**L. H. BACK**
Member of Technical Staff. Mem. ASME

**E. J. ROSCHKE**
Member of Technical Staff.

Jet Propulsion Laboratory,
California Institute of Technology,
Pasadena, Calif.

# Shear-Layer Flow Regimes and Wave Instabilities and Reattachment Lengths Downstream of an Abrupt Circular Channel Expansion[1]

*An experimental investigation of water flow through an abrupt circular-channel expansion is described over a Reynolds number range between 20 and 4200. The shear layer between the central jet and the reverse flow region along the wall downstream behaved differently in the various flow regimes that were observed. With increasing Reynolds number these regimes changed progressively from a laminar flow to an unstable vortex sheetlike flow and then to a more random fluctuating flow. The distance between the step and the reattachment location downstream correspondingly increased, reached a maximum, and then decreased. Of particular significance are the shear layer wave instabilities observed in the shear flow and their relationship to rettachment which apparently has not received much attention previously. Visual observations aided in understanding the results.*

## Introduction

The purpose of this investigation was to study the flow field downstream of an abrupt expansion in a circular channel and in particular to examine the growth of the shear layer between the central jet and the reverse flow along the wall in the region of separated flow. It is is the lateral growth or spread of the shear layer that determines the location of flow reattachment and thus the axial extent of the flow separation region. At relatively small Reynolds numbers for which the flow is laminar, the reattachment location has been found to move downstream with increasing Reynolds number, e.g., see Macagno and Hung [1][2] for an abrupt circular channel expansion, or Goldstein, et al. [2], and Mueller and O'Leary [3] for flow over backward-facing steps in channels. For example, the measurements by Macagno and Hung indicate a reattachment location of about 12 step heights at the largest Reynolds number $\mathrm{Re}_{D_0}$ of 135 for which their measurements were made; $\mathrm{Re}_{D_0}$ was based on the upstream tube diameter. Yet, it is known that for larger Reynolds number turbulent flows through abrupt circular channel expansions [4–9], through abrupt rectangular channel expansions [10], and over backward-facing steps in channels [11, 12], the reattachment lengths are smaller, varying between 6–12 step heights according to different investigators. In the turbulent regime there is a relatively small influence of either Reynolds number or upstream to downstream diameter ratio $D_0/D$ on the reattachment location in terms of step heights [9] as inferred from heat transfer measurements.

Clearly, there is an intermediate regime where the increase in the reattachment location with Reynolds number must reach a maximum and then decrease to merge eventually with the higher Reynolds number turbulent flow results. A hint that indeed this must be the case is afforded by the flow observations of Meisner and Rushmer [13] in an abrupt rectangular channel expansion which showed that the reattachment location moved back toward

---

[1] This paper presents the results of one phase of research carried out in the Propulsion Research and Advanced Concepts Section at the Jet Propulsion Laboratory, California Institute of Technology, under Contract No. NAS 7-100, sponsored by the National Aeronautics and Space Administration.

Contributed by the Applied Mechanics Division and presented at the Applied Mechanics Summer Conference, University of California, La Jolla, Calif., June 26–28, 1972, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion on this paper should be addressed to the Editorial Department, ASME, United Engineering Center, 345 East 47th Street, New York, N. Y. 10017, and will be accepted until October 20, 1972. Discussion received after this date will be returned. Manuscript received by ASME Applied Mechanics Division, February 8, 1971; final revision, July 22, 1971. Paper No. 72-APM-2.

[2] Numbers in brackets designate References at end of paper.

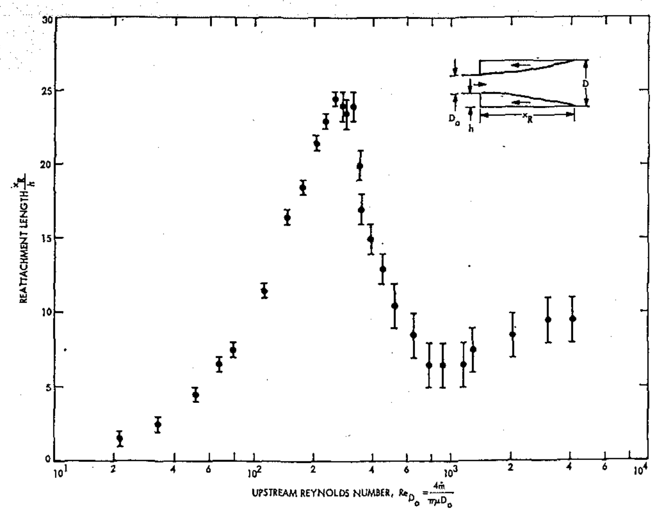


**Fig. 1 Reattachment lengths; $D_0$ = 0.375 in., $D$ = 0.975 in., and $h$ = 0.30 in.**

the step as the flow rate was increased. This trend has also been observed by Moore for the reattachment of a laminar boundary layer downstream of a backward-facing step [14]. The authors became aware of the small amount of information available on reattachment in the intermediate flow regime in connection with an investigation in a hot gas facility [15], and this in turn provided the motivation for this investigation.

In the present investigation reattachment lengths were measured and flow regimes were revealed by visual observation in water flows. The Reynolds numbers $\mathrm{Re}_{D_0}$ ranged from 20–4200, which spanned the laminar to turbulent shear flow regime. The Reynolds number $\mathrm{Re}_{D_0}$ was based on the diameter of the flow channel located just upstream of the abrupt expansion. A few applications where the present results are believed to be useful include fluid flow in the human body and in other mammals, and in modeling of high temperature, relatively low Reynolds number, gas flows.

## System and Measurements

The system consisted of an upstream chamber 1.25-in. dia where water was injected radially, a contraction section with a conical half angle of 30 deg, a short length of 0.375-in-dia tube (1.0-dia long), an abrupt transition to a 0.975-in-dia tube (9.7-dia long), and a convergent nozzle. This particular system was a half-scale model of a hot gas facility [15] in which argon was heated by an electric arc discharge between the tip of the centerbody (cathode) in the contraction section and the 0.375-in-dia tube (anode). Because of the contraction section and short tube length the boundary layer was expected to be relatively thin at the end of the short tube compared to the tube radius. The wake from the center body was also confined by the contraction section. The flow at the entrance of the abrupt channel expansion therefore is believed to have been nearly uniform. The step $h$ in the channel expansion was 0.30 in.

Small holes (0.030-in. dia) were located along the larger diameter tube one step height apart. Reattachment locations were determined by slowly metering dye through these holes and observing which direction the dye moved, toward the step in the reverse flow region or downstream in the reattachment region. Dye injected through these holes and other holes located upstream of the abrupt expansion was also used for flow visualization studies. In this connection it should be noted that reattachment locations were not inferred by visual observation of the dye filaments that were introduced into the flow upstream.

To accurately measure the lower flow rates ($\mathrm{Re}_{D_0}$ from 20 to about 350) the water discharge was collected and weighed over a time interval. At higher flow rates the water flow rate was measured with a calibrated rotameter. Water temperature, measured by insertion of a thermometer in the water discharge, ranged from 68–72 deg F.

## Results

Reattachment lengths are shown in Fig. 1 for the range of Reynolds numbers investigated. On an overall basis they are seen to first increase with Reynolds number, reach a maximum value, decrease to a minimum value, and then increase again somewhat. This variable behavior for which the flow reattaches at the same location at different Reynolds numbers, is associated with the various flow regimes.

At lower Reynolds numbers the reattachment location moves downstream with increasing Reynolds number as expected if the shear layer between the central jet and reverse flow along the wall spreads by *molecular diffusion of momentum*. For example, for laminar mixing of a stream with uniform velocity $u_0$ with a fluid at rest, velocity profiles depend upon a similarity variable [16]. In this case, the half width of the shear layer $b$ is given by

$$\frac{b}{x}\left(\frac{u_0 x}{\nu}\right)^{1/2} = c$$

where $c$ = constant, $x$ is the downstream distance, and $\nu$ is the kinematic viscosity. This relation can be used to estimate the dependence of reattachment location on Reynolds number by taking $b$ equal to the step height $h$ at the reattachment point $x = x_R$ to give

$$\frac{x_R}{h} = \frac{1}{c^2}\left(\frac{u_0 h}{\nu}\right) = \frac{1}{c^2}\left(\frac{h}{D_0}\right)\left(\frac{u_0 D_0}{\nu}\right) \qquad (1)$$

Mixing theory indicates a linear variation of reattachment location in terms of step heights with upstream Reynolds number $\frac{u_0 D_0}{\nu}$ which is $\frac{4\dot{m}}{\pi\mu D_0}$ for flow through a circular tube; $\dot{m}$ is the mass flow rate, $\mu$ the viscosity, and $D_0$ the tube diameter upstream of the expansion. The factor $h/D_0$ depends upon the abrupt expansion configuration. In this connection the present experimental results at lower Reynolds numbers are seen to better advantage in Fig. 2, where they are shown in linear coördinates. The reattachment location in terms of step heights increases with Reynolds number in a manner not basically different from that indicated by mixing theory.

The lateral growth of the laminar shear layer at these lower Reynolds numbers can be seen in Fig. 3(*a*). Dye injected continuously just downstream of the step flows back toward the step and then is entrained into the shear layer near the step to reveal its growth. There is difficulty in determining precisely the dependence of the half width of the shear layer spreading toward the wall at this relatively low Reynolds number, especially in the region farther along the tube where the dye which is slightly heavier than water begins to bend downward because of gravita-

## Nomenclature

$b$ = half width of shear layer
$c$ = constant
$D$ = tube diameter
$D_0$ = tube diameter upstream of expansion
$h$ = step height
$\dot{m}$ = mass flow rate
$\mathrm{Re}_{D_0}$ = upstream Reynolds number
$u_0$ = upstream flow speed
$u_r$ = reverse flow speed
$x$ = downstream distance
$x_R$ = reattachment length
$\mu$ = viscosity
$\nu$ = kinematic viscosity

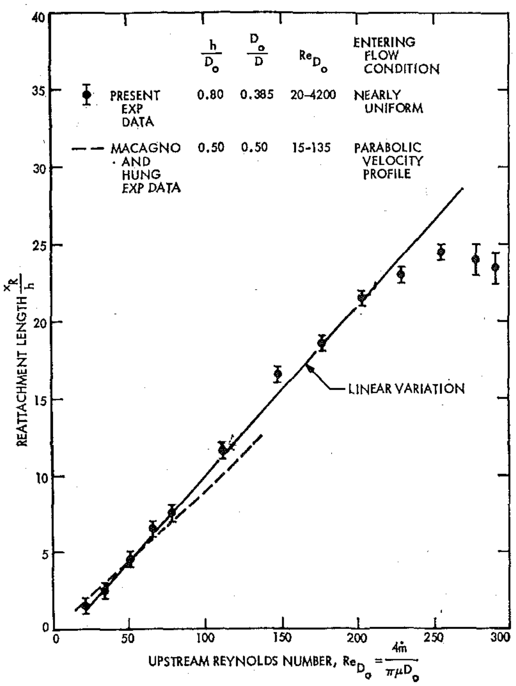


**Fig. 2 Reattachment lengths, laminar shear-layer regime**

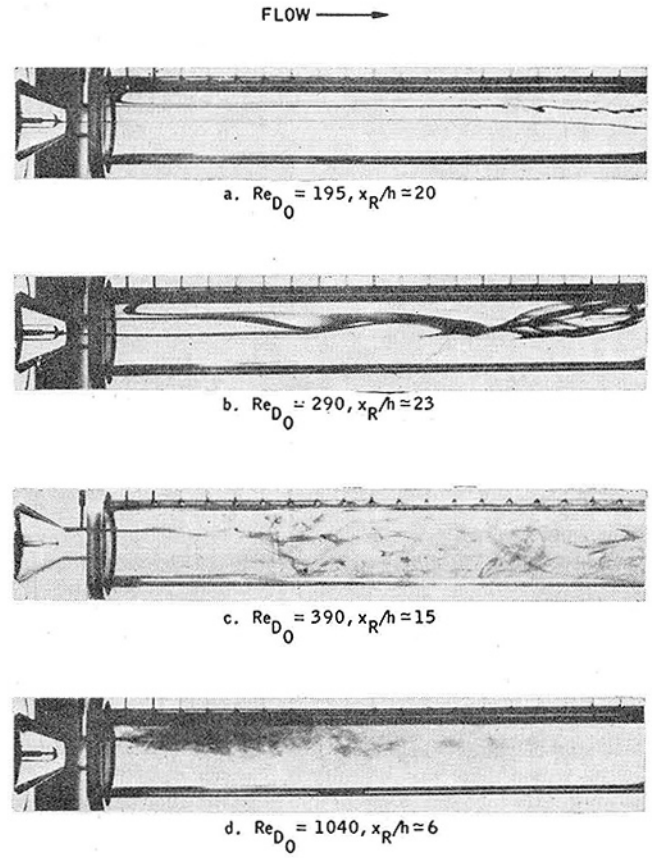


**Fig. 3 Visual observation of shear-layer growth**

tional effects. Dye slowly metered into the flow in the short tube preceding the abrupt channel expansion does not yield any information on the shear layer growth in this flow regime. The perturbed condition of this filament farther downstream is associated with gravitational effects. Of note is the negligible disturbance produced by the center body upstream in the contraction section as revealed by the undisturbed state of the dye filament introduced at the tip of the center body.

Also shown in Fig. 2 by the dashed curve are the measurements by Macagno and Hung which span a relatively small Reynolds number range. The slope of their results appears to be less than that of the present measurements. This difference is consistent with that inferred from laminar mixing theory since $h/D_0$ was smaller in their circular channel expansion than the value employed in this investigation. However, the difference in the slopes of the two sets of data is less than would be indicated by the configuration factor $h/D_0$ in equation (1). Differences between the entering flow profiles also may have influenced the results. It is not clear in which direction the subsequent shear layer growth would change, for example, as might be obtained by using a more appropriate velocity in equation (1). For an entering parabolic velocity profile (Macagno and Hung) the centerline velocity is twice that for a nearly uniform flow (this investigation) but velocities are less near the wall.

It should be mentioned that Macagno and Hung also found very good agreement between numerical solutions of the Navier-Stokes equations and their measurements. The predicted reattachment lengths increased with Reynolds number nearly linearly, as also indicated by mixing theory. Numerical solutions of the Navier-Stokes equation by Kawaguti [17] for an abrupt rectangular channel expansion also indicate a nearly linear increase. Simple mixing theory provides no information on the details of the reattachment process, but only indicates the position at which the thickening shear layer would intersect the wall. That part of the shear layer that does not have enough momentum to overcome the pressure rise associated with reattachment is then turned rearward to form the reverse flow [18].

As the Reynolds number increases, the reattachment lengths deviate from the linear variation and reach a peak value. This peak value is as large as 25 step heights (7.7 tube dia) and occurs at a Reynolds number of about 250. This behavior signifies the onset of a different flow regime where shear-layer waves and their stability play the dominant role. However, small waves were observed in the shear layer already at Reynolds numbers of about 200. These waves propagated downstream with the flow and began to grow in amplitude as the inertia forces were increased relative to the viscous forces, i.e., as the Reynolds number was increased. Fig. 3(*b*) depicts the shear-layer wave growth at a Reynolds number of 290. Of note is the lateral extent of the wave which appears to contain the shear layer itself. This behavior resembles a vortex sheet in that the viscous layer is sheared in one direction by the faster moving core flow and in the opposite direction by the slower moving reverse flow near the wall. In this flow regime reattachment is no longer determined by the growth of the laminar shear layer due to molecular diffusion, but instead is determined when the lateral extent of the undulating vortex sheetlike region, which also spreads by diffusion, extends to the wall. Within this regime, increases in Reynolds number caused undulations of increasing amplitude in the shear layer and a more rapid thickening of the vortex sheet like region. The result is that the reattachment length decreases with increasing Reynolds number, Fig. 1.

Further increases in Reynolds number above 290, Fig. 3(*b*), caused the undulation of the vortex sheetlike region to become more pronounced in both amplitude and frequency. At a Reynolds number of about 390, Fig. 3(*c*), these undulations began to lose definition and to form eddies of more random behavior. The portion of the reattachment curve in the Reynolds number range of about 400–1000, Fig. 1, was characterized by flow in which smooth undulations in the shear layer were replaced by this more random fluctuating behavior.

At a Reynolds number of about 1000 the flow reattached at a relatively short distance of about six step heights. The shear

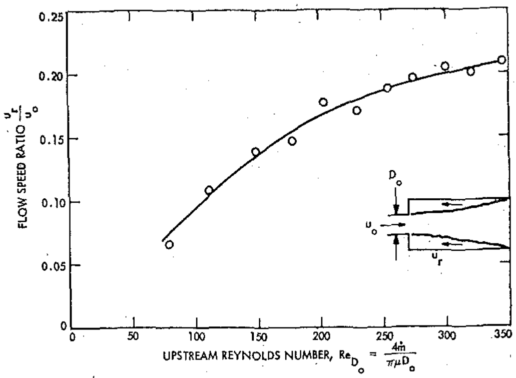


**Fig. 4 Reverse flow speeds**

layer spreads rapidly in the lateral direction in this case, Fig. 3(*d*), because of the enhanced lateral transport by more random, and relatively more pronounced, fluctuations in the flow. At this Reynolds number the spread of a dye filament introduced upstream appeared much more rapid than was the case at a Reynolds number of 390, Fig. 3(*c*).

At Reynolds number above about 1000 the reattachment point moves downstream again and then remains at a distance of about 9–10 step heights at the largest Reynolds numbers investigated. In this region there was difficulty in observing the flow visually because of the higher flow speeds and rapid diffusion of dye caused by random eddying motion of fluid which at the largest Reynolds numbers appeared to be "turbulent" in nature. Consequently, little information is available on those mechanisms in the flow that caused the reattachment location to move downstream again. For turbulent mixing of a uniform stream with a fluid at rest, the half width of the shear layer $b$ has been found experimentally [19] to be

$$b = cx \quad \text{with} \quad c = 0.27$$

This relation which is independent of Reynolds number or the factor $h/D_0$ indicated for laminar mixing, however, gives a reattachment length of only about four step heights.

In the lower Reynolds number regime where the flow speeds were smaller it was also possible to obtain information on the magnitude of the flow speeds $u_r$ in the reverse flow region near the wall. This was accomplished by observing and measuring the displacement of a small droplet of dye during an elapsed time. These results are shown in Fig. 4, where $u_r$ is given relative to the flow speed $u_0$ upstream of the abrupt channel expansion, i.e., $u_r/u_0$. The relative reverse flow speeds increase with Reynolds number and reach a value of about 0.2. This value is of the same magnitude as observed by Seban [12] for turbulent air flow over a backward-facing step at much higher Reynolds numbers. The reverse flow speeds shown were obtained in the central portion of the reverse flow region where they were reasonably uniform. Of course the reverse flow speed is zero at flow reattachment and at the step.

As in all experimental investigations there are certain questions that arise with regard to the quality of the data and the effect of various aspects of the particular configuration studied. The reattachment lengths shown in Figs. 1 and 2 were reproducible from day to day within the bands shown for the data points. In the laminar shear-layer regime ($Re_{D_0} < 250$) reattachment locations could be determined more accurately than at higher Reynolds numbers in the vortex-sheet instability regime and the more random fluctuation regime where the flow was basically unsteady. In particular, careful observation of the dye metered in through the holes along the tube was necessary in these latter regimes, and the extremities of the bands shown do indicate definite dye movement in either direction. Reattachment locations could not be inferred accurately by visual observation of dye filaments introduced into the flow upstream, Fig. 3. The convergent nozzle located a distance of 32 step heights along the tube is not believed to have influenced the largest reattachment location of about 25 step heights observed. However, the maximum reattachment lengths may have been influenced to some extent by the nature of the flow upstream of the channel expansion and disturbances present in the flow at that location. Reattachment appeared to be symmetric circumferentially for the results presented. It should also be mentioned that continuous visual observations are clearly more instructive than the few instantaneous flow pictures that are shown herein. These continuous observations formed the basis for the previous discussion. It is realized that other flow visualization techniques employing additives such as metallic powders or birefringent fluids, or the hydrogen bubble technique, etc., [20] are better suited to reveal a truer picture of the overall flow field, but dye injection at discrete locations does have the advantage of isolating particular regions in the flow that are of interest.

## Concluding Remarks

In an experimental investigation reattachment lengths were determined over a large range of Reynolds numbers in an abrupt circular-channel expansion. There were basially three flow regimes observed. At lower Reynolds numbers reattachment was essentially determined by the growth of the laminar shear layer between the central jet and reverse flow along the wall. In this regime the reattachment location moved downstream with increasing Reynolds number. At intermediate Reynolds numbers an instability developed in the vortex sheetlike shear layer and reattachment was determined when the lateral extent of this undulating motion extended to the wall. The reattachment location moved back toward the step with increasing Reynolds number in this regime. At higher Reynolds numbers the shear layer spread rapidly because of enhanced lateral transport by more random fluctuations and reattachment occurred a relatively short distance downstream of the step.

The results presented indicate the large variation of the reattachment location with Reynolds number that can occur in an abrupt channel expansion. In particular the separation region extended over a region up to 25 step heights long (7.7 tube diameters) at an upstream Reynolds number of about 250. Caution is directed toward the use of any techniques other than unsteady methods in the numerical calculation of such flows in which instabilities occur.

The present results are useful in applications such as fluid flow in the human body, and in modeling of high temperature, relatively low Reynolds number, gas flows. Local restrictions to blood flow in the human vascular system due to growths at the wall of a vessel occur in various cardiovascular diseases. The modified hemodynamics in flow reattachment regions is an important aspect of diseases such as arteriosclerosis and atherosclerosis [21, 22]. In high-temperature gas flows knowledge of the flow reattachment location is important because the peak heat flux is found in the reattachment region, and this in turn, establishes the kind of heat transfer distribution downstream of reattachment in the redevelopment region and upstream in the separated flow region [15].

## References


1 Macagno, E. O., and Hung, T. K., "Computational and Experimental Study of a Captive Annular Eddy," *Journal of Fluid Mechanics*, Vol. 28, Part 1, 1967, pp. 43–64.

2 Goldstein, R. J., et al., "Laminar Separation, Reattachment, and Transition of the Flow Over a Downstream-Facing Step," *Journal of Basic Engineering*, Trans. ASME, Series D, Vol. 92, No. 4, Dec. 1970, pp. 732–741.

3 Mueller, T. J., and O'Leary, R. A., "Physical and Numerical Experiments in Laminar Incompressible Separating and Reattaching Flows," AIAA Paper No. 70-763, July 1970.

4 Ede, A. J., Hislop, E. I., and Morris, R., "Effect on the Local Heat Transfer Coefficient in a Pipe of an Abrupt Disturbance of the Fluid Flow: Abrupt Convergence and Divergence of Diameter Ratio 2/1," *Proceedings of the Institution of Mechanical Engineers*, London, England, Vol. 170, No. 38, 1956, pp. 1113–1126.

5 Ede, A. J., "Effect of an Abrupt Disturbance of the Flow on the Local Heat-Transfer Coefficient in a Pipe: Progress Report, January 1959," NEL Report Heat 164, East Kilbride, Glasgow, Scotland, National Engineering Laboratory, 1959.

6 Ede, A. J., Morris, R., and Birch, Miss E. S., "The Effect of Abrupt Changes of Diameter on Heat Transfer in Pipes," NEL Report No. 73, East Kilbride, Glasgow, Scotland, National Engineering Laboratory, 1962.

7 Emerson, W. H., "Heat Transfer in a Duct in Regions of Separated Flow," *Proceedings, 3rd International Heat Transfer Conference* AICHE, Vol. 1, Aug. 1966, pp. 267–275.

8 Krall, K. M., and Sparrow, E. M., "Turbulent Heat Transfer in the Separated, Reattached, and Redevelopment Regions of a Circular Tube," *Journal of Heat Transfer*, TRANS. ASME, Series C, Vol. 88, No. 1, Feb. 1966, pp. 131–136.

9 Zemanick, P. P., and Dougall, R. S., "Local Heat Transfer Downstream of Abrupt Circular Channel Expansion," *Journal of Heat Transfer*, TRANS. ASME, Series C, Vol. 92, No. 1, Feb. 1970, pp. 53–60.

10 Filetti, E. G., and Kays, W. M., "Heat Transfer in Separated, Reattached, and Redevelopment Regions Behind a Double Step at Entrance to a Flat Duct," *Journal of Heat Transfer*, TRANS. ASME, Series C, Vol. 89, No. 2, May 1967, pp. 163–168.

11 Seban, R. A., Emery, A., and Levy, A., "Heat Transfer to Separated and Reattached Subsonic Turbulent Flows Obtained Downstream of a Surface Step," *Journal of the Aerospace Sciences*, Vol. 26, No. 12, Dec. 1959, pp. 809–814.

12 Seban, R. A., "Heat Transfer to the Turbulent Separated Flow of Air Downstream of a Step in the Surface of a Plate," *Journal of Heat Transfer*, TRANS. ASME, Series C, Vol. 86, No. 2, May 1964, pp. 259–264.

13 Meisner, J. E., and Rushmer, R. F., "Eddy Formation and Turbulence in Flowing Liquids," *Circulation Research*, Vol. 12, May 1963, pp. 455–463.

14 Moore, T. W. F., "Some Experiments on the Reattachment of a Laminar Boundary Layer Separating From a Rearward Facing Step on a Flat Plate Aerofoil," *Journal of the Royal Aeronautical Society*, Vol. 64, Nov. 1960, pp. 668–672.

15 Back, L. H., Massier, P. F., and Roschke, E. J., "Partially Ionized Gas Flow and Heat Transfer in the Separation, Reattachment, and Redevelopment Regions Downstream of an Abrupt Circular Channel Expansion," *Journal of Heat Transfer*, TRANS. ASME, Series C, Vol. 94, No. 1, Feb. 1972, pp. 119–127.

16 Schlichting, H., *Boundary-Layer Theory*, 6th ed., McGraw-Hill, New York, 1968, p. 175.

17 Kawaguti, M., "Numerical Solutions of the Navier-Stokes Equations for the Flow in a Channel With a Step," Report No. MCR-TSR-574, Mathematics Research Center, University of Wisconsin, Madison, May 1965 (AD-620108).

18 Chapman, D. R., Kuehn, D. M., and Larson, H. K., "Investigation of Separated Flows in Supersonic and Subsonic Streams With Emphasis on the Effect of Transition," NACA R-1356, 1958.

19 Abramovich, G. N., *The Theory of Turbulent Jets*, Massachussets Institute of Technology Press, Cambridge, Mass., 1963.

20 Clayton, B. R., and Massey, B. S., "Flow Visualization in Water: A Review of Techniques," *Journal of Scientific Instruments*, Vol. 44, 1967, pp. 2–11.

21 Young, D. F., and Shih, C. C., "Some Experiments on the Effect of Protuberances on Flow Through Tubes," *Experimental Mechanics*, Vol. 9, 1969, pp. 225–229.

22 Forrester, J. H., and Young, D. F., "Flow Through a Converging-Diverging Tube and Its Implications in Occlusive Vascular Disease—I. Theoretical Development, —II. Theoretical and Experimental Results and Their Implications," *Journal of Biomechanics*, Vol. 3, 1970, pp. 297–316.