound speed at the bottom of the water column, a quantity which is relatively insensitive to changes in the temperature of the environment.

Theoretical plots like those in Fig. 1 form the basis of the technique for determining the geo-acoustic parameters of a sediment from measurements of the vertical coherence of ambient noise in the water column. Suppose that the noise measurement returned a sediment sound speed of 1650 m/s, corresponding to a sound speed ratio of 1.1 when the seawater sound speed is 1500 m/s. According to Fig. 1a, the porosity of the sediment is 0.41, from which the sound attenuation is 0.19 dB/m/kHz (Fig. 1b), the shear speed is 96.2 m/s (Fig. 1c) and the shear attenuation is 38 dB/m/kHz (Fig. 1d). In expressing the two (intrinsic) attenuations in units of dB/m/kHz it is implicit that each varies linearly with the frequency, which is consistent with the predictions of the GS theory.

With the porosity now known, the grain size may be estimated from Eq. (5) and is found to be  $u_g \approx 88 \mu\text{m}$ , which is characteristic of a very fine sand. The bulk density of the sediment,  $\rho_o$ , may be calculated from the weighted mean

$$\rho_o = N\rho_w + (1 - N)\rho_g \quad , \quad (6)$$

where  $\rho_w = 1005 \text{ kg/m}^3$  is the density of seawater and  $\rho_g = 2730 \text{ kg/m}^3$  is the density of the mineral grains. Eq. (6) yields  $\rho_o = 2023 \text{ kg/m}^3$ . Similarly, the bulk modulus of the medium,  $\kappa_o$ , may be evaluated from the weighted mean

$$\frac{1}{\kappa_o} = N\frac{1}{\kappa_w} + (1 - N)\frac{1}{\kappa_g} \quad , \quad (7)$$

where  $\kappa_w = 2.37 \times 10^9 \text{ Pa}$  is the bulk modulus of seawater and  $\kappa_g = 3.36 \times 10^{10} \text{ Pa}$  is the bulk modulus of the mineral grains. Eq. (7) yields  $\kappa_o = 5.26 \times 10^9 \text{ Pa}$ . Finally, Wood's sound speed may be deduced, since  $c_o = (\kappa_o/\rho_o)^{1/2} = 1612 \text{ m/s}$ . This is the value of the compressional wave speed in the limit of low frequency, when the two-phase medium acts as a simple suspension in which grain-to-grain interactions are negligible.

## 5. CONCLUDING REMARKS

A shallow-water technique has been discussed for estimating the geo-acoustic parameters of a marine sediment from the critical angle of the seabed. The critical angle itself is obtained from a broadband measurement of the vertical coherence of the ambient noise in the water column. From the critical angle, the sound speed in the sediment is estimated, and all the remaining geo-acoustic parameters are deduced using the dispersion relations for the compressional wave and the shear wave, as predicted from a recently developed theory of wave propagation in saturated porous media.

## 6. ACKNOWLEDGEMENTS

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