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An EOQ Model for a Deteriorating Item with Time Dependent Quadratic Demand and Variable Deterioration under Permissible Delay in Payment

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Abstract

In a recent paper, Khanra, Ghosh and Chaudhuri's (2011) presented an EOQ model for a deteriorating item with time dependent quadratic demand under permissible delay in payment. Deterioration considered in most of the EOQ models is constant, while in most of the practical cases the deterioration rate increases with time. This work is motivated by Khanra, Ghosh and Chaudhuri's (2011) paper extending their model to allow for a variable rate of deterioration when delay in payment is permissible. The time varying demand rate is taken to be a quadratic function of time. For settling the account, the model is developed under two circumstances: case-1: The credit period is less than or equal to the cycle time and case-2: the credit period is greater than the cycle time. A numerical example is provided to illustrate the model. Sensitivity analysis has also been conducted to study the effect of the parameters.

Keywords: EOQ model, quadratic demand, permissible delay in payment, variable deterioration rate

1 Introduction

The classical economic order quantity (EOQ) inventory models were developed under the assumptions of constant demand rate. Later, many researchers developed EOQ models taking linearly increasing or decreasing demand and exponentially increasing or decreasing demand. The study of inventory model comes into force in 1915. Harris [1] was the first mathematician who studied on inventory problems. He established the simple but famous EOQ formula that was also derived, apparently independently, by Wilson [2]. Gradually, demand of goods may vary with time or with price or with the instantaneous inventory level displayed in a market. In recent years, inventory modelers are working for finding the economic replenishment policy for an inventory system having time dependent demand pattern. Silver and Meal [3] first developed a heuristic approach to determine EOQ in the general case of a time varying-demand pattern. Donaldson [4] first come out with a full analytic solution of the inventory replenishment problem with a linear trend in demand pattern over a finite-time horizon. Wagner and Whitin [5], Ritchie [6, 7, 8], Kicks and Donaldson [9], Buchanan [10], Mitra et al. [11], Ritchie and Tsado [12], Goyal [13], Goyal et al. [14] etc. made valuable contributions in this direction. Researchers like Dave and Patel [15], Bahari-Kasani [16], Goswami and Chaudhuri [17], Chung and Ting [18], Hariga [19], Jalan, Giri and Chaudhuri [20], Giri, Goswami and Chaudhuri [21], Jalan and Chaudhuri [22] etc. developed the inventory models for deteriorating items with trended demand. Khanra, Ghosh and Chaudhuri [23] developed inventory model considering time-quadratic demand rate.

During a delay period (or trade credit period) suppliers usually offer their retailers a certain credit period with interest during the permissible delay period. Goyal [13] first developed the EOQ model under the conditions of permissible delay in payment. Shinn, Hwang and Sung [24], Chu, Chang and Lan [25], Chung, Chang and Yang [26] also entered Goyal's model for the case of deteriorating items. Other notable works in this direction come from Davis and Gaither [27], Mandal and Phaujder [28], Aggarwal and Jaggi [29], Chang and Dye [30], Salmeh, Abboud, Ei-Kassar and Ghattas [31], Chung and Lio [32], Sana and Chaudhuri [33] etc. Recently, Khanna, Ghosh and Chaudhuri [23] developed an EOQ model for a deteriorating item with quadratic demand rate under permissible delay in payment. In real life situations, we see that items like fruits and vegetables whose deterioration rate increases with time. Ghare and Schrader [34] were the first to use the concept of deterioration followed by Covert and Philip [35] who formulated an inventory model with variable rate of deterioration with two-parameter Weibull distribution.

This study is related to an EOQ model for a deteriorating item with time dependent quadratic demand and variable deterioration under permissible delay in payment, which is the extension of author's earlier paper having quadratic demand pattern constant rate of deterioration. The motivation behind developing an EOQ model in the present paper is to introduce time dependent rate of deterioration when the demand is taken as quadratic function of time. The proposed inventory model is based on deteriorating items like fruits and vegetables whose deterioration rate increases with time. Among the various time-varying demands in EOQ models, the more realistic demand approach is to consider a quadratic demand rate along with variable rate of deterioration. For setting the account, the model is developed under two circumstances: case-1: The credit period is less than or equal to the cycle time and case-2: the credit period is greater than the cycle time. Main emphasis is laid on working out on exact solution for the model. An example is provided which stands in support of the developed model. The sensitivity of the solution with the changes of the values of the parameters associated with the model is discussed.

2 Assumptions

The following assumptions are made in developing the model:

(i) The demand rate for the item is represented by a quadratic and continuous function of time.

(ii) Replenishment rate is infinite, i. e., replenishment rate is instantaneous.

(iii) Shortage is not allowed.

(iv) The deterioration rate is variable rate of deterioration on the on-hand inventory per unit time and there is no repair or replenishment of the deteriorated items within the cycle.

(v) Time horizon is infinite.

3 Notations

The following notations have been used in developing the model:

(i) The time-dependent demand rate is $D(t) = a + bt + ct^2$, $a > 0$, $b \neq 0$ & $c \neq 0$. Here a is the initial rate of demand, b is the rate with which the demand rate increases. The rate of change in the demand rate itself changes at a rate c.

(ii) p is the unit purchase cost of item.

(iii) h_p is the inventory holding cost (excluding interest charges) per rupee of unit purchase cost per unit time.

(iv) $\theta(t) = \theta t$ where $0 < \theta << 1$ is the variable rate of deterioration of an item.

(v) K is the replenishment cost.

(vi) I_p is the interest charges per rupee investment in stock per year.

(vii) I_e is the interest earned per rupee in a year.

(viii) t_1 is permissible period (in year) of delay in settling the accounts with the supplier.

(ix) T is the time interval (in year between two successive orders).

4 Mathematical Formulation and Solution of the Model

The instantaneous inventory level $I(t)$ at any time t during the cycle time t is governed by the following differential equation

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(t), 0 \leq t \leq T, \tag{4.1}$$

where $I(0) = I_0$, $I(T) = 0$ and $D(T) = a + bt + ct^2$.

The solution of Eq. (4.1) is

$$I(t) = \left[a\left(T + \frac{\theta T^3}{6}\right) + b\left(\frac{T^2}{2} + \frac{\theta T^4}{8}\right) + c\left(\frac{T^3}{3} + \frac{\theta T^5}{10}\right)\right] e^{-\frac{\theta t^2}{2}}$$

$$-\left[a\left(t + \frac{\theta t^3}{6}\right) + b\left(\frac{t^2}{2} + \frac{\theta t^4}{8}\right) + c\left(\frac{t^3}{3} + \frac{\theta t^5}{10}\right)\right] e^{-\frac{\theta t^2}{2}}, 0 \le t \le T. \tag{4.2}$$

(neglecting the higher power of θ as $0 < \theta << 1$)

If $c = 0$, then Eq. (4.2) represents the instantaneous inventory level at any time t for the linear demand rate. Also, putting $b = c = 0$ in Eq. (4.2) represents the instantaneous inventory level at any time t for the constant demand rate.

Thus, the initial order quantity is

$$I_0 = I(0) = a\left(T + \frac{\theta T^3}{6}\right) + b\left(\frac{T^2}{2} + \frac{\theta T^4}{8}\right) + c\left(\frac{T^3}{3} + \frac{\theta T^5}{10}\right). \tag{4.3}$$

The total demand during the cycle period $[0, T]$ is

$$\int_0^T D(t)dt = \int_0^T (a + bt + ct^2)dt = T\left(a + \frac{bT}{2} + \frac{cT^2}{3}\right).$$

The number of deteriorating units is

$$I_0 - \int_0^T D(t)dt = \frac{\theta T^3}{120}\left(20a + 15bT + 12cT^2\right).$$

The deterioration cost for the cycle $[0, T] = p\times$ (number of deteriorated units)

$$= \frac{p\theta T^3}{120}\left(20a + 15bT + 12cT^2\right). \tag{4.4}$$

The total holding cost for the cycle $[0, T]$ is

$$HC = h\int_0^T I(t)dt = h\int_0^T \left[a\left(T + \frac{\theta T^3}{6}\right) + b\left(\frac{T^2}{2} + \frac{\theta T^4}{8}\right) + c\left(\frac{T^3}{3} + \frac{\theta T^5}{10}\right)\right] e^{-\frac{\theta t^2}{2}} dt$$

$$-h\int_0^T \left[a\left(t + \frac{\theta t^3}{6}\right) + b\left(\frac{t^2}{2} + \frac{\theta t^4}{8}\right) + c\left(\frac{t^3}{3} + \frac{\theta t^5}{10}\right)\right] e^{-\frac{\theta t^2}{2}} dt$$

$$= h\left[a\left(\frac{T^2}{2} + \frac{\theta T^4}{12}\right) + b\left(\frac{T^3}{3} + \frac{\theta T^5}{15}\right) + c\left(\frac{T^4}{4} + \frac{\theta T^6}{18}\right)\right] \tag{4.5}$$

where $h = h_p p$.
(neglecting the higher power of θ as $0 < \theta << 1$)

Case 1: Let $T > t_1$.

Since the interest is payable during the time $(T - t_1)$, the interest payable in any cycle $[0, T]$ is

$$IP_1 = pI_p \int_{t_1}^{T} I(t)dt$$

$$= pI_p \int_{t_1}^{T} \left[a\left(T + \frac{\theta T^3}{6}\right) + b\left(\frac{T^2}{2} + \frac{\theta T^4}{8}\right) + c\left(\frac{T^3}{3} + \frac{\theta T^5}{10}\right) \right] e^{-\frac{\theta t^2}{2}} dt$$

$$-pI_p \int_{t_1}^{T} \left[a\left(t + \frac{\theta t^3}{6}\right) + b\left(\frac{t^2}{2} + \frac{\theta t^4}{8}\right) + c\left(\frac{t^3}{3} + \frac{\theta t^5}{10}\right) \right] e^{-\frac{\theta t^2}{2}} dt$$

$$= pI_p \left[a\left(T + \frac{\theta T^3}{6}\right) + b\left(\frac{T^2}{2} + \frac{\theta T^4}{8}\right) + c\left(\frac{T^3}{3} + \frac{\theta T^5}{10}\right) \right] \left[(T - t_1) - \frac{\theta(T^3 - t_1^3)}{6} \right]$$

$$-pI_p \left[a\left(\frac{T^2 - t_1^2}{2} - \frac{\theta(T^4 - t_1^4)}{12}\right) + b\left(\frac{T^3 - t_1^3}{6} - \frac{\theta(T^5 - t_1^5)}{40}\right) \right]$$

$$-pI_p \left[c\left(\frac{T^4 - t_1^4}{12} - \frac{\theta(T^6 - t_1^6)}{90}\right) \right]. \tag{4.6}$$

(neglecting the higher power of θ as $0 < \theta << 1$)

Interest earned in the cycle period $[0, T]$ is

$$IE_1 = pI_e \int_0^T tD(t)dt = pI_e \int_0^T t(a + bt + ct^2)dt$$

$$= \frac{pI_e T^2}{12}(6a + 4bT + 3cT^2). \tag{4.7}$$

Total variable cost per cycle = replenishment cost + inventory holding cost + deterioration cost + interest payable during the permissible period − interest earned during the cycle.

So, the total variable cost per cycle per unit time is

$$Z_1(T) = \frac{K}{T} + \frac{h}{T}\left[a\left(\frac{T^2}{2} + \frac{\theta T^4}{12}\right) + b\left(\frac{T^3}{3} + \frac{\theta T^5}{15}\right) + c\left(\frac{T^4}{4} + \frac{\theta T^6}{18}\right) \right] + \frac{p\theta T^2}{120}\left(20a + 15bT + 12cT^2\right)$$

$$+\frac{pI_p}{T}\left[a\left(T+\frac{\theta T^3}{6}\right)+b\left(\frac{T^2}{2}+\frac{\theta T^4}{8}\right)+c\left(\frac{T^3}{3}+\frac{\theta T^5}{10}\right)\right]\left[(T-t_1)-\frac{\theta(T^3-t_1^3)}{6}\right]$$

$$-\frac{pI_p}{T}\left[a\left(\frac{T^2-t_1^2}{2}-\frac{\theta(T^4-t_1^4)}{12}\right)+b\left(\frac{T^3-t_1^3}{6}-\frac{\theta(T^5-t_1^5)}{40}\right)+c\left(\frac{T^4-t_1^4}{12}-\frac{\theta(T^6-t_1^6)}{90}\right)\right]$$

$$-\frac{pI_eT}{12}(6a+4bT+3cT^2). \tag{4.8}$$

Our aim is to find minimum variable cost per unit time.

The necessary and sufficient conditions to minimize $Z_1(T)$ for a given value of t_1 are respectively $\frac{dZ_1(T)}{dT}=0$ and $\frac{d^2Z_1(T)}{dT^2}>0$.

Now $\frac{dZ_1(T)}{dT}=0$ gives the following non-linear equation in T:

$$\frac{dZ_1(T)}{dT}=(a+bT+cT^2)\left[h\left(1+\frac{\theta T^3}{3}\right)+\frac{p\theta T}{2}\right]$$

$$+\frac{(a+bT+cT^2)}{T}\left[pI_p\left(1+\frac{\theta T^2}{2}\right)\left(T-t_1-\frac{\theta(T^3-t_1^3)}{6}\right)-pI_eT\right]-\frac{Z_1(T)}{T}=0. \tag{4.9}$$

(neglecting the higher power of θ as $0<\theta<<1$)

To get the optimal cycle length $T=T_1$, we have to solve Eq. (4.9) provided it satisfies the following condition $\frac{d^2Z_1(T)}{dT^2}>0$.

The EOQ in this case is as follows:

$$I_0(T_1)=a\left(T_1+\frac{\theta T_1^3}{6}\right)+b\left(\frac{T_1^2}{2}+\frac{\theta T_1^4}{8}\right)+c\left(\frac{T_1^3}{3}+\frac{\theta T_1^5}{10}\right). \tag{4.10}$$

The minimum annual variable cost $Z_1(T_1^*)$ is obtained from Eq. (4.8) for $T=T_1$.

Case 2: Let $T<t_1$.

In this case, the customer earns interest on the sales revenue up to the permissible delay period and no interest is payable during the period for the item kept in stock.

Interest earned for the period $[0,T]$ is

$$pI_e\int_0^T tD(t)dt=\frac{pI_eT^2}{12}\left(6a+4bT+3cT^2\right). \tag{4.11}$$

Interest earned for the permissible delay period $[T,t_1]$ is

$$pI_e(t_1-T)\int_0^T D(t)dt=\frac{pI_eT(t_1-T)}{6}(6a+3bT+2cT^2). \tag{4.12}$$

Hence total interest earned during the cycle = Interest earned for the period $[0, T]$ + Interest earned for the permissible delay period $[T, t_1]$, i. e. ,

$$IE_2 = pI_e \int_0^T tD(t)dt + pI_e(t_1 - T) \int_0^T D(t)dt$$

$$= pI_eT \left[\left(a + \frac{bT}{2} + \frac{cT^2}{3} \right) t_1 - \left(\frac{aT}{2} + \frac{bT^2}{6} + \frac{cT^3}{12} \right) \right] \tag{4.13}$$

In this case, the total variable cost per cycle = replenishment cost + inventory holding cost + deteriorating cost − interest earned during the cycle.

Hence, the total variable cost per unit time is

$$Z_2(T) = \frac{K}{T} + \frac{h}{T} \left[a \left(\frac{T^2}{2} + \frac{\theta T^4}{12} \right) + b \left(\frac{T^3}{3} + \frac{\theta T^5}{15} \right) + c \left(\frac{T^4}{4} + \frac{\theta T^6}{18} \right) \right]$$

$$-pI_e \left[\left(a + \frac{bT}{2} + \frac{cT^2}{3} \right) t_1 - \left(\frac{aT}{2} + \frac{bT^2}{6} + \frac{cT^3}{12} \right) \right]$$

$$+\frac{p\theta T^2}{120}(20a + 15bT + 12cT^2). \tag{4.14}$$

As before, we have to minimize $Z_2(T)$ for a given value of t_1.

The necessary and sufficient condition to minimize $Z_2(T)$ for a given value of t_1 are respectively $\frac{dZ_2(T)}{dT} = 0$ and $\frac{d^2Z_2(T)}{dT^2} > 0$.

Now $\frac{dZ_2(T)}{dT} = 0$ gives the following non-linear equation in T:

$$\frac{dZ_2(T)}{dt} = (a + bT + cT^2) \left[h \left(1 + \frac{\theta T^2}{3} \right) + \frac{p\theta T}{2} \right]$$

$$-\frac{pI_e}{T} \left[(a + bT + cT^2)t_1 - \left(aT + \frac{bT^2}{2} + \frac{cT^3}{3} \right) \right] - \frac{Z_2(T)}{T} = 0. \tag{4.15}$$

The EOQ in this case is as follows:

$$I_0(T_2) = a \left(T_2 + \frac{\theta T_2^3}{6} \right) + b \left(\frac{T_2^2}{2} + \frac{\theta T_2^4}{8} \right) + c \left(\frac{T_2^3}{3} + \frac{\theta T_2^5}{10} \right).$$

The minimum annual cost $Z_2(T_2^*)$ is obtained from equation (4.14) for $T = T_2$.

Case 3: Let $T = t_1$.

For $T = t_1$, both the cost function $Z_1(T)$ and $Z_2(T)$ are identical and the cost function is obtained by putting $T = t_1$ either in Eq.(4.8) or in Eq.(4.14) and is given by

$$Z_3(t_1) = \frac{K}{t_1} + \frac{h}{t_1}\left[a\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{12}\right) + b\left(\frac{t_1^3}{3} + \frac{\theta t_1^5}{15}\right) + c\left(\frac{t_1^4}{4} + \frac{\theta t_1^6}{18}\right)\right]$$

$$+\frac{p\theta t_1^2}{120}(20a + 15bt_1 + 12ct_1^2) - \frac{pI_e t_1}{12}(6a + 4bt_1 + 3ct_1^2). \tag{4.16}$$

The EOQ in this case is as follows:

$$I_0(t_1) = a\left(t_1 + \frac{\theta t_1^3}{6}\right) + b\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{8}\right) + c\left(\frac{t_1^3}{3} + \frac{\theta t_1^5}{10}\right). \tag{4.17}$$

5 Solution Procedure for Economic Order Quantity: Algorithm

The following steps are to be followed to find the optimum cost and economic order order quantity unless $T = T_1$.

Step 1: Determine T_1^* from Eq. (4.9). If $T_1^* > t_1$, evaluate $Z_1(T_1^*)$ from Eq. (4.8).

step 2: Determine T_2^* from Eq. (4.15). If $T_2^* < t_1$, evaluate $Z_2(T_2^*)$ from Eq. (4.14).

step 3: If the condition $T_1^* > t_1 > T_2^*$ is satisfied, then go to step 4. Otherwise go to step 5.

step 4: Compare $Z_1(T_1^*)$ & $Z_2(T_2^*)$ and find the minimum cost.

step 5: If the condition $T_1^* > t_1$ is satisfied but $T_2^* > t_1$, then $Z_1(T_1^*)$ is the minimum cost , else if $T_1^* < t_1$ but $T_2^* < t_1$ then, $Z_2(T_2^*)$is the minimum cost.

step 6: Compare $I_0^*(t_1)$ or $I_0^*(t_2)$ for the respective minimum cost .

6 Numerical Example

In this section, we provide a numerical example to illustrate the above theory.

Example 1: Let us consider the values of the system as $a = 1000$ units per year,$b = 150$ units per year, $c = 15$ units per year, $K = Rs.200$ per order, $I_p = 0.15$ per year, $I_e = 0.13$ per year, $h = Rs.0.12$ per year, $p = Rs.20$ per unit, $\theta = 0.20$ and $t_1 = 0.25$ year.

Solving Eq.(4.9), we have $T_1^* = 0.535024$ year and the minimum average cost is $Z_1(T_1^*) = Rs.121.76$.

Again, solving Eq.(4.15), we have $T_2^* = 0.331048$ year and the minimum average cost is $Z_2(T_2^*) = Rs.471.718$.

Here $T_2^* > t_1$ this contradicts Case-II. Only Case-I holds as $T_1^* > t_1$. Hence the minimum average cost in this case is $Z_1(T_1^*) = Rs.121.76$ where the optimum cycle length is $T_1^* = 0.535024$ year.

The economic order quantity is given by $I_0^*(T_1^*) = Rs.562.684$.

7 Sensitivity Analysis

We now study the effects of changes of values of the system parameters $a, b, c, K, I_p, I_e, h, p, \theta$ and t_1 on the optimal total cost and number of reorder. The sensitivity analysis is performed by changing each of parameters by $+50\%, +10\%, -10\%$ and -50% taking one parameter at a time and keeping the remaining parameters unchanged.

The analysis is based on the example-1 and the results are shown in the Table-1. The following points are observed.

(i) T_1^*, T_2^*, $Z_1(T_1^*)$ & $Z_2(T_2^*)$ decrease with increase in the value of the parameter a. Here T_1^*, T_2^* & $Z_2(T_2^*)$ are moderately sensitive to change in a while $Z_1(T_1^*)$ is highly sensitive to change in a.

(ii) T_1^*, T_2^*, $Z_1(T_1^*)$ & $Z_2(T_2^*)$ decrease with increase in the value of the parameter b. Here T_1^*, T_2^*, $Z_1(T_1^*)$ & $Z_2(T_2^*)$ are all lowly sensitive to change in b.

(iii) T_1^*, $Z_1(T_1^*)$ & $Z_2(T_2^*)$ decrease while T_2^* increases with increase in the value of the parameter c. Here T_1^*, T_2^*, $Z_1(T_1^*)$ & $Z_2(T_2^*)$ are all insensitive to change in c.

(iv) T_1^*, T_2^*, $Z_1(T_1^*)$ & $Z_2(T_2^*)$ increase with increase in the value of the parameter K. Here T_1^* & T_2^* are moderately sensitive to change in K while $Z_1(T_1^*)$ & $Z_2(T_2^*)$ are highly sensitive to change in K.

(v) T_1^* decreases and $Z_1(T_1^*)$ increases while T_2^* & $Z_2(T_2^*)$ remain same with increase in the value of the parameter I_p. Here T_1^* is moderately sensitive, $Z_1(T_1^*)$ is highly sensitive to change in I_p while T_2^* & $Z_2(T_2^*)$ are insensitive to change in I_p.

(vi) T_1^* increases while T_2^*, $Z_1(T_1^*)$ & $Z_2(T_2^*)$ decrease with increase in the value of parameter I_e. Here T_1^*, T_2^* & $Z_2(T_2^*)$ are moderately sensitive to change in I_e while $Z_1(T_1^*)$ is highly sensitive to change in I_e.

(vii) T_1^* & T_2^* decrease while $Z_1(T_1^*)$ & $Z_2(T_2^*)$ increase with increase in the value of the parameter h. Here T_1^* & $T_2^*$are lowly sensitive to change in h while $Z_1(T_1^*)$ & $Z_2(T_2^*)$ are moderately sensitive to change in h.

(viii) T_1^*, T_2^*, $Z_1(T_1^*)$ & $Z_2(T_2^*)$ decrease with increase in the value in the parameter p. Here T_1^*, T_2^* & $Z_2(T_2^*)$ are moderately sensitive to change in p while $Z_1(T_1^*)$ is highly sensitive to change in p.

(ix) T_1^* & T_2^* decrease while $Z_1(T_1^*)$ & $Z_2(T_2^*)$ increase with increase in the value of the parameter θ. Here T_1^*, T_2^* & $Z_2(T_2^*)$ are moderately sensitive to change in θ while $Z_1(T_1^*)$ is highly sensitive to change in θ.

(x) T_1^* & T_2^* increase while $Z_1(T_1^*)$ & $Z_2(T_2^*)$ decrease with increase in the value of the parameter t_1. Here T_1^* is moderately sensitive, T_2^* is lowly sensitive to change in t_1 while $Z_1(T_1^*)$ & $Z_2(T_2^*)$ are highly sensitive to change in t_1.

Table 1: Sensitivity analysis

parameter	% Change	T_1^*	$Z_1(T_1^*)$	T_2^*	$Z_2(T_2^*)$	Remark	Solution
a	50	0.491045	...	...	...	...	...
	10	0.523973	97.1051	...	...	$T_1^* > t_1$	$Z_1(T_1^*)$
	-10	0.547718	145.628	0.346299	483.035	$T_1^* > t_1$	$Z_1(T_1^*)$
	-50	0.626004	228.864	0.440076	491.969	$T_1^* > t_1$	$Z_1(T_1^*)$
b	50	0.532152	117.317	0.330155	468.896	$T_1^* > t_1$	$Z_1(T_1^*)$
	10	0.534434	120.875	0.330867	471.155	$T_1^* > t_1$	$Z_1(T_1^*)$
	-10	0.535622	122.643	0.331229	472.281	$T_1^* > t_1$	$Z_1(T_1^*)$
	-50	0.538098	126.154	0.331968	474.53	$T_1^* > t_1$	$Z_1(T_1^*)$
c	50	0.534902	121.636	0.33105	471.643	$T_1^* > t_1$	$Z_1(T_1^*)$
	10	0.534999	121.735	0.331048	471.703	$T_1^* > t_1$	$Z_1(T_1^*)$
	-10	0.535048	121.785	0.331047	471.733	$T_1^* > t_1$	$Z_1(T_1^*)$
	-50	0.535146	121.885	0.331045	471.793	$T_1^* > t_1$	$Z_1(T_1^*)$
K	50	0.59099	299.165	0.394302	747.148	$T_1^* > t_1$	$Z_1(T_1^*)$
	10	0.547173	158.71	0.345041	530.879	$T_1^* > t_1$	$Z_1(T_1^*)$
	-10	0.522287	83.941	0.316171	409.92	$T_1^* > t_1$	$Z_1(T_1^*)$
	-50	0.463718	...	...	...	$T_1^* > t_1$	...
I_p	50	0.451271	215.611	0.331048	471.718	$T_1^* > t_1$	$Z_1(T_1^*)$
	10	0.513535	144.883	0.331048	471.718	$T_1^* > t_1$	$Z_1(T_1^*)$
	-10	0.559861	95.6372	0.331048	471.718	$T_1^* > t_1$	$Z_1(T_1^*)$
	-50	0.707724	...	0.331048	471.718	$T_1^* > t_1$	$Z_2(T_2^*)$
I_e	50	0.704461	...	...	...	$T_1^* > t_1$	...
	10	0.562188	46.3757	...	...	$T_1^* > t_1$	$Z_1(T_1^*)$
	-10	0.51057	193.406	0.341698	493.928	$T_1^* > t_1$	$Z_1(T_1^*)$
	-50	0.434042	449.661	0.395231	566.946	$T_1^* > t_1$	$Z_1(T_1^*)$
h	50	0.528993	138.769	0.328515	481.983	$T_1^* > t_1$	$Z_1(T_1^*)$
	10	0.533805	125.179	0.330537	473.778	$T_1^* > t_1$	$Z_1(T_1^*)$
	-10	0.536249	118.334	0.331561	469.655	$T_1^* > t_1$	$Z_1(T_1^*)$
	-50	0.541215	104.542	0.333635	461.37	$T_1^* > t_1$	$Z_2(T_2^*)$
p	50	0.493901	...	...	...	$T_1^* > t_1$	...
	10	0.524581	92.812	0.318017	455.248	$T_1^* > t_1$	$Z_1(T_1^*)$
	-10	0.547131	149.999	0.345962	485.735	$T_1^* > t_1$	$Z_1(T_1^*)$
	-50	0.625549	251.372	0.439352	506.555	$T_1^* > t_1$	$Z_1(T_1^*)$
θ	50	0.478985	212.568	0.314673	507.729	$T_1^* > t_1$	$Z_1(T_1^*)$
	10	0.521537	141.697	0.327449	479.227	$T_1^* > t_1$	$Z_1(T_1^*)$
	-10	0.550136	100.68	0.334841	464.037	$T_1^* > t_1$	$Z_1(T_1^*)$
	-50	0.637499	0.156279	...	...	$T_1^* > t_1$	$Z_1(T_1^*)$
t_1	50	0.608972	...	0.333074	138.445	$T_1^* > t_1$	$Z_2(T_2^*)$
	10	0.548769	80.2409	0.33145	405.068	$T_1^* > t_1$	$Z_1(T_1^*)$
	-10	0.52198	165.961	0.330646	538.367	$T_1^* > t_1$	$Z_1(T_1^*)$
	-50	0.478973	373.229	0.329054	804.94	$T_1^* > t_1$	$Z_1(T_1^*)$

... indicates the infeasible solution.

8 Conclusion

This study can help substantially retailers or buyers in deciding their payment time, considering the benefits of the permissible delay in payment. The model considered above is suited for items having variable deterioration rate, earlier models have considered items having constant rate of deterioration. This model can be used for items like fruits and vegetables whose deterioration rate increases with time. With the help of this model total cost is obtained. The practical aspects of inventory management like opportunity cost and the effect of permissible delay in payment are also considered. The total cost obtained then can be used to obtain an average inventory variable cost, which can be optimized using calculus techniques. A numerical illustration proves the applicability of the suggested model.

The suggested model can be extended for items having constant demand, linear increasing demand, stock dependent demand, price dependent demand or power demand. This model can further be extended a three-parameter Weibull distribution or Gamma distribution. This study will act as a catalyst for the study of permissible delay in payment.

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