# Equilibrium strategic overbuying

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**Abstract**

We consider two firms competing both to sell their output and purchase their input from an upstream firm, to which they offer non-linear contracts. Firms may engage in strategic overbuying, purchasing more of the input when the supplier is capacity constrained than when it is not in order to exclude their competitor from the final market. Warehousing is a special case in which a downstream firm purchases more input than it uses and disposes of the rest. We show that both types of overbuying happen in equilibrium. The welfare analysis leads to ambiguous conclusions.

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## 1 Introduction

Most of modern industrial organization focuses on competition between firms to sell their products. However, firms also compete to purchase inputs and this type of competition raises specific issues, probably the most important being overbuying. Overbuying happens when a firm enjoying buyer power inflates its purchases for strategic purposes. Salop (2005) provides the following definition: "Anticompetitive 'overbuying' conduct by power buyers involves increasing the purchases of a particular input with the purpose and effect of gaining (or maintaining) either monopsony power in the input market or market power in the output market, or both". This is a broad definition that includes two types of overbuying, namely raising rivals' cost and predatory overbuying. Predatory overbuying

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consists in paying a higher price for the input in order to drive other purchasers out of the input market. It is actually an instance of predation, but while predation is usually analyzed on markets in which firms sell, it happens here on a market in which firms purchase. Raising rivals' cost overbuying consists in increasing purchases in order to raise the price competitors have to pay for their inputs and consequently weaken their position on the final market. This is the type of overbuying we are considering here. This type of overbuying is also discussed in Salop and Scheffman (1987), in which the authors base their definition of (raising rivals' cost) overbuying on the Alcoa case (United States v. Aluminum Co. of Am., 1945): Judge Learned Hand, writing the opinion for the U.S. Circuit Court of Appeal for the Second Circuit summarizes part of the plaintiff's accusations in the first trial as follows : "The plaintiff attempted to prove, and asserts that it did prove, that 'Alcoa' bought up bauxite deposits, both in Arkansas- the chief source of the mineral in the United States- and in Dutch, and British, Guiana, in excess of its needs, and under circumstances which showed that the purchases were not for the purpose of securing an adequate future supply, but only in order to seize upon any available supply and so assure its monopoly." It should be noted that plaintiffs do not simply point at a raising rivals' cost effect, but rather at the fact that there is no supply left at all for competitors. The plaintiffs' claims however were found convincing neither by the trial judge nor by the U.S. Circuit Court of Appeal. In fact, while allegations of overbuying appear in several cases, they hardly if ever prevailed.[1]

In the Wanadoo Interactive[2] case (European Commission (2003)), the European Commission indicates, as an element of context but without direct legal implications, that "the supply of ADSL modems was probably used jointly by France Télécom and Wanadoo Interactive to slow the development of competitors in the start-up phase at least in the first few months of 2001. France Télécom had authority to approve ADSL modems for use on its network. This power seems to have been used to postpone any widening of the range of potential modem suppliers, and to consolidate the shortage that obtained on the market at the beginning of 2001. On the one hand, France Télécom delayed the approval of modems manufactured by ECI [...]. On the other hand, orders for modems placed jointly by France Télécom and Wanadoo Interactive had the effect of taking up almost all of the production capacity of the only supplier authorized at the time, Alcatel, and this made it difficult for competitors to obtain supplies.[...] The "closing off" of the modems market in the first few months of 2001 had a strong inhibiting effect on the initial development of competitors. By way of example, in December 2000 Wanadoo Interactive already had at least [...]* modems in stock at France Télécom shops, while in January 2001 TOnline had succeeded in obtaining only one-tenth of that figure." While the refusal to grant approval of modems manufactured by ECI clearly smells like an abuse of a dominant position,

[1] See Salop (2005) for other cases not discussed here.

[2] Wanadoo Interactive was a subsidiary of France Télécom, the incumbent in the French telecommunication industry.

artificially reducing the supply, it is not clear if Wanadoo Interactive also inflated its purchases from Alcatel to deprive competitors from access to modems or if it acted in order to secure an “adequate future supply” (following the terms used in the Alcoa decision). It is in general a difficulty in these cases and in the analysis of overbuying to define what “adequate” means. The UK Competition Commission faced precisely this challenge in its groceries market investigation (Competition Commission (2008)). The issue then was to assert whether retailers engaged in “land banking” to limit entry on the market or land purchases were consistent with anticipated needs. The French antitrust authority had to solve a similar issue in a case opposing two retailing firms specialized in sports accessories, Decathlon and Go Sport (Conseil de la Concurrence (2004)). Go Sport was then accusing Decathlon, the dominant firm on this market, of opening new shops or extending existing shops in some areas to prevent Go Sport from getting the required authorizations to open shops in the same areas. In that case, the problem was thus not land banking but banking of legal authorizations.[3] In both cases, competition authorities did not find any evidence of such anticompetitive practices.

While the lack of success of plaintiffs in overbuying cases may stem from the fact that their claims were not founded, we believe that this mainly results from the difficulty to identify overbuying and more generally from the absence of a convincing theory of overbuying. While overbuying strategies are briefly mentioned in Salop and Scheffman (1983), it is in Salop and Scheffman (1987) that the authors provide the first detailed discussion of overbuying. However, they do so under the assumption that the “predator” faces a competitive fringe that is deprived of market power both on the final market and on the market for inputs. There is thus no real strategic interaction between the predator and its competitors. In the present paper we want to assert whether overbuying may show up when the distribution of market power between competing firms is more balanced on the final as well as on the intermediate market. We develop a model in which two firms compete both to sell their output on the final market and to purchase their input from a price-taking supplier. We identify overbuying by comparing firms’ purchases when the supplier has no capacity constraint with their purchases when the supplier faces a strict capacity constraint. Overbuying takes place when firms purchase more from a constrained supplier than from an unconstrained supplier. We show that overbuying actually happens in equilibrium and takes two different forms. Indeed, an overbuying firm may use all of the input it purchases to produce the final good, but it may also purchase units of input that it does not transform into the final good and thus remain unused. Following Salop (2005), we refer to the latter situation as “warehousing”. We finally show that overbuying as well as the existence of an upstream capacity constraint have an ambiguous impact on welfare.

The paper is organized as follows. In section 2, we present the model and discuss

[3] From 1996 to 2008 in France, the Raffarin law required that any firm wanting to open a retailing outlet of 300 square meters or more ask for an authorization by specific commissions.

related contributions from the literature. In section 3, we characterize market equilibria on the final market and present illustrative examples of overbuying with and without warehousing. Section 4 presents the resolution of the general model and discusses welfare implications of an upstream capacity constraint. Section 5 concludes.

# 2 Theoretical setting and literature review

In this section, we present our model and discuss the related literature. We highlight the theoretical issues raised by models in which firms compete to purchase inputs, we explain how we deal with these issues and compare with alternative approaches.

## 2.1 The game

Consider an industry composed of one upstream firm, $U$, and two downstream firms: an incumbent firm $I$ and an entrant $E$. Both downstream firms produce a homogeneous final good. Total demand is linear, and the inverse demand function is given by $P = 1 - X$, where $X$ is the total output offered on the final market. Downstream firms compete à la Cournot on the final market.

The production process is as follows. The upstream firm $U$ produces an input, which is then transformed by downstream firms into the output on a one-to-one basis. Therefore, if downstream firm $i$ ($i \in \{I, E\}$) decides to put $x_i$ on the final market, it has to purchase an amount of input $q_i \geq x_i$. Beyond the cost of purchasing the input, which derives from a mechanism described later, downstream firms face no transformation cost. However, whenever a firm sells an output $x_i$ strictly lower than the input $q_i$ it purchased, it cannot recover the cost corresponding to the $q_i - x_i$ units of unused input it owns.

The upstream firm $U$ can only produce input up to its capacity constraint, denoted by $\overline{Q}$. It produces at marginal cost $c \in [0, 1]$ up to $\overline{Q}$, and faces a marginal cost equal to $+\infty$ above $\overline{Q}$. Importantly, we do not put any restriction on the value of $\overline{Q}$. In particular, it can be higher than the total output that the two downstream firms would produce if there was no capacity constraint. For $\overline{Q} = +\infty$, the input supplier is not constrained.

Prior to competition on the final market, downstream firms compete to buy input from the upstream firm. As opposed to the downstream competition stage, where firms play simultaneously, we assume that firms are asymmetric as regards the input purchase phase. More precisely, downstream firms make take-it-or-leave-it offers to the upstream firm sequentially: $I$ first makes an offer, that is then either accepted or refused by $U$; then $E$ makes an offer, that is again either accepted or refused by $U$. This modelling choice is dictated by problems of non-existence of an equilibrium when offers are simultaneous.[4]

[4] The supplier has to reject at least one of the contracts when the total quantity requirements exceed its production capacity. So, a retailer may undercut its rival in the sense of making an offer to the supplier such that the supplier will accept it and stop supplying the rival. This creates deviations incentives that

An offer of downstream firm $i$ is composed of the quantity of input it wants to buy, $q_i$, and a lump-sum payment to $U$, $t_i$, in exchange for the supply of $q_i$. Obviously, any offer by $I$ such that $q_I > \overline{Q}$ will systematically be refused, and if $I$'s offer has been accepted, any offer by $E$ such that $q_E > \overline{Q} - q_I$ will be refused too.

The game is therefore composed of three stages, that we summarize here:

1. $I$ offers to $U$ a contract $(q_I, t_I)$. If U accepts the contract, it delivers the quantity $q_I$ to $I$ and receives from $I$ the transfer $t_I$.

2. After observing actions in stage 1, $E$ offers $(q_E, t_E)$ to $U$. If $U$ accepts the contract, it delivers the quantity $q_E$ to $E$ and receives from $E$ the transfer $t_E$.

3. Both firms know what happened in stages 1 and 2. They compete à la Cournot on the final market. We denote respectively by $x_I$ and $x_E$ the incumbent's and entrant's outputs on the final market.

It is worth noting here that in equilibrium, firm $i$ will always offer a transfer $t_i = cq_i$. Indeed, the upstream firm's outside option in its bargaining with $i$ is always 0 (net of the profit it may have already earned in a previous stage): when $E$ makes an offer to $U$, it knows that $U$ awaits no other offer, and would thus earn no additional profit if it were to refuse its offer, regardless of what $I$ offered $U$ in the first stage. Then, $E$ offers the lowest possible transfer such that $U$ earns a non-negative profit from its sales to $E$, that is $t_E = cq_E$. Consequently, when $I$ makes an offer to $U$, it also knows that $U$ will earn no additional profit in the next stage, regardless of its own offer. $I$ thus offers $t_I = cq_I$, that again leaves $U$ with no profit. In the next sections, we will thus take for granted that downstream firm $i$'s offer is always of the form $(q_i, cq_i)$, and will thus only have to determine the equilibrium value of $q_i$.

As a useful reference for what follows, consider the following alternative two-stage game. In stage 1, the incumbent purchases $q_I$ from the upstream firm (which has no capacity constraint) and puts $x_I$ on the market. In stage 2, the entrant purchases $q_E$ from the upstream firm and puts $x_E$ on the market. This game is exactly identical to a standard Stackelberg duopoly game in which both firms have a marginal cost $c$. In equilibrium, the entrant purchases and puts on the market $x_E^S(q_I) = q_E^S(q_I) = \frac{1-q_I-c}{2}$ in stage 2. In stage 1, the incumbent purchases and puts on the market $x_I^S = q_I^S = \frac{1-c}{2}$. Finally, $q_E^S(q_I^S) = \frac{1-c}{4}$.

## 2.2 Related literature

Since Salop and Scheffman's seminal work, several contributions have dealt with the issue of overbuying. We discuss these contributions and their relations to our model in this section. A key issue is the way competition for purchases is modelled. Stahl II (1988)

lead to the non-existence of an equilibrium in many situations if offers are simultaneous.

presents an interesting model in which merchants compete first to purchase inputs and then to sell outputs in a two-stage game. On the market for inputs, merchants put bids and the merchant with the highest bid gets all the supply corresponding to this price. This leads to winner-take-all competition for inputs. Of course, the author has to define a tie-breaking rule determining the distribution of the input between merchants when the highest bid is offered simultaneously by several of them. This turns out to have significant consequences on the equilibrium. The author's objective is not to analyze overbuying but rather to assert whether the introduction of competition for inputs may lead to the emergence of a walrasian outcome in a Bertrand setting with capacity constraints. Winner-take-all competition for inputs may be a convenient assumption for this purpose, but we do not believe that it is a very convincing assumption in an analysis of strategic overbuying. Actually, it *assumes* overbuying since a merchant either gets nothing or the total supply of inputs. The coexistence of several merchants in equilibrium is permitted only by some (exogenously imposed) tie-breaking rules. In an analysis of strategic overbuying, we need more flexibility in a firm's choice of the quantity of input it is purchasing.

One way to achieve this is to assume that firms choose the quantity of input they purchase and the price is then determined by the market clearing condition of the input market. Along this line, Gabszewicz and Zanaj (2008) show that an incumbent can deter entry through strategic overbuying in a model in which the entrant and the incumbent are price-takers on the intermediate market and compete for inputs by addressing their demand to an upstream industry.[5] For this to work, a market clearing price must exist whatever the demand of downstream firms is. One thus needs to assume that the (competitive) industry producing the input is willing to produce any quantity of the product as soon as the price is sufficiently large. Conversely, if there is an upper bound to the input supply, $\overline{Q}$, there is a real difficulty to determine the distribution of input when the total demand from downstream firms exceeds $\overline{Q}$. Since Gabszewicz and Zanaj (2008) assume a finite inelastic supply, they are in the latter situation with the consequence that the model is not a properly defined game, as profits cannot be calculated when total demand from downstream firms exceeds $\overline{Q}$.[6]

[5]The authors further assume price rigidity when the incumbent deters entry and price adjustment when the incumbent accommodates. They note that the assumption of price rigidity in the deterrence case is crucial for deterrence to happen in equilibrium. In the present paper, we have no assumption of price rigidity.

[6]The same problem arises in Salinger (1988). The model is a two-stage game. Upstream firms make quantity offers in stage 1. In stage 2 downstream firms determine their demand for inputs. In this stage, the input supply is fixed. Implictly, it is assumed that an auctioneer chooses a price such that the total demand of downstream firms is equal to the supply. However, this does not solve the problem because if a downstream firm deviates from such a situation by increasing its demand, the input market is in a disequilibrium and the model does not specify what happens in this case. Allain and Souam (2006) offer one solution to this problem buy introducing a "market maker" who buys the whole supply from the upstream firms and commits to supply the whole demand of the downstream firms at a price $w$ that it sets. If the supply of upstream firms is lower than the demand of downstream firms, then the market

A possible solution to the above mentioned problem is to assume that the input supply curve is not bounded (as in Riordan (1998) and Christin (2011)). Here, we take a different approach. We assume that the input is produced by a supplier that is able to produce at constant marginal cost up to a finite $\overline{Q}$, but cannot produce more than $\overline{Q}$. However, the downstream firms do not announce a quantity of input they would like to purchase at any price. They offer a contract to the supplier specifying the quantity of the good they want to get and the payment they are ready to make to the supplier as a counterpart. If the supplier is not able to produce the required quantity, it rejects the contract. If it is able to produce this quantity, it may still reject the contract if the payment is too low. Allowing for these more sophisticated contracts, that are however standard in the vertical relations literature (e.g. Hart and Tirole (1990), Rey and Tirole (2007)), solves the difficulties raised by the existence of a capacity constraint in input production.[7]

A third solution is adopted in Eső, Nocke and White (2010). This article analyzes the distribution of an exogenous total capacity between $n$ firms which in the following stage compete à la Cournot. The modeling choice is to assume that the capacity is efficiently allocated between firms through some mechanism such as an efficient auction. An efficient allocation is defined as an allocation that maximizes industry profits. Consequently, if firms have linear production costs, all the capacity is allocated to just one firm. Then, this firm is in a monopoly position, which clearly allows the maximization of industry profits. Actually, the authors want to analyze the allocation of capacity between firms and in most of the paper assume that production costs are convex, which leads to a much wider variety of capacity allocations. This approach differs from ours in two ways. Obviously, the mechanism of capacity allocation is different. We do not assume an efficient allocation and in general industry profits are not maximized in equilibrium. More importantly, in Eső, Nocke and White (2010), the input (the "capacity" in their model) is already produced and the discussion bears only on its allocation between firms. In our model, the production of the input is endogenous. The upstream firm produces only the quantities required by downstream firms offering acceptable contracts. This is why the input production cost plays a central role in our analysis, while it is absent from the analysis in Eső, Nocke and White (2010). To illustrate the difference, consider the result in Eső, Nocke and White (2010) for linear costs and a very large production capacity. The capacity has to be entirely distributed and efficiency requires that it is allocated to just one firm. This firm will not use all of this capacity. In this sense, there is overbuying in their model, but it is costless. As we will see, in the same conditions, we have in general different results because in our model overbuying is costly. When a firm purchases units of the input to

maker must purchase the additional necessary quantity on some external market to which firms do not have access. This prevents demand from ever being larger than total supply.

[7]Avenel (2010) also considers a finite upstream production capacity and quantity-transfer contracts, but assumes that the upstream firm makes offers to downstream firms. The issue is whether the upstream firm can or cannot extend its monopoly power to the final market.

divert them from its competitors, it has to pay at least the production cost of these units. So, if the upstream capacity is very large and the input marginal production cost is strictly positive, overbuying becomes prohibitively costly and does not happen in equilibrium.[8]

# 3 Downstream competition & illustrative examples

Before developing the complete resolution of the model defined above, we consider here the equilibrium that emerges for specific values of the two parameters, the marginal cost $c$ and the production capacity $\overline{Q}$. These equilibria illustrate two typical outcomes of the general model: overbuying without warehousing and overbuying with warehousing. As a preliminary to the discussion of examples, we need to solve the third stage of the game. Since this stage is influenced neither by $c$ nor by $\overline{Q}$, the solution presented here is general and will be used for the resolution of the general model.

## 3.1 Downstream competition equilibrium

In stage 3, firm $i$ ($i \in \{I, E\}$) owns $q_i \geq 0$ units of output. If downstream firms faced no capacity constraint, firm $i$ would set $x_i$ so as to maximize its profit $\pi_i = P(x_I + x_E)x_i$ and its unconstrained best reply to its rival's output $x_j$ ($j \in \{I, E\}$, $j \neq i$) would be $x_i^{BR}(x_j) = \max\{0, \frac{1-x_j}{2}\}$. Then the constrained best reply of $i$ is $\min\{x_i^{BR}(x_j), q_i\}$. The resulting equilibrium is given in the following lemma.

**Lemma 1.** *The equilibrium outputs of the incumbent and the entrant in stage 3, respectively $x_I^*(q_I, q_E)$ and $x_E^*(q_I, q_E)$, are as follows:*

- *If $q_I \geq \frac{1}{3}$ and $q_E \geq \frac{1}{3}$, then downstream firms play the unconstrained Cournot solution, namely $x_I^*(q_I, q_E) = x_E^*(q_I, q_E) = \frac{1}{3}$.*
- *If there exists $i \in \{I, E\}$ such that $q_i < \frac{1}{3}$, then firm $i$ always plays $x_i^*(q_I, q_E) = q_i$, whereas $j \neq i$ plays its unconstrained best reply $x_j^*(q_I, q_E) = \frac{1-q_i}{2}$ as long as $q_i > 1 - 2q_j$ and plays $x_j^*(q_I, q_E) = q_j$ otherwise.*

Note that at this stage, depending on the values of $q_I$ and $q_E$, it may well be that the final profit of a downstream firm is negative. However, as all costs are sunk, each downstream firm is still better off following the previously described equilibrium strategy than leaving the market.

[8] Exclusive purchasing agreements avoid this cost and allow firms to reduce or suppress their competitors' access to the input without supporting the cost of actually producing the input. Indeed, a firm purchases exclusivity rights rather than the input. However, such contracts would quite certainly be challenged by antitrust authorities. Our objective in this paper is precisely to determine to what extent firms still have the possibility to limit their competitors' access to the input without signing exclusive purchasing agreements.

## 3.2 Overbuying without warehousing

Assume that $c = 1/2$. Given the demand function, it is a relatively high marginal cost. Let us first consider that there is no constraint on the upstream production capacity: $\overline{Q} = +\infty$. In stage 2, the entrant anticipates the equilibrium in the next stage. Consider first $q_I \geq \frac{1}{2}$. The entrant anticipates that whatever the quantity of input it purchases in stage 2, it will not make positive profits in stage 3. Indeed, for $0 \leq q_E < 1/3$, $\pi_E = \left(1 - q_E - \frac{1-q_E}{2}\right) q_E - \frac{1}{2} q_E = -\frac{q_E^2}{2}$. The final good is sold at a price lower than the input's marginal cost of production. This happens because in stage 2 the cost of purchasing the input is a sunk cost, so the perceived marginal cost of the final good in stage 3 is zero. Alternatively, for $q_E \geq 1/3$, $\pi_E = \frac{1}{9} - \frac{1}{2} q_E$. Again, in stage 3, firms play the Cournot duopoly equilibrium for zero marginal cost, that is, $x_I = x_E = 1/3$ and the product is sold below the input's marginal cost of production. The best strategy for the entrant in stage 2, when $q_I \geq \frac{1}{2}$, is thus $q_E = 0$. Consider now $q_I \in [\frac{1}{2}, \frac{1}{3}]$. The entrant still makes negative profits for $q_E \geq 1 - 2q_I$, but now it is possible to choose $q_E < 1 - 2q_I$. Then, both the incumbent's and the entrant's constraints on the final market are binding, so that $\pi_E = (1 - q_E - q_I)\, q_E - \frac{1}{2} q_E = \left(\frac{1}{2} - q_E - q_I\right) q_E$. If $q_E$ is sufficiently small, the final good is sold at a price above the input's marginal cost and the entrant makes profits. The optimal purchases are $q_E = \frac{1}{4} - \frac{1}{2} q_I$, leading to $\pi_E = \left(\frac{1}{4} - \frac{1}{2} q_I\right)^2 > 0$. Finally, consider $q_I < \frac{1}{3}$. The entrant anticipates that in stage 3 its best reply to $q_I$ will be $\frac{1-q_I}{2}$. However, this best reply ignores the input's marginal cost and leads to negative profits. Thus, the entrant will choose $q_E < \frac{1-q_I}{2}$, which leads to $\pi_E = \left(\frac{1}{2} - q_E - q_I\right) q_E$. Then, it is optimal for the entrant to purchase $q_E = \frac{1}{4} - \frac{1}{2} q_I$ in stage 2. The entrant makes positive profits in stage 3. To sum up, the equilibrium in stage 2 is as follows: for $q_I \geq 1/2$, $q_E^* = 0$, while for $q_I < 1/2$, $q_E^* = \frac{1}{4} - \frac{1}{2} q_I$.

In stage 1, the incumbent makes its purchase decision. Of course, it anticipates the entrant's strategy in stage 2 and the market outcome in stage 3. Choosing $q_I \geq 1/2$ keeps the entrant out of the market. The incumbent is thus in a monopoly position in stage 3. Based on the perceived cost of the final good, it plays $x_I = 1/2$. It means that the final good is sold at a price equal to the input's marginal cost. Thus profits are zero for $q_I = 1/2$ and strictly negative for $q_I > 1/2$. Indeed, $q_I - 1/2$ are purchased in stage 1 at a cost of $c\,(q_I - 1/2) > 0$ and remain in the incumbents inventory. Clearly, the incumbent will stick to values of $q_I$ below $1/2$. Then, $\pi_I = \left(1 - q_I - \left(\frac{1}{4} - \frac{1}{2} q_I\right)\right) q_I - \frac{1}{2} q_I = \frac{1}{2} q_I \left(\frac{1}{2} - q_I\right)$ and thus the incumbent purchases $q_I^* = \frac{1}{4}$. This results in the following market outcome and profits: $q_I^* = 1/4; q_E^* = 1/8; \pi_I^* = 1/32; \pi_E^* = 1/64$.

Now assume that the upstream production capacity is equal to $\overline{Q} = 3/8$. Stage 3 is identical to the previous case. However, in stage 2, the upstream capacity constraint makes a difference because the entrant may not be able to purchase the quantity of input it would like. Since $q_I \leq \overline{Q} < 1/2$, the entrant's optimal purchases are given by $q_E^* = \min\{\frac{1}{4} - \frac{1}{2} q_I, \overline{Q} - q_I\}$. For $\overline{Q} = 3/8$, this means that $q_E^* = \frac{1}{4} - \frac{1}{2} q_I$ for $q_I \leq \frac{1}{4}$ and $q_E^* = \overline{Q} - q_I$

for $q_I \geq \frac{1}{4}$. In stage 1, the incumbent thus bases its decision on $\pi_I = \frac{1}{2}q_I\left(\frac{1}{2} - q_I\right)$ for $q_I \leq \frac{1}{4}$ and $\pi_I = \left(1 - q_I - \left(\overline{Q} - q_I\right) - \frac{1}{2}\right) q_I = \frac{1}{8}q_I$ for $q_I \geq \frac{1}{4}$. This is an increasing function of $q_I$ and the incumbent thus purchases $q_I^* = \overline{Q}$, leaving no remaining capacity for the upstream firm to supply the entrant with input. Market outcomes and profits are as follows: $q_I^* = 3/8; q_E^* = 0; \pi_I^* = 3/64; \pi_E^* = 0$. It is interesting to note that for $q_I \geq \frac{1}{4}$, the incumbent does not have to trade-off between a price effect and a quantity effect because there is no price effect. Indeed, the entrant purchases every unit of the input that can be produced and that the incumbent did not purchase in stage 1, so that the total output of the final market is the same whatever the value of $q_I$. Obviously, the best strategy for the incumbent is then to purchase $\overline{Q}$.

If we now compare the equilibrium without a constraint and the equilibrium with $\overline{Q} = 3/8$, we see that the incumbent actually purchases more in the presence of the constraint in order to drive the entrant out of the market. Actually, for the values of parameters considered here, this leaves the total output on the final market, as well as the sum of firms' profits, unchanged. However, the distribution of output and profits between the incumbent and the entrant is dramatically modified. This modification is induced by the increase in the incumbent's purchases. Because this increase is driven by these strategic considerations, it is a typical instance of strategic buying.

### 3.3 Overbuying with warehousing

In the previous example, there is no warehousing as defined in Salop (2005) in equilibrium. Warehousing happens when a firm purchases more of the input than it actually uses. So, part of the input remains in its inventories. This may be costly. The firm may also prefer to get rid of this product by destroying it or selling it at scrap value. We ignore these issues here and assume that keeping the product in inventory is costless. However, purchasing the input is in general costly. This is why in the previous example, warehousing does not happen in equilibrium. The marginal cost is too high and $\overline{Q}$ is not large enough for a firm to engage in warehousing. Conversely, let us now look at a situation where the marginal cost is low and $\overline{Q}$ is large. Assume $c = 0$ and, provisionally, that the production capacity $\overline{Q}$ is infinite. In stage 2, the entrant's optimal purchases are easy to determine. Indeed, for $q_I \geq \frac{1}{3}$, the entrant purchases $q_E^* = \frac{1}{3}$.[9] This leads to $x_I^* = x_E^* = \frac{1}{3}$ and $\pi_E = \frac{1}{9}$. Choosing $q_E < \frac{1}{3}$ would induce a larger $x_I$, a lower $x_E$ and finally lower profits for the entrant. For $q_I < \frac{1}{3}$, the entrant purchases its best reply to $q_I$, namely $q_E^* = \frac{1-q_I}{2}$. This clearly leads to positive profits. In fact, this example is simpler than the previous one because we do not have to compare the final price with the input's production cost. If the final price is positive, it is larger than the input cost and profits are positive. Moving back to stage 1, it is clear that the incumbent purchases $q_I^* = \frac{1}{3}$. In stage 3, $x_I^* = x_E^* = \frac{1}{3}$. The market outcome is exactly the outcome we would get with a duopoly of vertically integrated firms

[9] In fact, since $c = 0$, any value of $q_E^* \geq \frac{1}{3}$ leads to the same profit.

producing their own input at zero marginal cost.

Now assume that the production capacity is finite and larger than $\frac{1}{2}$, the monopoly outcome on the final market. It is straightforward that the incumbent's optimal strategy is to purchase $\overline{Q}$ in stage 1. This is costless and keeps the entrant out of the final market. In stage 3, the incumbent is in a monopoly position and puts $\frac{1}{2}$ on the final market. Because there is a constraint on the upstream production capacity, even if the capacity is very large, the incumbent is able to monopolize the final market through strategic buying. The difference with the previous example is that here part of the input ($\overline{Q} - \frac{1}{2}$ units) stays in the incumbent's warehouses. So, we have an instance of overbuying with warehousing.

### 3.4 Concluding remarks

While we hope these two examples are helpful to get the intuition of what happens in our model, they are clearly too specific to draw general conclusions. In general, the input's marginal production cost is positive, so that warehousing is costly. It does not necessarily imply that positive marginal costs are incompatible with warehousing, but it raises the question of the level of marginal cost compatible with equilibrium warehousing behavior. Also, when $\overline{Q}$ is larger than the total output on the final market in the absence of a capacity constraint, while lower than $\frac{1}{2}$, purchasing $\overline{Q}$ in stage 1 to keep the entrant out of the market implies for the incumbent producing more of the final good than both firms together in the absence of a constraint. This is clearly a drawback of strategic buying that reduces the profitability of this behavior. In what follows, we solve the game played by the incumbent and the entrant in the general case.

## 4 The general case

In this section, we solve stages 1 and 2 of the game in the general case, first when the upstream firm does not face any capacity constraint and then when it faces a capacity constraint given by $\overline{Q}$. In both cases, we first determine the purchase decision of the entrant given the incumbent's purchase $q_I$, and then determine the purchase decision of the incumbent. Comparing purchases with and without a capacity constraint allows us to characterize equilibrium overbuying situations. The section ends with a discussion of welfare implications of the existence of an upstream capacity constraint.

### 4.1 Input purchases in the absence of an upstream capacity constraint

**The entrant's purchase decision** Taking as given the incumbent's input purchases $q_I$, the entrant sets $q_E$ so as to maximize its profit, taking into account the cost of purchasing the input. The entrant's equilibrium purchase decision $q_E^*$ thus solves the program:

$$\max_{q_E} P(x_I^*(q_I, q_E) + x_E^*(q_I, q_E))x_E^*(q_I, q_E) - cq_E. \tag{1}$$

Lemma 1 is very helpful to understand the various strategies that the entrant can adopt in stage 2. First, if the incumbent purchased less than $\frac{1}{3}$, the entrant knows that the incumbent will actually put $q_I$ on the market whatever its decision in stage 2 may be. So, the entrant plays its best reply to $q_I$. Note that there is no incentive here for the entrant to engage in a warehousing strategy. In fact, the entrant is in the situation of a follower in a standard Stackelberg duopoly game.

For $q_I \geq \frac{1}{3}$, the entrant has more strategic options because it can influence the equilibrium in stage 3. In this sense, in the subgames starting at stage 2, the entrant is a leader. To analyze its strategy, we can refer to the taxonomy introduced by Fudenberg and Tirole (1984). In a "puppy-dog" strategy, the entrant purchases a small quantity of product.[10] Then, the incumbent would like to have a large output. Because of that, it will be capacity constrained in stage 3 and play $x_I = q_I$. Of course, when adopting the puppy-dog strategy, the entrant would like to play its best reply to $q_I$, as the follower in the standard Stackelberg game, but that may be too much, since a puppy-dog cannot purchase more than $1-2q_I$ in stage 2. The best reply to $q_I$ may also be zero, which means that a puppy-dog simply stays out of the market. Alternatively, the entrant can adopt a "top dog" strategy and purchase a large quantity of the product.[11] Then, the incumbent will not be capacity constrained and thus will play its best reply to $q_E$ in stage 3. Of course, in a top dog strategy, there is no point purchasing strictly more than $\frac{1}{3}$, because this would increase the purchasing cost and have no impact on stage 3: the two firms will play $\frac{1}{3}$. A top dog strategy forces the incumbent to keep in its inventories part of the product purchased in stage 1.

The entrant's best strategy depends on $c$ and $q_I$. Essentially, the larger $c$, the larger the cost of a top dog strategy. The nature of the top dog strategy also depends on $c$. For $c$ larger than $\frac{1}{6}$, the entrant will renounce to implement the $\left(\frac{1}{3}, \frac{1}{3}\right)$ equilibrium in stage 3 and prefer to limit its purchases to $\frac{1-2c}{2} < \frac{1}{3}$. As regards $q_I$, the larger $q_I$, the smaller the standard Stackelberg best reply to $q_I$ and the price at which this best reply is sold. This reduces the profits of a puppy-dog and raises the incentives to switch to a top dog strategy. Lemma 2 provides the threshold values between the puppy-dog and the top dog strategies.

**Lemma 2.** *The optimal purchase strategy of the entrant depends on $c$ and $q_I$ as follpws:*

- *If $c < \frac{1}{6}$, then $q_E^*(q_I) = \frac{1-q_I-c}{2}$ for $q_I \leq 1 - c - \frac{2\sqrt{1-3c}}{3}$ and $q_E^*(q_I) = \frac{1}{3}$ otherwise;*
- *If $c \in [\frac{1}{6}, \frac{1}{2}]$, then $q_E^*(q_I) = \frac{1-q_I-c}{2}$ for $q_I \leq 1-c-\frac{1-2c}{\sqrt{2}}$ and $q_E^*(q_I) = \frac{1-2c}{2}$ otherwise;*
- *If $c \in [\frac{1}{2}, 1]$, then $q_E^*(q_I) = \frac{1-q_I-c}{2}$ for $q_I \leq 1 - c$ and $q_E^*(q_I) = 0$ otherwise.*

[10] More precisely, a puppy-dog purchases less than $1-2q_I$. Consequently, a puppy-dog strategy is possible only when the incumbent purchases less than $\frac{1}{2}$ in stage 1.

[11] Here, a large quantity is a quantity above $1 - 2q_I$.

*Proof.* See Appendix A.1. □

It should be noted that the capacity choice of $E$ is not continuous: switching from the puppy-dog strategy to the top dog strategy induces a discontinuous increase of the quantity purchased by $E$. This jump comes from the fact that the entrant's profit function has two local maxima, one corresponding to the optimal puppy-dog strategy and the other to the optimal top dog strategy. Unless $c = \frac{1}{2}$ and $q_I = \frac{1}{2}$, the top dog purchases strictly more than the puppy-dog.[12] On the bold curve represented in figure 1, $E$ is indifferent between these two strategies. We assume that, on the curve, $E$ plays the puppy-dog strategy as it does below the curve. When crossing the curve from below, a discontinuous increase in $E$'s purchase takes places and induces a parallel discontinuous decrease in the quantity sold by $I$ in the next stage.

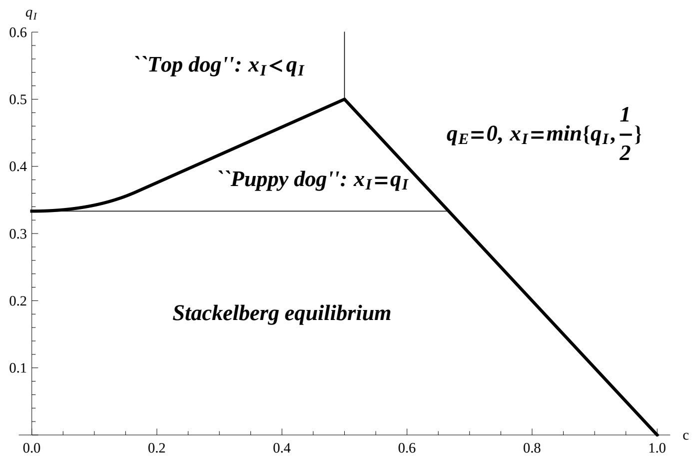


Figure 1: Purchase strategy of the entrant, and effect of this strategy on the output competition stage, depending on the cost $c$ and the quantity purchased by the incumbent $q_I$.

**The incumbent's purchase decision** In stage 1, anticipating the entrant's decision in stage 2 and the equilibrium of stage 3, the incumbent sets $q_I$ to the profit maximizing value $q_I^*$, thus solving the program:

$$\max_{q_I} P(x_I^*(q_I, q_E^*(q_I)) + x_E^*(q_I, q_E^*(q_I)))x_I^*(q_I, q_E^*(q_I)) - cq_I.$$

The following proposition presents the equilibrium of the game in the absence of an upstream capacity constraint.

[12]For $c = \frac{1}{2}$ and $q_I = \frac{1}{2}$, both the puppy-dog and the top dog purchase $1 - 2q_I = 0$.

**Proposition 1.** *When there is no upstream capacity constraint, the quantity of input purchased by each firm is as follows:*

- *If* $c < \frac{1}{6}$*, then the incumbent purchases* $q_I^* = 1 - c - \frac{2\sqrt{1-3c}}{3}$ *and the entrant* $q_E^* = \frac{\sqrt{1-3c}}{3}$*;*
- *If* $c \in [\frac{1}{6}, \frac{\sqrt{2}-1}{2\sqrt{2}-1}]$*, then the incumbent purchases* $q_I^* = 1 - c - \frac{1-2c}{\sqrt{2}}$ *and the entrant* $q_E^* = \frac{1-2c}{2\sqrt{2}}$*;*
- *If* $c \in [\frac{\sqrt{2}-1}{2\sqrt{2}-1}, 1]$*, then the incumbent purchases* $q_I^* = \frac{1-c}{2}$ *and the entrant* $q_E^* = \frac{1-c}{4}$*.*

*In stage 3, both firms sell exactly the amount they purchase from the upstream firm:* $x_i^* = q_i^*$ *for* $i \in \{I, E\}$*.*

*Proof.* See Appendix A.2. □

The incumbent would like to play the Stackelberg equilibrium, in which $E$ would buy and sell $q_E^S(q_I) = \frac{1-q_I-c}{2}$ in stage 2 and anticipating this, $I$ would buy and sell $q_I = \frac{1-c}{2}$. However, the firms are not playing the standard Stackelberg game. As Lemma 2 shows, $E$ may have an incentive not to play a puppy-dog strategy and thus buy the Stackelberg follower quantity $q_E^S(q_I)$ in stage 2, but rather to play a top dog strategy that will leave the incumbent with useless units of the intermediate good. This actually happens whenever $c < \left(\sqrt{2}-1\right) / \left(2\sqrt{2}-1\right)$. For these values of $c$, $\frac{1-c}{2} > 1 - c - \frac{1-2c}{\sqrt{2}}$, so that, from Lemma 2, if $I$ purchases $\frac{1-c}{2}$, $E$ purchases $\min\left\{\frac{1}{3}, \frac{1-2c}{2}\right\}$, and $I$ cannot sell all of its purchases in the next stage.

It turns out that the incumbent always prefers to prevent triggering a top dog strategy and, when necessary to achieve this goal, reduces its purchases in stage 1. For $c > \left(\sqrt{2}-1\right) / \left(2\sqrt{2}-1\right)$, $I$ purchases $\frac{1-c}{2}$ because it anticipates that $E$ will then buy $\frac{1-c}{4}$ and that both firms will sell all their capacity on the final market. Conversely, for $c < \left(\sqrt{2}-1\right) / \left(2\sqrt{2}-1\right)$, the incumbent purchases the highest possible quantity so that the entrant plays a puppy-dog strategy and both firms sell their whole capacity in stage 3. This has two consequences. First, when there is no upstream capacity constraint, it is never optimal for any firm to buy more on the upstream market than it sells on the final market. Second, the incumbent's purchases are increasing in $c$ for $c < \left(\sqrt{2}-1\right) / \left(2\sqrt{2}-1\right)$ and decreasing only for $c > \left(\sqrt{2}-1\right) / \left(2\sqrt{2}-1\right)$. When $c$ is very low, the top dog strategy is very attractive for the entrant, so the incumbent has to reduce its purchases a lot to prevent the entrant from playing this strategy. For $c = 0$, the incumbent purchases $\frac{1}{3}$, while in the standard Stackelberg game it would purchase $\frac{1}{2}$. As $c$ increases, the top dog strategy becomes more costly and the entrant's incentives to play this strategy decrease. As a consequence, the incumbent can increase its purchases while still inducing the puppy-dog strategy. As can be seen in figure 2, the incumbent's purchases increase until they reach the purchases the incumbent would make in a standard Stackelberg game, which happens for $c = \left(\sqrt{2}-1\right) / \left(2\sqrt{2}-1\right)$. Further increases in $c$ result in a reduction of the incumbent's purchases, as in the standard Stackelberg game. The entrant's purchases are always decreasing in $c$. For $c < \left(\sqrt{2}-1\right) / \left(2\sqrt{2}-1\right)$, this decrease is induced by two effects. Indeed, $q_E^* = \frac{1}{2}\left(1 - q_I^*(c) - c\right)$ and $q_I^*(c)$ is increasing in $c$ for these low values of $c$. For $c < \left(\sqrt{2}-1\right) / \left(2\sqrt{2}-1\right)$, $q_I^*(c)$ is decreasing in $c$, but the direct cost effect is stronger and, as in the standard Stackelberg game, $q_E^*$ is decreasing. Finally, the total output on the final market, $q_I^* + q_E^*$ is always decreasing in $c$, although at a slower rate than in the standard Stackelberg game for low values of $c$.

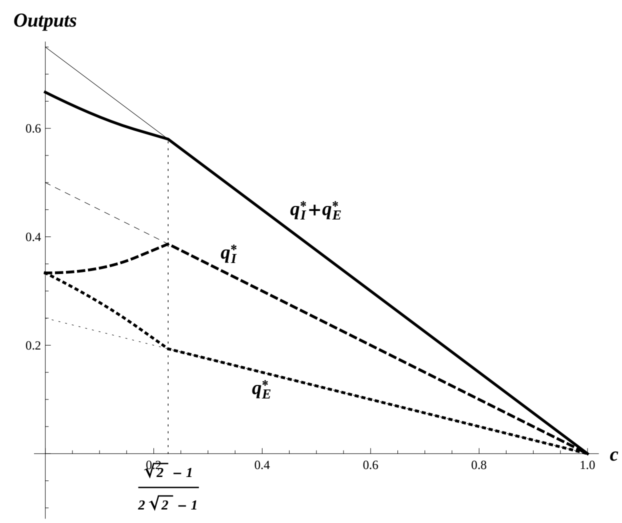


Figure 2: Quantity purchased and sold in equilibrium (in bold) by firm $I$ (dashed), firm $E$ (dotted) and by both downstream firms (plain). In thin line, we give their values in the equilibrium of the standard Stackelberg game. For $c > \frac{\sqrt{2}-1}{2\sqrt{2}-1}$, the two equilibria are identical.

## 4.2 Input purchases with an upstream capacity constraint

We now consider the case in which the upstream firm has a capacity constraint, namely $\overline{Q} < +\infty$.

**The entrant's purchase decision** As in the previous case, the entrant sets its demand for capacity taking $q_I$ as given, and therefore solves the program (1), subject to $q_E \leq \overline{Q} - q_I$. The following Lemma presents the solution to this program, denoted $\overline{q}_E^*$, while figure 3 illustrates this solution for $c = 0.2$ and $\overline{Q} \in [0, 0.8]$. The proof in Appendix provides a more detailed presentation of the entrant's strategy.

**Lemma 3.** *In the presence of an upstream capacity constraint, $q_E^*(q_I) \leq \overline{Q} - q_I$ and thus $\overline{q}_E^*(q_I, \overline{Q}) = q_E^*(q_I)$, whenever:*

$$
\begin{aligned}
& c \in \left[0, \tfrac{1}{6}\right] \text{ and } q_I \leq \min\{\max\{0, 2\overline{Q} + c - 1\}, \max\{1 - c - \tfrac{2\sqrt{1-3c}}{3}, \overline{Q} - \tfrac{1}{3}\}\}, \\
\text{or } & c \in \left[\tfrac{1}{6}, \tfrac{1}{2}\right] \text{ and } q_I \leq \min\{\max\{0, 2\overline{Q} + c - 1\}, \max\{1 - c - \tfrac{1-2c}{\sqrt{2}}, \tfrac{2\overline{Q}+2c-1}{2}\}\}, \\
\text{or } & c \in \left[\tfrac{1}{2}, 1\right] \text{ and } q_I \leq \min\{\max\{0, 2\overline{Q} + c - 1\}, \overline{Q}\}.
\end{aligned}
$$

*E's strategy consists in purchasing $\overline{q}_E^*(q_I, \overline{Q}) = \frac{1-q_I-c}{2}$ as long as:*

$$
\begin{aligned}
& c \in \left[0, \tfrac{1}{6}\right],\ \overline{Q} < \tfrac{4}{3} - c - \tfrac{2\sqrt{1-3c}}{3} \text{ and } q_I \in \left[1 - c - \tfrac{2\sqrt{1-3c}}{3}, \min\{\tfrac{c+2\overline{Q}}{3} - \tfrac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}, 2\overline{Q} + c - 1\}\right], \\
\text{or } & c \in \left[\tfrac{1}{6}, \tfrac{1}{2}\right],\ \overline{Q} < \tfrac{3-4c}{2} - \tfrac{1-2c}{\sqrt{2}} \text{ and } q_I \in \left[1 - c - \tfrac{1-2c}{\sqrt{2}}, \min\{\tfrac{c+2\overline{Q}}{3} - \tfrac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}, 2\overline{Q} + c - 1\}\right].
\end{aligned}
$$

*Finally, $\overline{q}_E^*(q_I, \overline{Q}) = \overline{Q} - q_I$ otherwise.*

*Proof.* See Appendix A.3. □

Unsurprisingly, whenever the constraint is relaxed enough so that $E$ can still purchase the unconstrained quantity, it has no incentive to change its strategy as compared to the unconstrained case. By contrast, when the constraint prevents the entrant from buying $q_E^*(q_I)$, $E$ is forced to buy less than $q_E^*(q_I)$, and may even decide to buy less than the remaining capacity $\overline{Q} - q_I$. More precisely, depending on the value of $q_E^*(q_I)$, two cases occur.

First of all, if the unconstrained optimum is $\frac{1-q_I-c}{2}$, that is, the entrant would like to play the puppy-dog strategy, then the optimum in the capacity constrained case consists in purchasing the whole capacity $\overline{Q} - q_I$. So, the entrant is a constrained puppy-dog that purchases as much of the input as it can.

By contrast, when the unconstrained optimum is to play the top dog strategy by buying either the Cournot quantity $q_E = \frac{1}{3}$ when $c < \frac{1}{6}$ or $q_E = \max\{0, \frac{1-2c}{2}\}$ when $c \geq \frac{1}{6}$, then it is not always optimal for $E$ to purchase the whole remaining capacity $\overline{Q} - q_I$. A constrained top-dog may stick to the top-dog strategy and purchase as much of the product as it can or switch to the puppy-dog strategy and reduce its purchases up to the point where the constraint may not be binding any more. In this case, the entrant prefers to play an unconstrained puppy-dog strategy than a constrained top-dog strategy. To get the intuition for this result, recall that in the unconstrained case shifting from the puppy-dog strategy to the top-dog strategy induces a jump in the quantity purchased by $E$. Now, as the top-dog strategy is constrained by the remaining capacity $\overline{Q} - q_I$, the resulting profit of $E$ in this strategy, $q_E(\frac{1-q_E}{2} - c)$, decreases and may become lower than the profit of an unconstrained puppy dog, $\frac{(1-c-q_I)^2}{4}$.

If $q_I$ is so close to $\overline{Q}$ that a puppy-dog is also constrained, then the difference between a puppy-dog and a top-dog vanishes and $E$ purchases $\overline{Q} - q_I$. Actually, when $\overline{Q} - q_I$ is sufficiently small, $E$ has no choice but to purchase $\overline{Q} - q_I$.

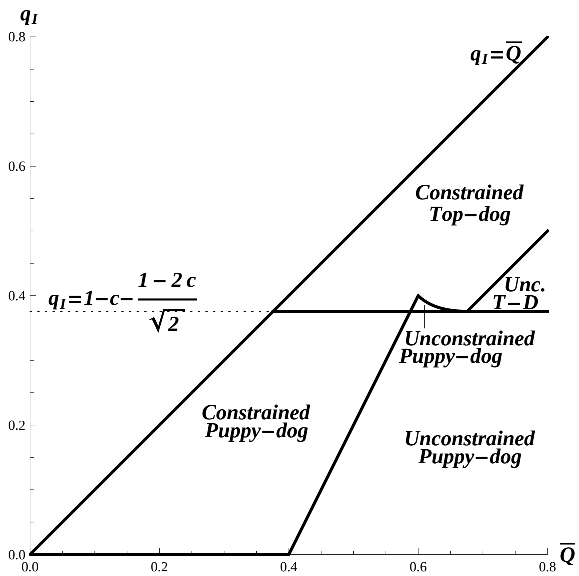


Figure 3: Purchase strategy of the entrant for an upstream cost $c = 0.2$ and a capacity constraint $\overline{Q} \in [0, 0.8]$. The dotted line represents the frontier between the puppy-dog strategy (below) and the top-dog strategy (above) in the unconstrained case.

Figure 3 illustrates the purchase strategy of the entrant in the presence of a capacity constraint for specific values of $c$ and $\overline{Q}$. Comparing it to the unconstrained case, we see that in most cases, if the entrant implemented a puppy-dog (respectively top-dog) strategy in the unconstrained case, then it also implements a puppy-dog (resp. top-dog) strategy in the constrained case, although now the constraint may be binding. However, for intermediate values of $\overline{Q}$ and $q_i$, the entrant may switch from a top-dog strategy to an unconstrained puppy-dog strategy.

**The incumbent's purchase decision** We now determine the purchase decision of the incumbent. The incumbent's optimal purchases in stage 1, $\overline{q}_I^*$, are the solution of the following program:

$$\max_{q_I} P(x_I^*(q_I, \overline{q}_E^*(q_I, \overline{Q})) + x_E^*(q_I, \overline{q}_E^*(q_I, \overline{Q})))x_I^*(q_I, \overline{q}_E^*(q_I, \overline{Q})) - cq_I, \quad s.t. \;\; q_I \leq \overline{Q}.$$

The following Lemma presents the equilibrium of the game in the presence of an upstream capacity constraint.

**Lemma 4.** *In the presence of an upstream capacity constraint, the quantity of input purchased by each firm is as follows:*

- *If* $c < \frac{1}{6}$ *and* $\overline{Q} \leq \frac{17-24c}{36c} - \frac{1-c}{3c}\sqrt{1-3c}$, *then* $\overline{q}_I^* = \overline{Q}$ *and* $\overline{q}_E^* = 0$;
- *If* $c < \frac{1}{6}$ *and* $\overline{Q} > \frac{17-24c}{36c} - \frac{1-c}{3c}\sqrt{1-3c}$, *then* $\overline{q}_I^* = 1 - c - \frac{2}{3}\sqrt{1-3c}$ *and* $\overline{q}_E^* = \frac{\sqrt{1-3c}}{3}$;
- *If* $c \in [\frac{1}{6}, \frac{\sqrt{2}-1}{2\sqrt{2}-1}]$ *and* $\overline{Q} \leq \frac{1-2c+2c^2}{2c} - \frac{(1-c)(1-2c)}{2\sqrt{2}c}$, *then* $\overline{q}_I^* = \overline{Q}$ *and* $\overline{q}_E^* = 0$;
- *If* $c \in [\frac{1}{6}, \frac{\sqrt{2}-1}{2\sqrt{2}-1}]$ *and* $\overline{Q} > \frac{1-2c+2c^2}{2c} - \frac{(1-c)(1-2c)}{2\sqrt{2}c}$, *then* $\overline{q}_I^* = 1 - c - \frac{1-2c}{\sqrt{2}}$ *and* $\overline{q}_E^* = \frac{1-2c}{2\sqrt{2}}$;
- *If* $c \in [\frac{\sqrt{2}-1}{2\sqrt{2}-1}, \sqrt{2}-1]$ *and* $\overline{Q} \leq \frac{1+2c-c^2}{8c}$, *then* $\overline{q}_I^* = \overline{Q}$ *and* $\overline{q}_E^* = 0$;
- *If* $c \in [\frac{\sqrt{2}-1}{2\sqrt{2}-1}, \sqrt{2}-1]$ *and* $\overline{Q} > \frac{1+2c-c^2}{8c}$, *then* $\overline{q}_I^* = \frac{1-c}{2}$ *and* $\overline{q}_E^* = \frac{1-c}{4}$;
- *If* $c \in [\sqrt{2}-1, 1]$ *and* $\overline{Q} \leq \frac{1-c}{2}\left(1 + \frac{1}{\sqrt{2}}\right)$, *then* $\overline{q}_I^* = \overline{Q}$ *and* $\overline{q}_E^* = 0$;
- *If* $c \in [\sqrt{2}-1, 1]$ *and* $\overline{Q} > \frac{1-c}{2}\left(1 + \frac{1}{\sqrt{2}}\right)$, *then* $\overline{q}_I^* = \frac{1-c}{2}$ *and* $\overline{q}_E^* = \frac{1-c}{4}$.

*In stage 3, both firms sell exactly the amount they purchased from the upstream firm:* $\overline{x}_i^* = \overline{q}_i^*$ *for* $i \in \{I, E\}$, *except for* $\{c < \frac{1}{6}$ *and* $\overline{Q} \in [\frac{1}{2}, \frac{17-24c}{36c} - \frac{1-c}{3c}\sqrt{(1-3c)}]\}$ *or* $\{c \in [\frac{1}{6}, \frac{\sqrt{2}-1}{2\sqrt{2}-1}]$ *and* $\overline{Q} \in [\frac{1}{2}, \frac{1-2c+2c^2}{2c} - \frac{(1-c)(1-2c)}{2\sqrt{2}c}]\}$ *or* $\{c \in [\frac{\sqrt{2}-1}{2\sqrt{2}-1}, \sqrt{2}-1]$ *and* $\overline{Q} \in [\frac{1}{2}, \frac{1+2c-c^2}{8c}]\}$. *For these values,* $\overline{x}_I^* = \frac{1}{2} < \overline{q}_I^*$ *and* $\overline{x}_E^* = 0$.

*Proof.* See Appendix A.4. □

The following proposition describes the impact of an upstream capacity constraint on the equilibrium.

**Proposition 2.** *There exists a decreasing function of* $c$, *denoted by* $\overline{Q}_{sup}(c)$, *such that* $\overline{Q}_{sup}(1) = 0$ *and:*

- *if* $\overline{Q} \leq \overline{Q}_{sup}(c)$, *then firm I purchases the whole capacity of the upstream firm* $\overline{Q}$ *in stage 1 and E purchases no input*
- *if* $\overline{Q} > \overline{Q}_{sup}(c)$, *I sticks to the unconstrained strategy. I and E purchase the same quantities as in the absence of an upstream capacity constraint.*

*Proof.* The proposition results from a comparison (calculations not provided here) between the equilibrium described in Lemma 4 and the equilibrium in the absence of an upstream capacity constraint described in Proposition 1. The threshold is defined as follows:

$$
\begin{aligned}
\overline{Q}_{sup}(c) &= \tfrac{17-24c}{36c} - \tfrac{(1-c)\sqrt{1-3c}}{3c} \quad \text{if} \quad c < \tfrac{1}{6}, \\
&= \tfrac{1-2c+2c^2}{2c} - \tfrac{(1-c)(1-2c)}{2\sqrt{2}c} \quad \text{if} \quad c \in [\tfrac{1}{6}, \tfrac{\sqrt{2}-1}{2\sqrt{2}-1}), \\
&= \tfrac{1+2c-c^2}{8c} \quad \text{if} \quad c \in [\tfrac{\sqrt{2}-1}{2\sqrt{2}-1}, \sqrt{2}-1), \\
&= \tfrac{1-c}{2}\left(1 + \tfrac{1}{\sqrt{2}}\right) \quad \text{otherwise.}
\end{aligned}
$$

$\square$

$I$ chooses between two different types of strategies: On the one hand, it can ensure that $q_I$ is low enough so that $E$ can still implement its unconstrained strategy. On the other hand, $I$ can buy a high enough amount of capacity, so that in Stage 2 $E$ would like to buy more capacity than is available, and therefore buys $\overline{Q} - q_I$ in the end.

The former strategy is only possible when the total capacity $\overline{Q}$ is large enough. Indeed, when the total capacity is lower than the standard Stackelberg leader quantity $\frac{1-c}{2}$, $E$'s best reply to any $q_I \in [0, \overline{Q}]$ is $q_E = \overline{Q} - q_I$, because in that case, the entrant anticipates that regardless of $q_E$, the incumbent will always sell all of its capacity $q_I$ on the final market, and $E$ would therefore want to sell its best reply to $q_I$, that is $\frac{1-q_I-c}{2} > \overline{Q} - q_I$. $E$ thus buys $\overline{Q} - q_I$. By contrast, with the latter strategy $I$ earns a negative profit for high enough $\overline{Q}$, because the amount that $I$ must buy in order to constrain $E$ becomes excessively high as $\overline{Q}$ increases.

In between, $I$ chooses $\overline{q}_I^*$ so that $E$ is capacity constrained as long as the total capacity is lower than the threshold $\overline{Q}_{sup}(c)$, for the cost of implementing this strategy increases with the total capacity available whereas the gain associated with the strategy only increases up to the point where $I$ can sell the monopoly quantity ($\frac{1-c}{2}$) on the final market. $\overline{Q}_{sup}(c)$ decreases with $c$ for similar reasons: for a given level of capacity, as the marginal cost of production increases, it becomes more costly for $I$ to buy the whole capacity of the upstream firm, whereas the benefits of using this strategy are unchanged.

It should be noted that there cannot be any partial strategic overbuying: if $I$ buys capacity to induce $\overline{q}_E^*(q_I) = \overline{Q} - q_I$, it is optimal for $I$ to buy $\overline{q}_I^* = \overline{Q}$. Indeed, note that $E$ always sells the whole quantity it purchased. Then two different cases may occur depending on the best reply of $I$ to $\overline{Q} - q_I$ in Stage 3. On the one hand, if the best reply of $I$ is $q_I$, then $I$'s profit is $q_I(1 - \overline{Q} - c)$, which increases with $q_I$ and therefore is maximized for $q_I = \overline{Q}$. On the other hand, if the best reply of $I$ is $\frac{1-q_E}{2} = \frac{1-\overline{Q}+q_I}{2}$, then $I$'s profit is $\frac{(1-\overline{Q}+q_I)^2}{4} - cq_I$, which is decreasing in $q_I$ up to $q_I = \overline{Q} - 1 + 2c$ and increasing in $q_I$ above this threshold. It is also maximized when $q_I = \overline{Q}$: the benefit of deterring entry and thus being able to set the monopoly output on the downstream market always offsets the cost of buying the $\overline{Q} - \frac{1}{2}$ more capacity than is necessary.

### 4.3 Overbuying v. adequate purchases

$I$'s decision to buy the whole capacity can result from two different effects and therefore is not always strategic buying. Indeed, $I$ may buy the whole capacity because it is lower than $I$'s output in the unconstrained case (and hence than the monopoly output). Then it is not strategic buying. Overbuying happens when $I$ purchases $\overline{Q}$, while $\overline{Q}$ is larger than $I$'s purchases in the unconstrained case. Then, the incumbent increases its purchases to deprive the entrant from access to the input.

**Proposition 3.** *Firm $I$ engages in strategic overbuying, i.e. $\bar{q}_I^* > q_I^*$, if and only if $\overline{Q} \in (q_I^*, \overline{Q}_{sup}(c)]$.*

*Proof.* Proposition 3 results from the comparison between the incumbent's purchasing strategy in the absence of a capacity constraint $q_I^*$, as described in Proposition 1, and $\overline{Q}$. □

In the interval $(q_I^*, \overline{Q}_{sup}(c)]$, $I$ buys the whole capacity while it is higher than its unconstrained output $q_I^*$, in which case it is strategic buying. Purchasing the whole capacity enables $I$ to deter entry and sell a quantity equal to $\min\{\overline{Q}, \frac{1}{2}\}$. Strategic buying may happen even when the upstream capacity is larger than the total output in the unconstrained case, i.e. $q_E^*(q_I^*) + q_I^* < \overline{Q}$. The following corollary is an immediate consequence of Proposition 3.

**Corollary 1.** *If $\overline{Q} \in \left(q_I^*, \overline{Q}_{sup}(c)\right]$, then strategic buying involves warehousing as long as $\overline{Q} > 1/2$: $I$ only sells the monopoly output $x^M = \frac{1}{2}$ on the final market. Otherwise, $I$ sells the whole capacity on the final market.*

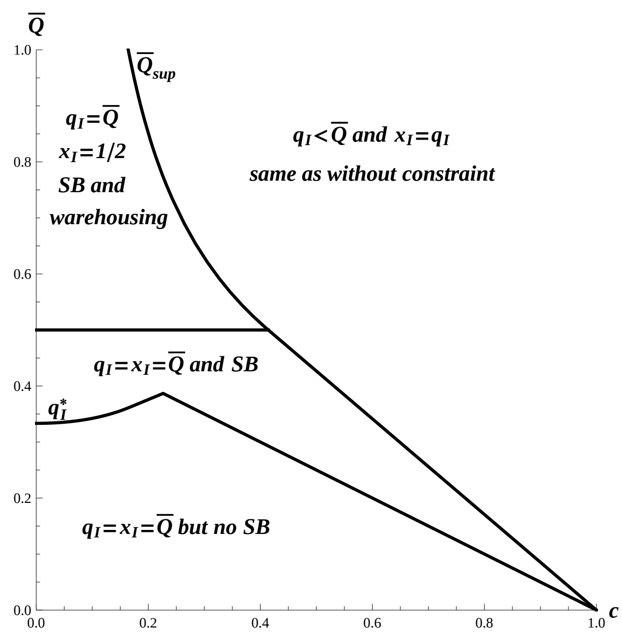


Figure 4: Purchasing strategy of the incumbent in the constrained capacity case.

The threshold above which there is warehousing does not depend on $c$, as it corresponds exactly to the monopoly output with 0 marginal cost. Indeed, the cost of buying the capacity is sunk when firm $I$ sets its output. As it has no other cost of production, $I$ wants to set the monopoly output with 0 marginal cost, that is $\frac{1}{2}$. Then, it will do so whenever $\overline{Q} > 1/2$, and will leave some of its capacity unused. Figure 4 summarizes the purchasing strategy of the incumbent in the constrained capacity case.

### 4.4 The impact of an upstream capacity constraint on welfare

When downstream firms purchase $Q$ from the upstream firm and put on the final market the quantity $X \leq Q$, the welfare is

$$W(X,Q) = \frac{X^2}{2} + X(1-X) - cQ,$$

where the first term of the right-hand side is consumer's surplus, the second term is firms' revenues and the third term is the input production cost. Welfare maximization requires $Q^0 = X^0$ and $X^0 = 1-c$. Essentially, a way to achieve welfare maximization would be to offer the product on the final market at a price equal to the marginal cost of production. Comparing $X^0$ with the equilibrium total output on the final market in the absence of an upstream capacity constraint, we find that $x_I^* + x_E^* \leq X^0$ with equality only for $c = 1$ and $x_I^* + x_E^* = X^0 = 0$. Because both the incumbent and the entrant enjoy some market power, the total output is distorted away from its welfare maximizing value. Note that one of the conditions for efficiency, namely $Q = X$, is satisfied in this equilibrium, so that the inefficiency lies entirely in the value of $X$.

In order to appreciate the impact of an upstream capacity constraint on welfare, we need to compare the equilibrium with and without such a constraint. Some aspects of this comparison are rather straightforward. First, if $\overline{Q} \leq x_I^* + x_E^*$, then the total outcome in the presence of the capacity constraint, $\overline{x}_I^* + \overline{x}_E^*$, is also lower than $x_I^* + x_E^*$ and this implies a lower welfare. The output is already lower than the optimum in the absence of a constraint and it is further reduced by the constraint. Second, if $\overline{Q} > \overline{Q}_{sup}(c)$, downstream firms' input purchases and final good output are just as in the absence of a constraint. Thus, such a (lax) constraint has no consequence on welfare. Third, when $x_I^* + x_E^* \geq 1/2$ and $\overline{Q} \in [1/2, \overline{Q}_{sup}]$, the incumbent implements a strategy of overbuying with warehousing, which reduces the output and increases production costs. Therefore, the welfare is lower in the constrained case.

We now come to the less obvious aspects of the welfare analysis. Consider $\overline{Q} \in \left(x_I^* + x_E^*, \min(\frac{1}{2}, \overline{Q}_{sup}(c))\right]$. For these values of the upstream production capacity, the incumbent purchases all the input the upstream firm is able to produce, namely $\overline{q}_I^* = \overline{Q}$. Of course, the entrant is driven out of the market because there is no input left to purchase. Then, the incumbent is in a monopoly position on the final market. Since input costs are sunk, the incumbent would like to act as a monopolist with zero production costs. That is, it would like to put $x_I = \frac{1}{2}$ on the market. This is larger than or equal to $\overline{q}_I^*$, so the incumbent puts exactly $\overline{q}_I^* = \overline{Q}$ on the market. The output on the final market is thus larger than in the absence of an upstream capacity constraint. The intuition for this result is as follows: the upstream constraint creates the opportunity for the incumbent to monopolize the final market through strategic buying. Once the incumbent is in a monopoly position with a rather large quantity of input, it has an incentive to put it all on the market. This is more profitable than purchasing $q_I^*$, ignoring the strategic opportunities created by the constraint as a myopic incumbent would do. Since we still have $\overline{q}_I^* = \overline{x}_I^*$ and $\overline{q}_I^*$ is strictly larger than $x_I^*+x_E^*$, while still below $X^0$, the existence of an upstream constraint increases the welfare for these values of $\overline{Q}$.

The situation is more intricate when $x_I^* + x_E^* < \frac{1}{2}$ and $\overline{Q}$ is strictly larger than $\frac{1}{2}$, but still below $\overline{Q}_{sup}(c)$. Then, the incumbent purchases $\overline{Q}$, thus driving the entrant out of the market, but puts only $\frac{1}{2}$ on the final market, keeping $\overline{Q} - \frac{1}{2}$ in its warehouses. There are two effects of the existence of the upstream constraint here. First, the output on the final market is larger, actually closer to the welfare maximizing output. This is welfare increasing. However, now we have $\overline{x}_I^* < \overline{q}_I^*$. Units of input are produced at a cost only to be stored by the incumbent, thus with no value to consumers. This is welfare decreasing. As $\overline{Q}$ increases, moving closer to $\overline{Q}_{sup}(c)$, this effect worsens, while the welfare increasing effect remains unchanged. Finally, the positive effect offsets the negative one whenever $\overline{Q} \in \left(\frac{1}{2}, \min\{\frac{3(5c(2-c)-1)}{32c}, \overline{Q}_{sup}(c)\}\right]$. Proposition 4 below summarizes the impact of a capacity constraint on welfare.

**Proposition 4.** *The impact on welfare of the existence of an upstream production capacity constraint $\overline{Q}$ depends on $c$ and $\overline{Q}$. For $\overline{Q} < x_I^*+x_E^*$ or $\overline{Q} \in \left[\max\{x_I^* + x_E^*, \frac{3(5c(2-c)-1)}{32c}\}, \overline{Q}_{sup}(c)\right]$, it reduces welfare. For $\overline{Q} \in \left[x_I^* + x_E^*, \min\{\frac{3(5c(2-c)-1)}{32c}, \overline{Q}_{sup}(c)\}\right]$, it increases welfare. For $\overline{Q} > \overline{Q}_{sup}(c)$, it has no impact on welfare.*

Figure 5 provides a representation of this impact depending on the values of $c$ and $\overline{Q}$.

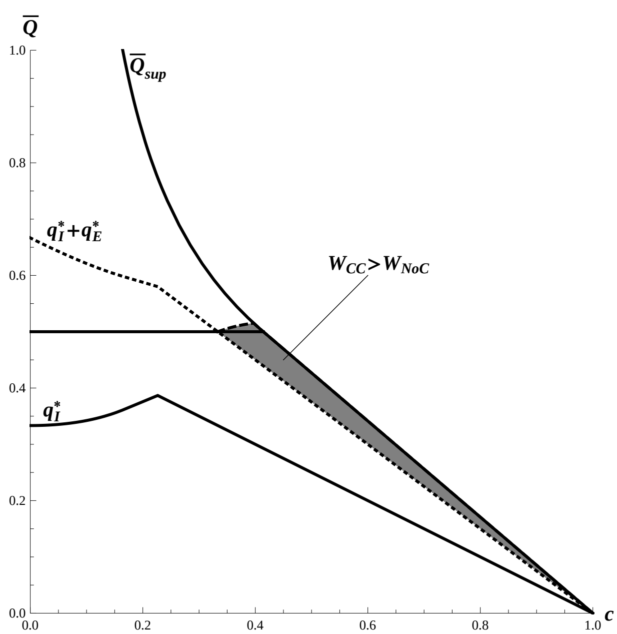


Figure 5: Impact of a capacity constraint on welfare.

## 5 Conclusion

Based on a simple model of vertical relations, we show that overbuying can emerge in equilibrium when the upstream supplier faces a capacity constraint. We find both overbuying with and without warehousing. Overbuying always leads to the exclusion of the entrant and to the monopolization of the final market by the incumbent. The impact on the final market output and price is ambiguous, but we identify cases in which overbuying clearly leads to higher prices for consumers and a lower welfare. This is all the more true in the case of warehousing due to the supplementary costs incurred to produce units of input that remain in the incumbent's inventories. We thus establish that overbuying is a practice that antitrust authorities should treat with much attention in oligopolistic sectors. Because of the ambiguity of the effect on welfare, overbuying should probably be subject to a rule of reason approach. A difficulty at this point is to recognize overbuying. It is not enough to observe that the incumbent purchases all the production capacity of the input supplier. Of course, warehousing is a usual suspect, but it may result from errors in the firm's anticipation of its future needs. As noted by Judge Learned Hand in the Alcoa case: "In the case at bar, the first issue was whether, when 'Alcoa' bought up the bauxite deposits, it really supposed that they would be useful in the future. It would be hard to imagine an issue in which the credibility of the witnesses should more depend upon the impressions derived from their presence." On this issue, our results suggest that if a firm purchases very large quantities of input at a high price, it is not part of an overbuying strategy but rather because this firm expects a high demand in the future. However, the issue of enforcing antitrust laws in alleged overbuying cases clearly deserves further research.

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# A Appendix

## A.1 Proof of Lemma 2

For $q_I \geq 1/3$,

$$\begin{aligned}
\pi_E &= \frac{1}{9} - cq_E \quad \text{for} \quad q_E \geq \frac{1}{3}, \\
&= q_E\left(\frac{1-q_E}{2} - c\right) \quad \text{for} \quad q_E \in [\max\{0, 1-2q_I\}, \frac{1}{3}], \\
&= q_E(1 - q_I - q_E - c) \quad \text{for} \quad q_E \leq \max\{0, 1-2q_I\}.
\end{aligned}$$

Profit maximization on each interval of $q_E$ leads to three local maxima.

- For $q_E \geq \frac{1}{3}$, $q_E = \frac{1}{3}$.

- For $q_E \in [\max\{0, 1-2q_I\}, \frac{1}{3}]$, the unconstrained optimum would be $q_E = \frac{1-2c}{2}$, which is not always within the relevant interval. Therefore, the solution to this problem is as follows: If $c < \frac{1}{6}$, then $q_E = \frac{1}{3}$. If $c \in [\frac{1}{6}, \frac{1}{2}]$, then $q_E = \frac{1-2c}{2}$ for $q_I \geq \frac{1+2c}{4}$ and $q_E = 1 - 2q_I$ otherwise. Finally, if $c > \frac{1}{2}$, then $q_E = 1 - 2q_I$ for $q_I < \frac{1}{2}$ and $q_E = 0$ otherwise.

- For $q_E \leq \max\{0, 1 - 2q_I\}$, if $c < \frac{1}{2}$, then the solution to this program is $q_E = \frac{1-q_I-c}{2}$ for $q_I < \frac{1+c}{3}$, $q_E = 1 - 2q_I$ for $q_I \in [\frac{1+c}{3}, \frac{1}{2}]$, and $q_E = 0$ otherwise. If $c \in [\frac{1}{2}, \frac{2}{3}]$, then $q_E = \frac{1-q_I-c}{2}$ for $q_I < 1 - c$ and $q_E = 0$ otherwise. Finally, if $c > \frac{2}{3}$ then $q_E = 0$.

Comparing the local maxima leads to the optimal value of $q_E$.

- If $c < \frac{1}{6}$, then $q_E = \frac{1-q_I-c}{2}$ if $q_I < 1 - \frac{2}{3}\sqrt{1-3c} - c$, and otherwise $q_E = \frac{1}{3}$.
- If $c \in [\frac{1}{6}, \frac{1}{2}]$, then $q_E = \frac{1-q_I-c}{2}$ if $q_I < 1 - c - \frac{1-2c}{\sqrt{2}}$, and otherwise $q_E = \frac{1-2c}{2}$.
- If $c \in [\frac{1}{2}, \frac{2}{3}]$, then $q_E = \frac{1-q_I-c}{2}$ if $q_I < 1 - c$ and $q_E = 0$ otherwise.
- If $c > \frac{2}{3}$, then $q_E = 0$.

For $q_I < \frac{1}{3}$,

$$\begin{aligned}\pi_E &= \left(\frac{1-q_I}{2}\right)^2 - cq_E \quad \text{for} \quad q_E \geq \frac{1-q_I}{2} \\ &= q_E(1 - q_I - q_E - c) \quad \text{otherwise.}\end{aligned}$$

We determine the two local maxima.

- For $q_E \geq \frac{1-q_I}{2}$, $q_E = \frac{1-q_I}{2}$.
- For $q_E < \frac{1-q_I}{2}$, $q_E = \max\{0, \frac{1-q_I-c}{2}\}$.

The latter strategy always yields a higher profit than the former.

**Summary and incumbent's profit** For $\{c < 1/6$ and $q_I < 1 - c - \frac{2\sqrt{1-3c}}{3}\}$, $q_E = x_E = \frac{1-q_I-c}{2}$, $x_I = q_I$ and $\pi_I = \frac{q_I(1-q_I-c)}{2}$. For $\{c < 1/6$ and $q_I \geq 1 - c - \frac{2\sqrt{1-3c}}{3}\}$, $q_E = x_E = x_I = 1/3$ and $\pi_I = \frac{1}{9} - cq_I$. For $\{c \in [1/6, 1/2]$ and $q_I < 1 - c - \frac{1-2c}{\sqrt{2}}\}$, $q_E = x_E = \frac{1-q_I-c}{2}$, $x_I = q_I$ and $\pi_I = \frac{q_I(1-q_I-c)}{2}$. For $\{c \in [1/6, 1/2]$ and $q_I \geq 1-c-\frac{1-2c}{\sqrt{2}}\}$, $q_E = x_E = \frac{1-2c}{2}$, $x_I = \frac{1+2c}{4}$ and $\pi_I = \left(\frac{1+2c}{4}\right)^2 - cq_I$. For $\{c \in [1/2, 1]$ and $q_I < 1 - c\}$, $q_E = x_E = \frac{1-q_I-c}{2}$, $x_I = q_I$ and $\pi_I = \frac{q_I(1-q_I-c)}{2}$. For $\{c \in [1/2, 1]$ and $q_I \in [1 - c, \frac{1}{2}]\}$, $q_E = x_E = 0$, $x_I = q_I$ and $\pi_I = (1 - q_I - c)q_I$. For $\{c \in [1/2, 1]$ and $q_I > \frac{1}{2}\}$, $q_E = x_E = 0$, $x_I = \frac{1}{2}$ and $\pi_I = \frac{1}{4} - cq_I$.

## A.2 Proof of proposition 1

The incumbent's profit is given by:

For $c < 1/6$, $\begin{cases} \pi_I = \frac{q_I(1-q_I-c)}{2} & \text{for} \quad q_I < 1-c-\frac{2\sqrt{1-3c}}{3} \\ \pi_I = \frac{1}{9} - cq_I & \text{for} \quad q_I \geq 1-c-\frac{2\sqrt{1-3c}}{3} \end{cases}$

For $c \in [1/6, 1/2]$, $\begin{cases} \pi_I = \frac{q_I(1-q_I-c)}{2} & \text{for} \quad q_I < 1-c-\frac{1-2c}{\sqrt{2}} \\ \pi_I = \left(\frac{1+2c}{4}\right)^2 - cq_I & \text{for} \quad q_I \geq 1-c-\frac{1-2c}{\sqrt{2}} \end{cases}$

For $c \in [1/2, 1]$, $\begin{cases} \pi_I = \frac{q_I(1-q_I-c)}{2} & \text{for} \quad q_I < 1-c \\ \pi_I = (1-q_I-c)q_I & \text{for} \quad q_I \in [1-c, \frac{1}{2}] \\ \pi_I = \frac{1}{4} - cq_I & \text{for} \quad q_I > \frac{1}{2} \end{cases}$

Profit maximization and Lemmas 1 and 2 lead to Proposition 1.

## A.3 Proof of Lemma 3

In this appendix, we determine the entrant's equilibrium strategy depending on $c$, $q_I$ and $\overline{Q}$. Lemma 3 results from a comparison (calculations not provided here) between this optimal strategy and the optimal strategy in the absence of an upstream constraint described in Lemma 2.

**Case 1:** $\overline{Q} > 2/3$

If $q_I \leq \overline{Q} - 1/3$, then $\overline{Q} - q_I > \max\{\frac{1}{3}, \frac{1-q_I}{2}\}$. The constraint is too relaxed to have an effect. Therefore, the profit of $E$ and its optimal strategy are the same as in the unconstrained case.

If $q_I \in [\overline{Q} - 1/3, \overline{Q}]$, then $\overline{Q} - q_I \in [1 - 2q_I, 1/3]$. Here, the constraint plays a role and

$$\begin{aligned} \pi_E &= q_E \left( \frac{1-q_E}{2} - c \right) \quad \text{for} \quad q_E \in [\max\{0, 1-2q_I\}, \overline{Q} - q_I], \\ &= q_E(1-q_I-q_E-c) \quad \text{for} \quad q_E \leq \max\{0, 1-2q_I\}. \end{aligned}$$

The two local maxima are as follows:

- For $q_E \in [\max\{0, 1-2q_I\}, \overline{Q} - q_I]$:
  - If $c < 1/6$, then $q_E = \overline{Q} - q_I$.
  - If $c \in [1/6, 1/2]$, then:
    - If $\overline{Q} < \frac{7+6c}{12}$, then $q_E = 1 - 2q_I$ for $q_I < \frac{1+2c}{4}$, $q_E = \frac{1-2c}{2}$ for $q_I \in [\frac{1+2c}{4}, c + Q - \frac{1}{2}]$, and $q_E = \overline{Q} - q_I$ otherwise.
    - If $\overline{Q} \geq \frac{7+6c}{12}$, then $q_E = \frac{1-2c}{2}$ for $q_I < c + Q - \frac{1}{2}$ and $q_E = \overline{Q} - q_I$ otherwise.
  - Finally, if $c > 1/2$ then $q_E = 0$.
- For $q_E \leq \max\{0, 1-2q_I\}$:

- If $c < \frac{1}{2}$, then $q_E = \frac{1-q_I-c}{2}$ for $q_I < \frac{1+c}{3}$, $q_E = 1-2q_I$ for $q_I \in [\frac{1+c}{3}, \frac{1}{2}]$, and $q_E = 0$ otherwise.
- If $c \in [\frac{1}{2}, \frac{2}{3}]$, then $q_E = \frac{1-q_I-c}{2}$ for $q_I < 1-c$ and $q_E = 0$ otherwise.
- Finally, if $c > \frac{2}{3}$ then $q_E = 0$.

Comparing these local maxima, we find that $q_E = \frac{1-2c}{2}$ for

- $c \in [\frac{1}{6}, \frac{4-\sqrt{2}}{12}]$ and $\left\{\left\{\overline{Q} \in [\frac{3-4c}{2} - \frac{(1-2c)}{\sqrt{2}}, \frac{4-3c}{3} - \frac{(1-2c)}{\sqrt{2}}] \text{ and } q_I \in [1-c-\frac{(1-2c)}{\sqrt{2}}, \overline{Q}+c-\frac{1}{2}]\right\}\right.$ or $\left.\left\{\overline{Q} > \frac{4-3c}{3} - \frac{(1-2c)}{\sqrt{2}} \text{ and } q_I \in [\overline{Q}-\frac{1}{3}, Q+c-\frac{1}{2}]\right\}\right\}$

  or
- $c \in [\frac{4-\sqrt{2}}{12}, 1/2]$ and $\left\{\left\{\overline{Q} \in [2/3, \frac{4-3c}{3} - \frac{(1-2c)}{\sqrt{2}}] \text{ and } q_I \in [1-c-\frac{(1-2c)}{\sqrt{2}}, \overline{Q}+c-\frac{1}{2}]\right\}\right.$ or $\left.\left\{\overline{Q} > \frac{4-3c}{3} - \frac{(1-2c)}{\sqrt{2}} \text{ and } q_I \in [\overline{Q}-\frac{1}{3}, Q+c-\frac{1}{2}]\right\}\right\}$.

It is optimal to set $q_E = \max\{0, \frac{1-q_I-c}{2}, 1-2q_I\}$ for

- $c < 1/6$ and $\overline{Q} \in [2/3, \frac{4-3c}{3} - \frac{2\sqrt{1-3c}}{3}]$ and $q_I \in \left[\overline{Q}-\frac{1}{3}, \frac{c+2Q}{3} - \frac{\sqrt{c^2-3(1-c)^2+6Q-8cQ-2Q^2}}{3}\right]$

  or
- $c \in [\frac{1}{6}, \frac{4-\sqrt{2}}{12}]$ and $\left\{\left\{\overline{Q} \in [2/3, \frac{3-4c}{2} - \frac{(1-2c)}{\sqrt{2}}] \text{ and } q_I \in [\overline{Q}-\frac{1}{3}, \frac{c+2Q}{3} - \frac{\sqrt{c^2-3(1-c)^2+6Q-8cQ-2Q^2}}{3}]\right\}\right.$ or $\left.\left\{\overline{Q} \in [\frac{3-4c}{2} - \frac{(1-2c)}{\sqrt{2}}, \frac{4-3c}{3} - \frac{(1-2c)}{\sqrt{2}}] \text{ and } q_I \in [\overline{Q}-\frac{1}{3}, 1-c-\frac{(1-2c)}{\sqrt{2}}]\right\}\right\}$

  or
- $c \in [\frac{4-\sqrt{2}}{12}, \frac{1}{2}]$ and $\overline{Q} \in [\frac{2}{3}, \frac{4-3c}{3} - \frac{2\sqrt{1-3c}}{3}]$ and $q_I \in [\overline{Q}-\frac{1}{3}, 1-c-\frac{(1-2c)}{\sqrt{2}}]$

  or
- $c \in [1/2, 1]$.

Finally, it is optimal to set $\overline{Q} - q_I$ otherwise.

**Case 2:** $\overline{Q} \in [1/2, 2/3]$

If $q_I < 1/3$, then $\overline{Q} - q_I > \frac{1-q_I}{2}$ and the profit as well as the purchases of $E$ are the same as in the unconstrained case. If $q_I \in [1/3, 1-\overline{Q}]$, then $\overline{Q} - q_I \in [0, 1-2q_I]$. The profit of $E$ is $\pi_E = q_E(1-q_I-q_E-c)$ for all $q_E \in [0, \overline{Q}-q_I]$. The optimal quantity is $\min\{\frac{1-q_I-c}{2}, \overline{Q}-q_I\}$. If $q_I \in [1-\overline{Q}, \overline{Q}]$, then $\overline{Q}-q_I \in [1-2q_I, 1/3]$ and

$$\begin{aligned} \pi_E &= q_E\left(\frac{1-q_E}{2} - c\right) \quad \text{if} \quad q_E \in [\max\{0, 1-2q_I\}, \overline{Q}-q_I], \\ &= q_E(1-q_I-q_E-c) \quad \text{if} \quad q_E \leq \max\{0, 1-2q_I\}. \end{aligned}$$

The two corresponding local maxima as well as the global maximum are described in case 1.

**Case 3:** $\overline{Q} \in [1/3, 1/2]$

If $q_I < 1/3$, then $\overline{Q} - q_I > \frac{1-q_I}{2}$ and the profit as well as the purchases of $E$ are the same as in the unconstrained case. If $q_I \in [1/3, \overline{Q}]$, then $q_I < 1 - \overline{Q}$, which implies that $\overline{Q} - q_I \in [0, 1-2q_I]$. The profit of $E$ is $\pi_E = q_E(1 - q_I - q_E - c)$ for $q_E \in [0, \overline{Q} - q_I]$. The optimal quantity is $\min\{\frac{1-q_I-c}{2}, \overline{Q} - q_I\}$.

**Case 4:** $\overline{Q} \in [0, 1/3]$

$1/3 > q_I > 0 > 2\overline{Q} - 1$ and $\overline{Q} - q_I < \frac{1-q_I}{2}$. The profit of $E$ is $\pi_E = q_E(1 - q_I - q_E - c)$ for $q_E \in [0, \overline{Q} - q_I]$. The optimal quantity is $\min\{\frac{1-q_I-c}{2}, \overline{Q} - q_I\}$.

**Summary and incumbent's profit**

We describe the outcome of Stage 2 for all values of $\overline{Q}$, $c$ and $q_I$.

**First case:** $c < 1/6$

For $\overline{Q} < \frac{2-c}{3}$ and $q_I \leq \max\{0, 2\overline{Q} - 1 + c\}$, $q_E = \frac{1-q_I-c}{2} = x_E$, $x_I = q_I$ and $\pi_I = q_I\left(\frac{1-q_I-c}{2}\right)$. For $\overline{Q} < \frac{2-c}{3}$ and $q_I \in [\max\{0, 2\overline{Q}-1+c\}, \min\{\overline{Q}, 1-\overline{Q}\}]$, $q_E = \overline{Q} - q_I = x_E$, $x_I = q_I$ and $\pi_I = q_I(1 - \overline{Q} - c)$. For $\overline{Q} < \frac{2-c}{3}$ and $q_I > \min\{\overline{Q}, 1-\overline{Q}\}$, $q_E = \overline{Q} - q_I = x_E$, $x_I = \frac{1-q_E}{2} = \frac{1-\overline{Q}+q_I}{2}$ and $\pi_I = \frac{(1-\overline{Q}+q_I)^2}{4} - cq_I$.

For $\overline{Q} \in [\frac{2-c}{3}, \frac{4}{3} - c - \frac{2\sqrt{1-3c}}{3}]$ and $q_I < \frac{c+2\overline{Q}}{3} - \frac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}$, $q_E = \frac{1-q_I-c}{2} = x_E$, $x_I = q_I$ and $\pi_I = q_I\left(\frac{1-q_I-c}{2}\right)$. For $\overline{Q} \in [\frac{2-c}{3}, \frac{4}{3} - c - \frac{2\sqrt{1-3c}}{3}]$ and $q_I \geq \frac{c+2\overline{Q}}{3} - \frac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}$, $q_E = \overline{Q} - q_I = x_E$, $x_I = \frac{1-q_E}{2} = \frac{1-\overline{Q}+q_I}{2}$ and $\pi_I = \frac{(1-\overline{Q}+q_I)^2}{4} - cq_I$.

For $\overline{Q} > \frac{4}{3} - c - \frac{2\sqrt{1-3c}}{3}$ and $q_I < 1 - c - \frac{2\sqrt{1-3c}}{3}$, $q_E = \frac{1-q_I-c}{2} = x_E$, $x_I = q_I$ and $\pi_I = q_I\left(\frac{1-q_I-c}{2}\right)$. For $\overline{Q} > \frac{4}{3} - c - \frac{2\sqrt{1-3c}}{3}$ and $q_I \in [1 - c - \frac{2\sqrt{1-3c}}{3}, \overline{Q} - \frac{1}{3}]$, $q_E = \frac{1}{3} = x_E = x_I$ and $\pi_I = \frac{1}{9} - cq_I$. For $\overline{Q} > \frac{4}{3} - c - \frac{2\sqrt{1-3c}}{3}$ and $q_I > \overline{Q} - \frac{1}{3}$, $q_E = \overline{Q} - q_I = x_E$, $x_I = \frac{1-q_E}{2} = \frac{1-\overline{Q}+q_I}{2}$ and $\pi_I = \frac{(1-\overline{Q}+q_I)^2}{4} - cq_I$.

**Second case:** $c \in [\frac{1}{6}, \frac{1}{2}]$

For $\overline{Q} < \frac{2-c}{3}$ and $q_I \leq \max\{0, 2\overline{Q} - 1 + c\}$, $q_E = \frac{1-q_I-c}{2} = x_E$, $x_I = q_I$ and $\pi_I = q_I\left(\frac{1-q_I-c}{2}\right)$. For $\overline{Q} < \frac{2-c}{3}$ and $q_I \in [\max\{0, 2\overline{Q}-1+c\}, \min\{\overline{Q}, 1-\overline{Q}\}]$, $q_E = \overline{Q} - q_I = x_E$, $x_I = q_I$ and $\pi_I = q_I(1 - \overline{Q} - c)$. For $\overline{Q} < \frac{2-c}{3}$ and $q_I > \min\{\overline{Q}, 1-\overline{Q}\}$, $q_E = \overline{Q} - q_I = x_E$, $x_I = \frac{1-q_E}{2} = \frac{1-\overline{Q}+q_I}{2}$ and $\pi_I = \frac{(1-\overline{Q}+q_I)^2}{4} - cq_I$.

For $\overline{Q} \in [\frac{2-c}{3}, \frac{3-4c}{2} - \frac{(1-2c)}{\sqrt{2}}]$ and $q_I < \frac{c+2\overline{Q}}{3} - \frac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}$, $q_I < 2\overline{Q} - 1 + c$, $q_E = \frac{1-q_I-c}{2} = x_E$, $x_I = q_I$ and $\pi_I = q_I\left(\frac{1-q_I-c}{2}\right)$. For $\overline{Q} \in [\frac{2-c}{3}, \frac{3-4c}{2} - \frac{(1-2c)}{\sqrt{2}}]$ and $q_I \geq \frac{c+2\overline{Q}}{3} - \frac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}$, $q_E = \overline{Q} - q_I = x_E$, $x_I = \frac{1-q_E}{2} = \frac{1-\overline{Q}+q_I}{2}$ and $\pi_I = \frac{(1-\overline{Q}+q_I)^2}{4} - cq_I$.

For $\overline{Q} > \frac{3-4c}{2} - \frac{(1-2c)}{\sqrt{2}}$ and $q_I < 1 - c - \frac{(1-2c)}{\sqrt{2}}$, $q_E = \frac{1-q_I-c}{2} = x_E$, $x_I = q_I$ and $\pi_I = q_I\left(\frac{1-q_I-c}{2}\right)$. For $\overline{Q} > \frac{3-4c}{2} - \frac{(1-2c)}{\sqrt{2}}$ and $q_I \in [1-c-\frac{(1-2c)}{\sqrt{2}}, \overline{Q}+c-\frac{1}{2}]$, $q_E = \frac{1-2c}{2} = x_E$, $x_I = \frac{1+2c}{4} = \frac{1-q_E}{2}$ and $\pi_I = \left(\frac{1+2c}{4}\right)^2 - cq_I$. For $\overline{Q} > \frac{3-4c}{2} - \frac{(1-2c)}{\sqrt{2}}$ and $q_I > \overline{Q} + c - \frac{1}{2}$, $q_E = \overline{Q} - q_I = x_E$, $x_I = \frac{1-q_E}{2} = \frac{1-\overline{Q}+q_I}{2}$ and $\pi_I = \frac{(1-\overline{Q}+q_I)^2}{4} - cq_I$.

**Third case:** $c \in [\frac{1}{2}, 1]$

For $\overline{Q} < 1-c$ and $q_I < \max\{0, 2\overline{Q}-1+c\}$, $q_E = \frac{1-q_I-c}{2} = x_E$, $x_I = q_I$ and $\pi_I = q_I\left(\frac{1-q_I-c}{2}\right)$. For $\overline{Q} < 1-c$ and $q_I > \max\{0, 2\overline{Q}-1+c\}$, $q_E = \overline{Q} - q_I = x_I$, $x_I = q_I$ and $\pi_I = q_I(1-\overline{Q}-c)$.

For $\overline{Q} \geq 1-c$ and $q_I < 1-c$, $q_E = \frac{1-q_I-c}{2} = x_E$, $x_I = q_I$ and $\pi_I = q_I\left(\frac{1-q_I-c}{2}\right)$. For $\overline{Q} \geq 1-c$ and $q_I > 1-c$, $q_E = 0 = x_E$, $x_I = q_I$ and $\pi_I = q_I(1-q_I-c)$.

## A.4 Proof of Lemma 4

In this appendix, we determine the incumbent's equilibrium strategy depending on $c$ and $\overline{Q}$. The entrant's equilibrium strategy follows from Lemma 3.

**First case:** $c < 1/6$

For $\overline{Q} < \frac{1-c}{2}$, $\pi_I = q_I(1-\overline{Q}-c)$, which is maximized for $q_I = \overline{Q}$.

For $\overline{Q} \in [\frac{1-c}{2}, \frac{1}{2}]$,

$$\begin{aligned}\pi_I &= q_I\left(\frac{1-q_I-c}{2}\right) \quad \text{for} \quad q_I \leq 2\overline{Q}-1+c,\\ &= q_I(1-\overline{Q}-c) \quad \text{for} \quad q_I \in [2\overline{Q}-1+c, \overline{Q}].\end{aligned}$$

We first maximize $\pi_I$ on each interval.

- For $q_I \leq 2\overline{Q}-1+c$, the unconstrained solution would be $q_I = \frac{1-c}{2}$. However, this is larger than $2\overline{Q}-1+c$. Therefore, the solution is $q_I = 2\overline{Q}-1+c$.
- For $q_I \in [2\overline{Q}-1+c, \overline{Q}]$, the solution is $q_I = \overline{Q}$.

The latter strategy always yields a higher profit than the former.

For $\overline{Q} \in [\frac{1}{2}, \frac{2-c}{3}]$,

$$\begin{aligned}\pi_I &= q_I\left(\frac{1-q_I-c}{2}\right) \quad \text{for} \quad q_I \leq 2\overline{Q}-1+c,\\ &= q_I(1-\overline{Q}-c) \quad \text{for} \quad q_I \in [2\overline{Q}-1+c, 1-\overline{Q}],\\ &= \frac{(1-\overline{Q}+q_I)^2}{4} - cq_I \quad \text{for} \quad q_I \in [1-\overline{Q}, \overline{Q}].\end{aligned}$$

The three local maxima are as follows:

- For $q_I \leq 2\overline{Q} - 1 + c$, $q_I = 2\overline{Q} - 1 + c$.
- For $q_I \in [2\overline{Q} - 1 + c, 1 - \overline{Q}]$, $q_I = 1 - \overline{Q}$.
- For $q_I \in [1 - \overline{Q}, \overline{Q}]$, $q_I = \overline{Q}$.

Comparing the three local maxima, we find that the global maximum is $q_I = \overline{Q}$.

For $\overline{Q} \in [\frac{2-c}{3}, \frac{4}{3} - c - \frac{2\sqrt{1-3c}}{3}]$,

$$\begin{aligned}\pi_I &= q_I\left(\frac{1-q_I-c}{2}\right) \quad \text{for} \quad q_I \leq \tfrac{c+2\overline{Q}}{3} - \tfrac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}, \\ &= \frac{(1-\overline{Q}+q_I)^2}{4} - cq_I \quad \text{for} \quad q_I \in [\tfrac{c+2\overline{Q}}{3} - \tfrac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}, \overline{Q}].\end{aligned}$$

We determine two local maxima:

- For $q_I \leq \frac{c+2\overline{Q}}{3} - \frac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}$, $q_I = \frac{c+2\overline{Q}}{3} - \frac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}$.
- For $q_I \in [\frac{c+2\overline{Q}}{3} - \frac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}, \overline{Q}]$, $q_I = \overline{Q}$.

The latter strategy always yields a higher profit than the former.

Finally, for $\overline{Q} > \frac{4}{3} - c - \frac{2\sqrt{1-3c}}{3}$,

$$\begin{aligned}\pi_I &= q_I\left(\frac{1-q_I-c}{2}\right) \quad \text{for} \quad q_I \leq 1 - c - \frac{2\sqrt{1-3c}}{3}, \\ &= \frac{1}{9} - cq_I \quad \text{for} \quad q_I \in [1 - c - \frac{2\sqrt{1-3c}}{3}, \overline{Q} - \frac{1}{3}] \\ &= \frac{(1-\overline{Q}+q_I)^2}{4} - cq_I \quad \text{for} \quad q_I \in [\overline{Q} - \frac{1}{3}, \overline{Q}].\end{aligned}$$

The three local maxima are as follows:

- For $q_I \leq 1 - c - \frac{2\sqrt{1-3c}}{3}$, $q_I = 1 - c - \frac{2\sqrt{1-3c}}{3}$.
- For $q_I \in [1 - c - \frac{2\sqrt{1-3c}}{3}, \overline{Q} - \frac{1}{3}]$, $q_I = 1 - c - \frac{2\sqrt{1-3c}}{3}$.
- For $q_I \in [\overline{Q} - \frac{1}{3}, \overline{Q}]$, $q_I = \overline{Q}$.

Comparing these local maxima, we find that the optimal value is $q_I = \overline{Q}$ for $\overline{Q} < \frac{17-24c}{36c} - \frac{(1-c)\sqrt{1-3c}}{3c}$. Otherwise, the optimal value is $q_I = 1 - c - \frac{2\sqrt{1-3c}}{3}$.

**Second case:** $c \in [\frac{1}{6}, \frac{1}{2}]$

For $\overline{Q} < \frac{1-c}{2}$, $\pi_I = q_I(1-\overline{Q}-c)$, which is maximized for $q_I = \overline{Q}$.

For $\overline{Q} \in [\frac{1-c}{2}, \frac{1}{2}]$,

$$\begin{aligned} \pi_I &= q_I\left(\frac{1-q_I-c}{2}\right) \quad \text{for} \quad q_I \leq 2\overline{Q}-1+c, \\ &= q_I(1-\overline{Q}-c) \quad \text{for} \quad q_I \in [2\overline{Q}-1+c, \overline{Q}] \end{aligned}$$

We determine two local maxima:

- For $q_I \leq 2\overline{Q}-1+c$, then the unconstrained local maximum would be $q_I = \frac{1-c}{2}$. For $\{c > 1/3$ and $\overline{Q} \in [\frac{3(1-c)}{4}, \frac{1}{2}]\}$, this interior solution applies. For $c \leq 1/3$ or $\{c > 1/3$ and $\overline{Q} \in [\frac{1-c}{2}, \frac{3(1-c)}{4}]\}$, $q_I = 2\overline{Q}-1+c$.
- For $q_I \in [2\overline{Q}-1+c, \overline{Q}]$, the optimal solution is $q_I = \overline{Q}$.

Comparing these two local maxima, we find that the optimal strategy is to set $q_I = \frac{1-c}{2}$ for $\{c > \sqrt{2}-1$ and $\overline{Q} > \frac{1-c}{2}\left(1+\frac{1}{\sqrt{2}}\right)\}$, and $q_I = \overline{Q}$ otherwise.

For $\overline{Q} \in [\frac{1}{2}, \frac{2-c}{3}]$,

$$\begin{aligned} \pi_I &= q_I\left(\frac{1-q_I-c}{2}\right) \quad \text{for} \quad q_I \leq 2\overline{Q}-1+c, \\ &= q_I(1-\overline{Q}-c) \quad \text{for} \quad q_I \in [2\overline{Q}-1+c, 1-\overline{Q}], \\ &= \frac{(1-\overline{Q}+q_I)^2}{4} - cq_I \quad \text{for} \quad q_I \in [1-\overline{Q}, \overline{Q}]. \end{aligned}$$

The three local maxima are as follows:

- For $q_I \leq 2\overline{Q}-1+c$, $q_I = \min\{\frac{1-c}{2}, 2\overline{Q}-1+c\}$.
- For $q_I \in [2\overline{Q}-1+c, 1-\overline{Q}]$, $q_I = 1-\overline{Q}$.
- For $q_I \in [1-\overline{Q}, \overline{Q}]$, $q_I = \overline{Q}$.

Comparing the profits obtained with each strategy, we find that it is optimal to set $q_I = \min\{\frac{1-c}{2}, 2\overline{Q}-1+c\}$ for $\{c \in [1-\sqrt{\frac{2}{5}}, \sqrt{2}-1]$ and $\overline{Q} \in [\frac{1+2c-c^2}{8c}, \frac{2-c}{3}]\}$ or $c \in [\sqrt{2}-1, 1]$. Otherwise, it is optimal to set $q_I = \overline{Q}$.

For $\overline{Q} \in [\frac{2-c}{3}, \frac{(1-2c)}{\sqrt{2}}]$,

$$\begin{aligned} \pi_I &= q_I\left(\frac{1-q_I-c}{2}\right) \quad \text{for} \quad q_I \leq \frac{c+2\overline{Q}}{3} - \frac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}, \\ &= \frac{(1-\overline{Q}+q_I)^2}{4} - cq_I \quad \text{for} \quad q_I \in [\frac{c+2\overline{Q}}{3} - \frac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}, \overline{Q}] \end{aligned}$$

The two local maxima are as follows:

- For $q_I \leq \frac{c+2\overline{Q}}{3} - \frac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}$, $q_I = \min\{\frac{1-c}{2}, \frac{c+2\overline{Q}}{3} - \frac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}\}$.

- For $q_I \in [\frac{c+2\overline{Q}}{3} - \frac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}, \overline{Q}]$, $q_I = \overline{Q}$.

Comparing the profits obtained with each strategy, we find that it is optimal to set $q_I = \min\{\frac{1-c}{2}, \frac{c+2\overline{Q}}{3} - \frac{\sqrt{c^2-3(1-c)^2+6\overline{Q}-8c\overline{Q}-2\overline{Q}^2}}{3}\}$ for $\{c \in [\frac{5-2\sqrt{2}-\sqrt{2(9-6\sqrt{2})}}{15-8\sqrt{2}}, 1-\sqrt{\frac{5}{2}}]$ and $\overline{Q} \geq \frac{1+2c-c^2}{8c}\}$ or $c \in [1-\sqrt{\frac{5}{2}}, 1]$. Otherwise, it is optimal to set $q_I = \overline{Q}$.

For $\overline{Q} > \frac{3-4c}{2} - \frac{(1-2c)}{\sqrt{2}}$,

$$\begin{aligned}
\pi_I &= q_I\left(\frac{1-q_I-c}{2}\right) \quad \text{for} \quad q_I \leq 1-c-\frac{(1-2c)}{\sqrt{2}}, \\
&= \left(\frac{1+2c}{4}\right)^2 - cq_I \quad \text{for} \quad q_I \in [1-c-\frac{(1-2c)}{\sqrt{2}}, \overline{Q}+c-\frac{1}{2}] \\
&= \frac{(1-\overline{Q}+q_I)^2}{4} - cq_I \quad \text{for} \quad q_I \in [\overline{Q}+c-\frac{1}{2}, \overline{Q}].
\end{aligned}$$

The three local maxima are as follows:

- For $q_I \leq 1-c-\frac{(1-2c)}{\sqrt{2}}$, $q_I = \min\{\frac{1-c}{2}, 1-c-\frac{(1-2c)}{\sqrt{2}}\}$.

- For $q_I \in [1-c-\frac{(1-2c)}{\sqrt{2}}, \overline{Q}+c-\frac{1}{2}]$, $q_I = 1-c-\frac{2\sqrt{1-3c}}{3}$.

- For $q_I \in [\overline{Q}+c-\frac{1}{2}, \overline{Q}]$, $q_I = \overline{Q}$.

Comparing these three local maxima, we find that it is optimal to set $q_I = \min\{\frac{1-c}{2}, 1-c-\frac{(1-2c)}{\sqrt{2}}\}$ for $\{c \in [\frac{1}{6}, \frac{\sqrt{2}-1}{2\sqrt{2}-1}]$ and $\overline{Q} > \frac{1-2c+2c^2}{2c} - \frac{(1-c)(1-2c)}{2\sqrt{2}c}\}$ or $\{c \in [\frac{\sqrt{2}-1}{2\sqrt{2}-1}, \frac{5-2\sqrt{2}-\sqrt{2(9-6\sqrt{2})}}{15-8\sqrt{2}}]$ and $\overline{Q} > \frac{1+2c-c^2}{8c}\}$ or $c \in [\frac{5-2\sqrt{2}-\sqrt{2(9-6\sqrt{2})}}{15-8\sqrt{2}}, 1]$. Otherwise it is optimal to set $q_I = \overline{Q}$.

**Third case:** $c \in [\frac{1}{2}, 1]$

For $\overline{Q} < \frac{1-c}{2}$, $\pi_I = q_I(1-\overline{Q}-c)$, which is maximized for $q_I = \overline{Q}$.

For $\overline{Q} \in [\frac{1-c}{2}, 1-c]$,

$$\begin{aligned}
\pi_I &= q_I\left(\frac{1-q_I-c}{2}\right) \quad \text{for} \quad q_I \leq 2\overline{Q}-1+c, \\
&= q_I(1-\overline{Q}-c) \quad \text{for} \quad q_I \in \left[2\overline{Q}-1+c, \overline{Q}\right]
\end{aligned}$$

The two local maxima are as follows:

- For $q_I \leq 2\overline{Q}-1+c$, the unconstrained local maximum would be $q_I = \frac{1-c}{2}$. If $\overline{Q} \in [\frac{3(1-c)}{4}, \frac{1}{2}]$, this interior solution applies. Otherwise, $q_I = 2\overline{Q}-1+c$.

- For $q_I \in \left[2\overline{Q} - 1 + c, \overline{Q}\right]$, the optimal solution is $q_I = \overline{Q}$.

Comparing the two local maxima, we find that the optimal strategy is to set $q_I = \frac{1-c}{2}$ for $\overline{Q} > \frac{1-c}{2}\left(1 + \frac{1}{\sqrt{2}}\right)$, and $q_I = \overline{Q}$ otherwise.

For $\overline{Q} > 1 - c$,

$$\begin{aligned} \pi_I &= q_I \left(\frac{1 - q_I - c}{2}\right) \quad \text{for} \quad q_I \leq 1 - c, \\ &= q_I(1 - q_I - c) \quad \text{for} \quad q_I \in [1 - c, \overline{Q}]. \end{aligned}$$

For $q_I \leq 1 - c$, the unconstrained optimum is $q_I = \frac{1-c}{2} < 1 - c$. For $q_I > 1 - c$, the unconstrained optimum would also be $q_I = \frac{1-c}{2}$, and therefore the local maximum is at $q_I = 1-c$. This latter strategy obviously leads to a lower profit than the former. Therefore, it is optimal to set $q_I = \frac{1-c}{2}$.